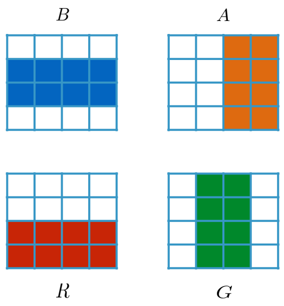


Informatics 1

Computation and Logic

Karnaugh Maps

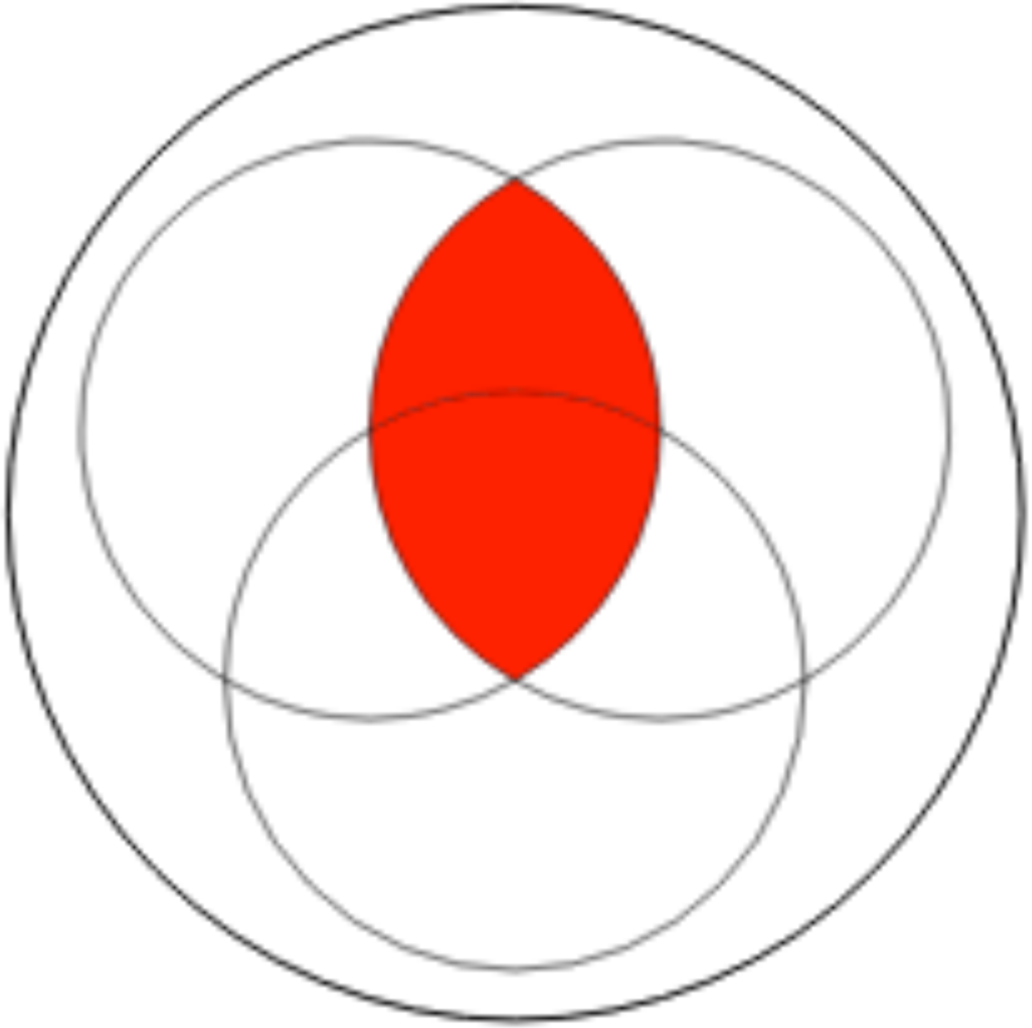
Michael Fourman

InfPals

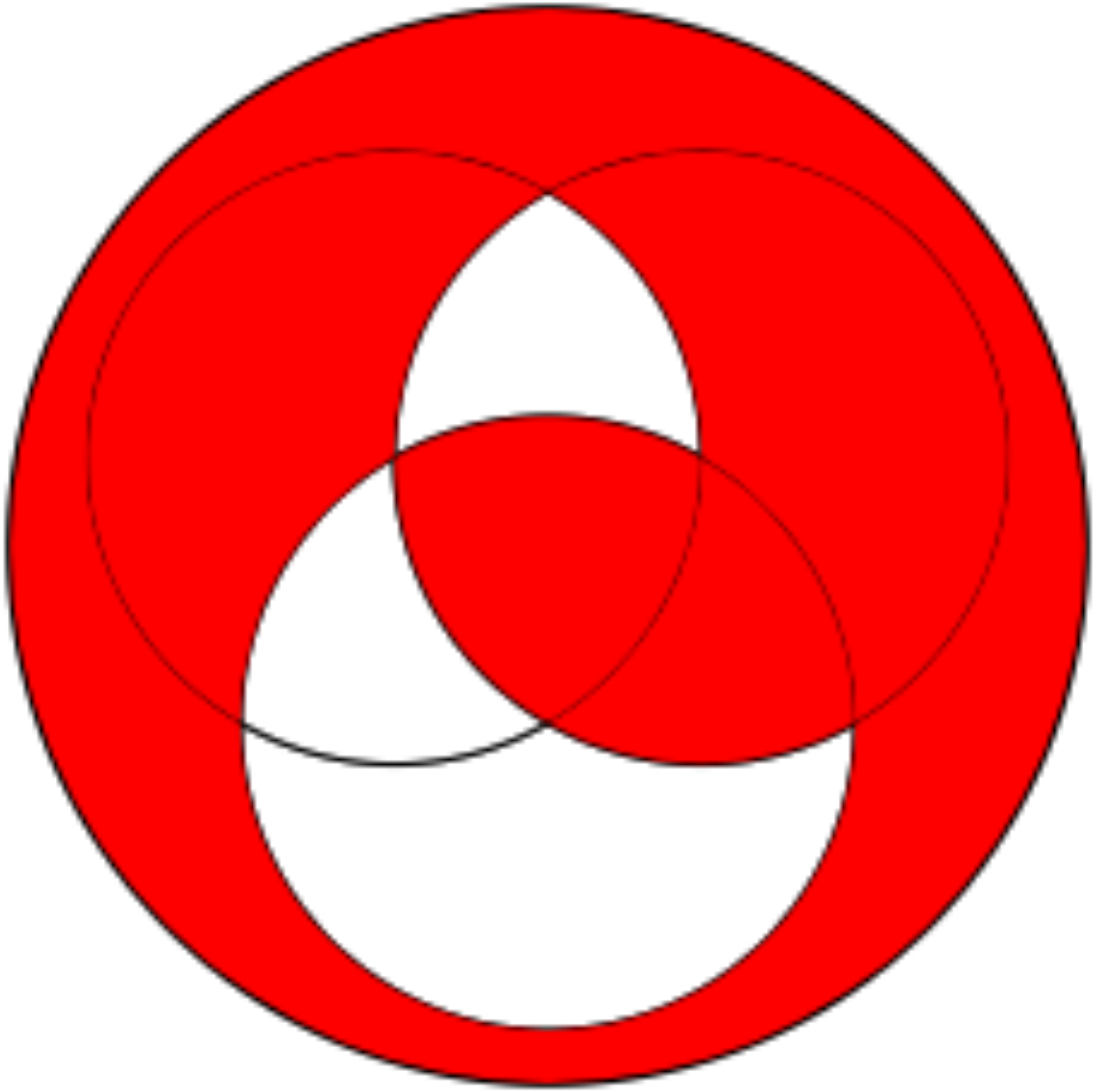
web.inf.ed.ac.uk/infweb/student-services/ito/students/year1/student-support/infpals

Action: Make a commitment and sign up ASAP and before 5pm on Wednesday the 4th of October, using your First Name and your Student Number at tinyurl.com/infpals2017

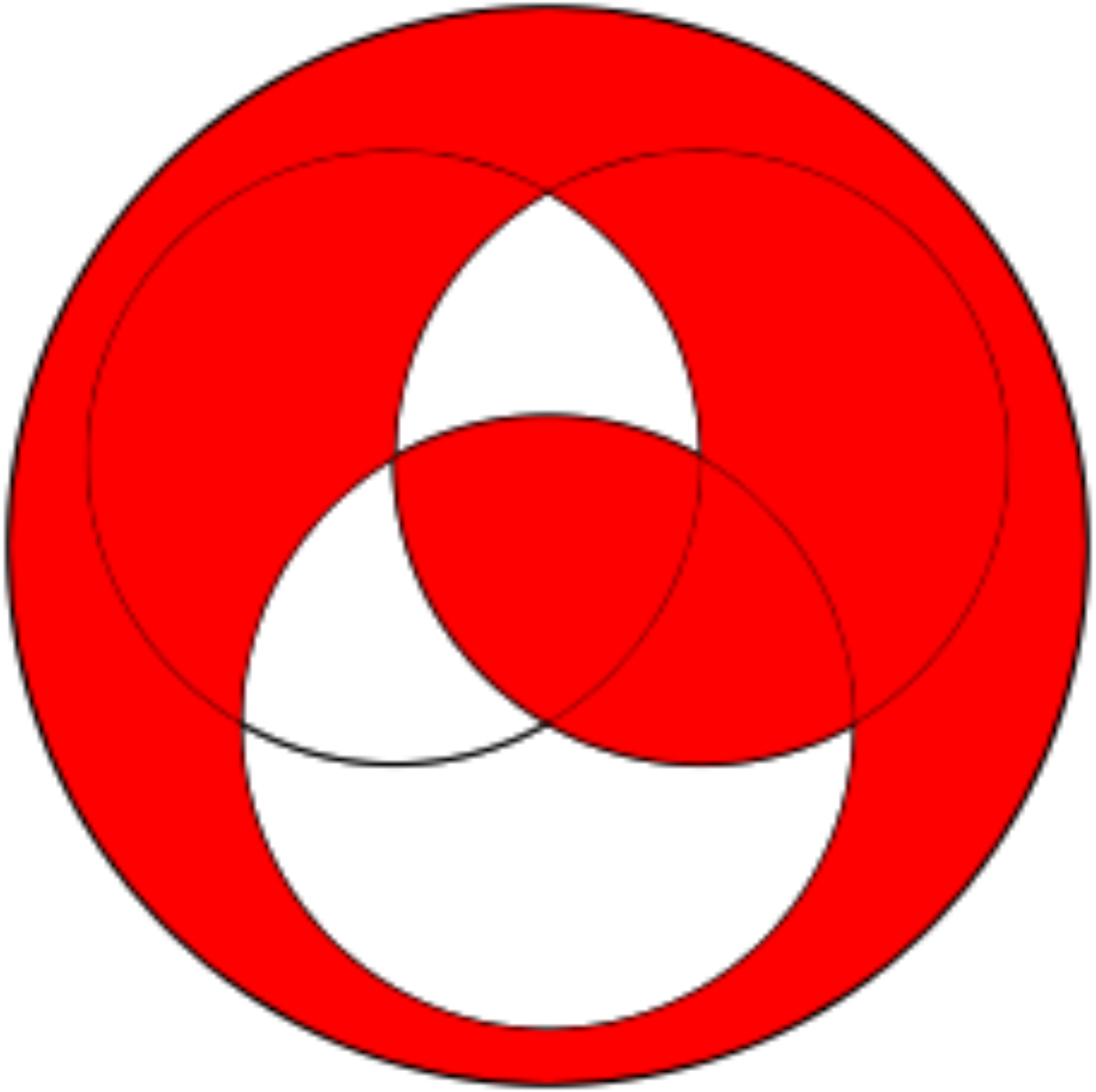
R && A



R	A	G	**R ∧ A**
0	0	0	**0**
0	0	1	**0**
0	1	0	**0**
0	1	1	**0**
1	0	0	**0**
1	0	1	**0**
1	1	0	**1**
1	1	1	**1**



R	A	G	??
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



R	A	G	??
0	0	0	**1**
0	0	1	**0**
0	1	0	**1**
0	1	1	**1**
1	0	0	**1**
1	0	1	**0**
1	1	0	**0**
1	1	1	**1**

$$(\neg R \wedge \neg A \wedge \neg G)$$

$$\vee$$

$$(\neg R \wedge A \wedge \neg G)$$

$$\vee$$

$$(\neg R \wedge A \wedge G)$$

$$\vee$$

$$(R \wedge \neg A \wedge \neg G)$$

$$\vee$$

$$(R \wedge A \wedge G)$$

R	A	G	**??**
0	0	0	**1**
0	0	1	**0**
0	1	0	**1**
0	1	1	**1**
1	0	0	**1**
1	0	1	**0**
1	1	0	**0**
1	1	1	**1**

disjunctive normal form DNF

engineers write + for ∨, × for ∧, A′ for ¬A

and call this a **sum of products SOP**

R′A′G′ + R′AG′ + R′AG + RA′G′ + RAG

we can also look at the states that do not satisfy the property

$$\neg\left(\begin{array}{l}(\neg R \wedge \neg A \wedge G) \\ \vee \\ (R \wedge \neg A \wedge G) \\ \vee \\ (R \wedge A \wedge \neg G)\end{array}\right)$$

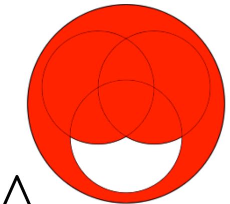
R	A	G	**??**
0	0	0	**1**
0	0	1	**0**
0	1	0	**1**
0	1	1	**1**
1	0	0	**1**
1	0	1	**0**
1	1	0	**0**
1	1	1	**1**

Then we apply de Morgan…

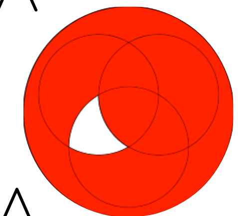
$$\neg\left(\begin{array}{l}(\neg R\wedge\neg A\wedge G)\\ \vee\\ (R\wedge\neg A\wedge G)\\ \vee\\ (R\wedge A\wedge\neg G)\end{array}\right)$$

R	A	G	??
0	0	0	**1**
0	0	1	**0**
0	1	0	**1**
0	1	1	**1**
1	0	0	**1**
1	0	1	**0**
1	1	0	**0**
1	1	1	**1**

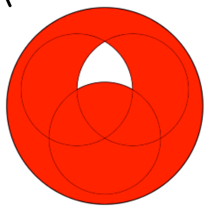
$$=\begin{array}{c}\neg(\neg R\wedge\neg A\wedge G)\\ \wedge\\ \neg(R\wedge\neg A\wedge G)\\ \wedge\\ \neg(R\wedge A\wedge\neg G)\end{array}=\begin{array}{c}(R\vee A\vee\neg G)\\ \wedge\\ (\neg R\vee A\vee\neg G)\\ \wedge\\ (\neg R\vee\neg A\vee G)\end{array}=$$



$\wedge$



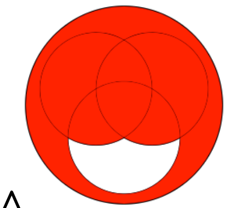
$\wedge$



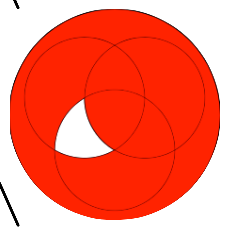
$$\neg\left(\begin{array}{l}(\neg R \wedge \neg A \wedge G) \\ \vee \\ (R \wedge \neg A \wedge G) \\ \vee \\ (R \wedge A \wedge \neg G)\end{array}\right)$$

R	A	G	**??**
0	0	0	**1**
0	0	1	**0**
0	1	0	**1**
0	1	1	**1**
1	0	0	**1**
1	0	1	**0**
1	1	0	**0**
1	1	1	**1**

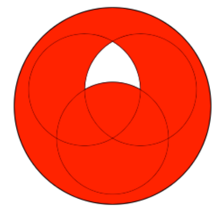
$$= \begin{array}{l}(R \vee A \vee \neg G) \\ \wedge \\ (\neg R \vee A \vee \neg G) \\ \wedge \\ (\neg R \vee \neg A \vee G)\end{array} =$$



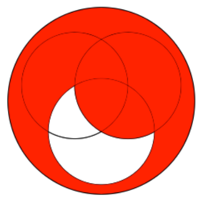
$\wedge$



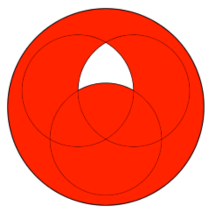
$\wedge$



$=$

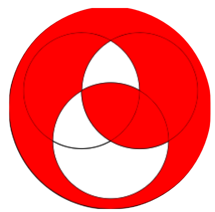


$\wedge$

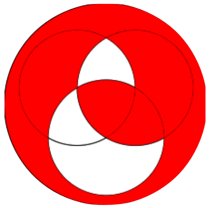


$$= \begin{array}{l}(A \vee \neg G) \\ \wedge \\ (\neg R \vee \neg A \vee G)\end{array}$$

$=$



conjunctive normal form CNF

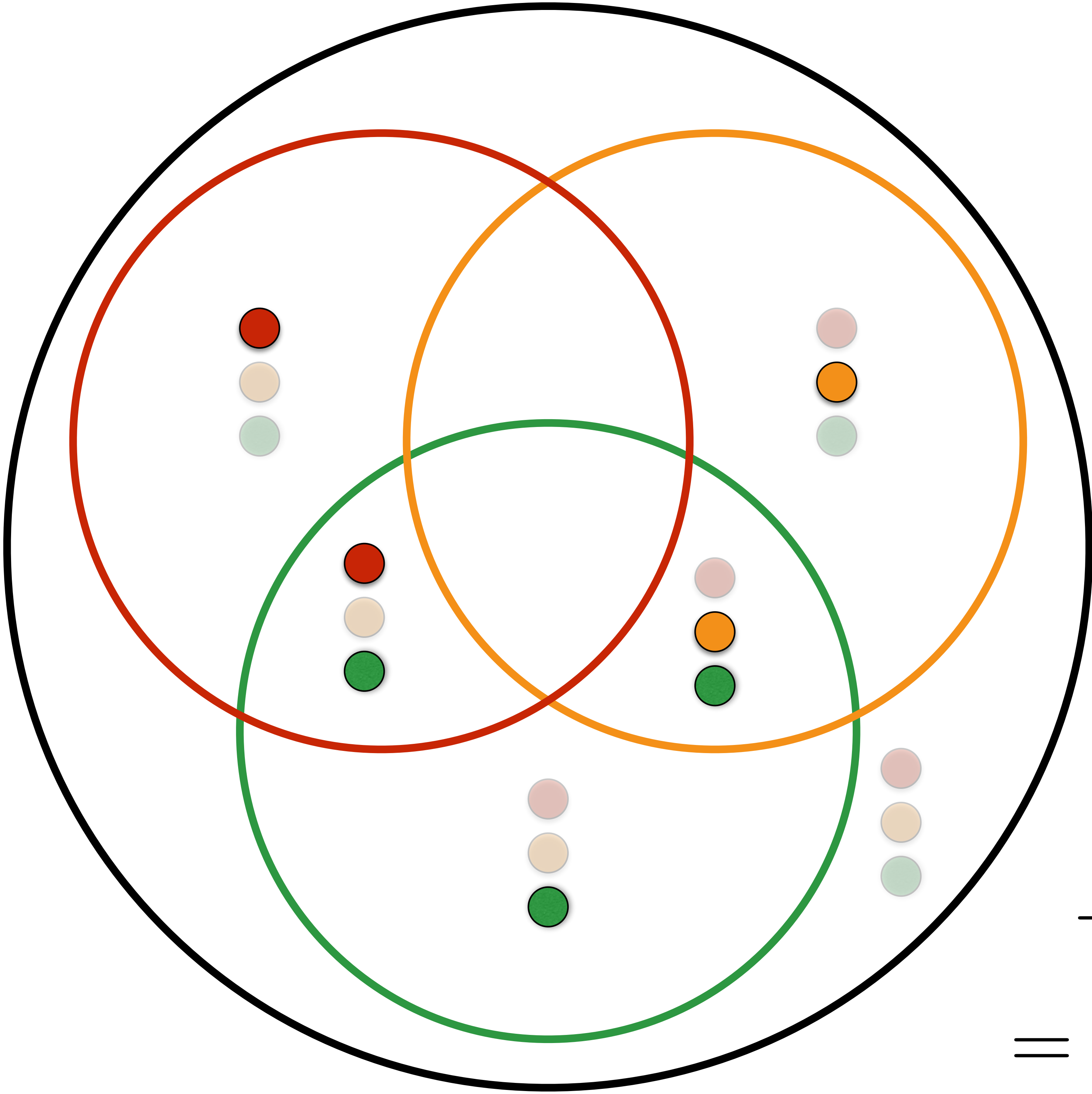


$$= \quad (A \vee \neg G) \wedge (\neg R \vee \neg A \vee G)$$

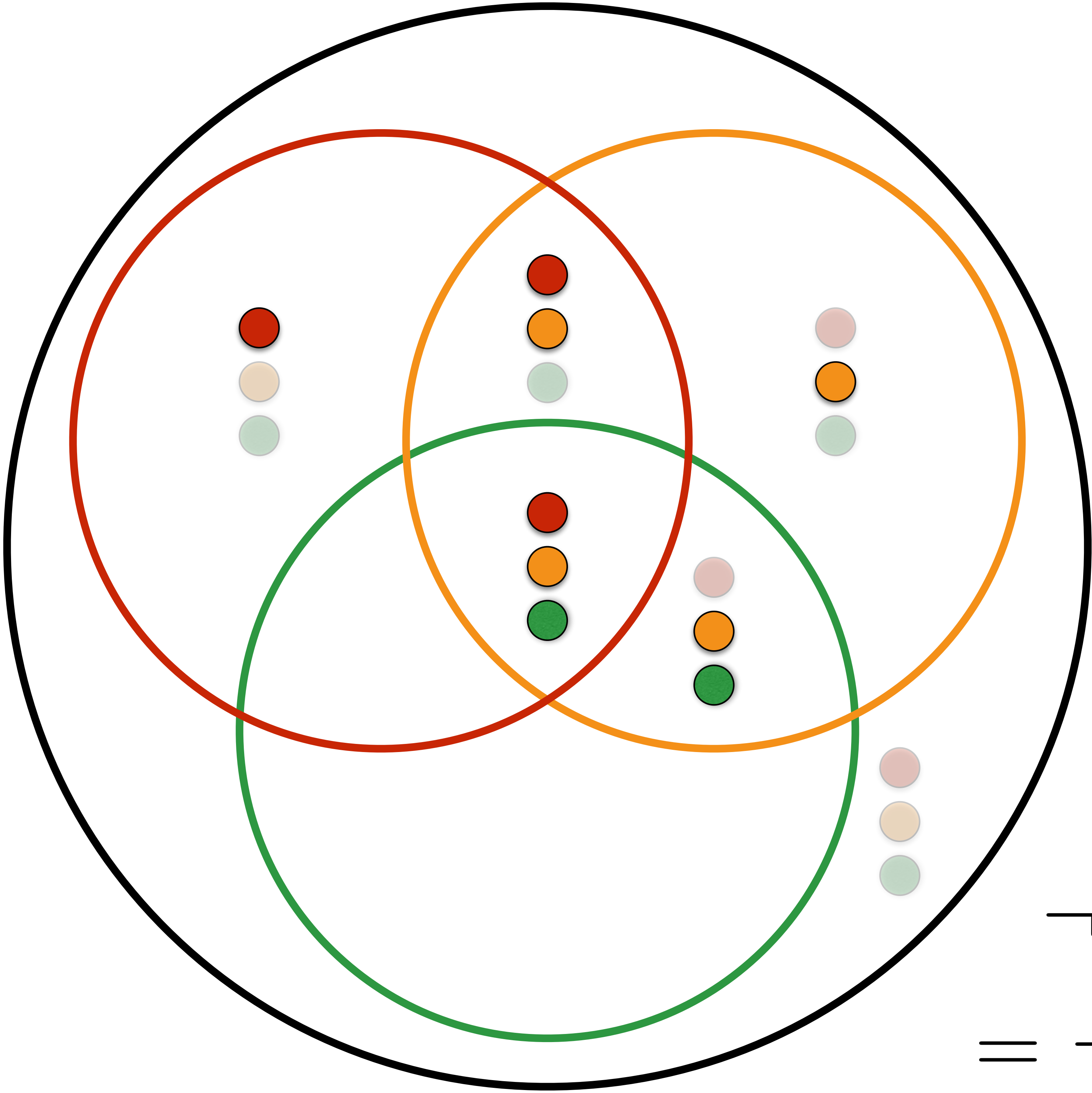
engineers write this as

(A + G′).(R′ + A′ + G)

a product of sums POS



$$\neg(R \wedge A)$$
$$= \neg R \vee \neg A$$



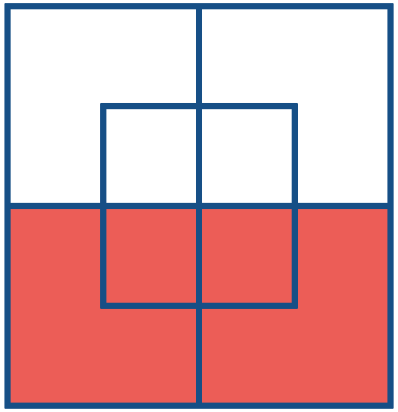
$$\neg(G \wedge \neg A)$$
$$= \neg G \vee A$$

Boolean Algebra

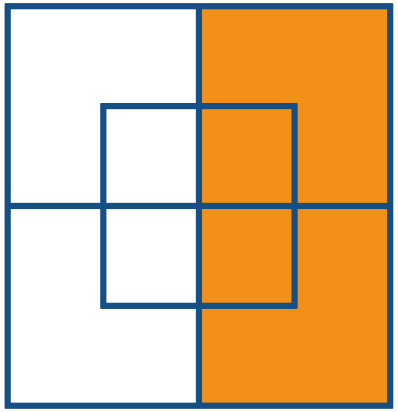
$x \vee (y \vee z) = (x \vee y) \vee z$	$x \wedge (y \wedge z) = (x \wedge y) \wedge z$	associative
$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$	$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$	distributive
$x \vee y = y \vee x$	$x \wedge y = y \wedge x$	commutative
$x \vee 0 = x$	$x \wedge 1 = x$	identity
$x \vee 1 = 1$	$x \wedge 0 = 0$	annihilation
$x \vee x = x$	$x \wedge x = x$	idempotent
$x \vee \neg x = 1$	$\neg x \wedge x = 0$	complements
$x \vee (x \wedge y) = x$	$x \wedge (x \vee y) = x$	absorbtion
$\neg(x \vee y) = \neg x \wedge \neg y$	$\neg(x \wedge y) = \neg x \vee \neg y$	de Morgan
$\neg\neg x = x$	$x \rightarrow y = \neg x \leftarrow \neg y$	

Lewis Carroll (The Rev. C.L. Dodgson)

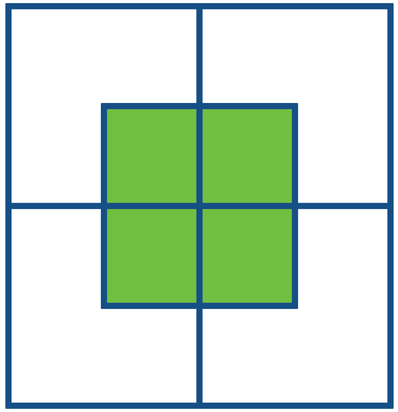
R



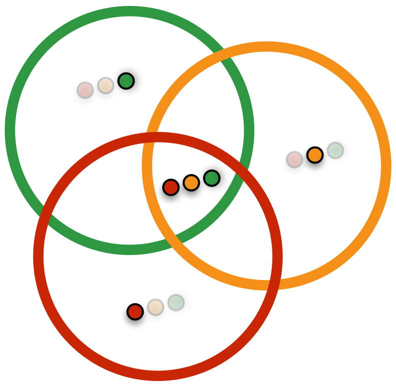
A

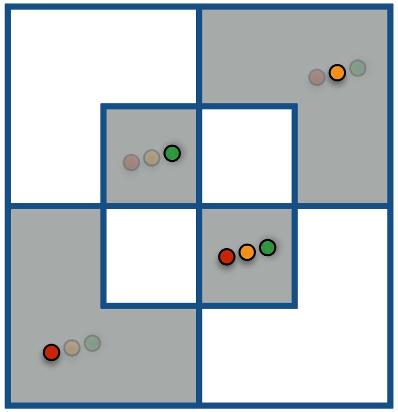


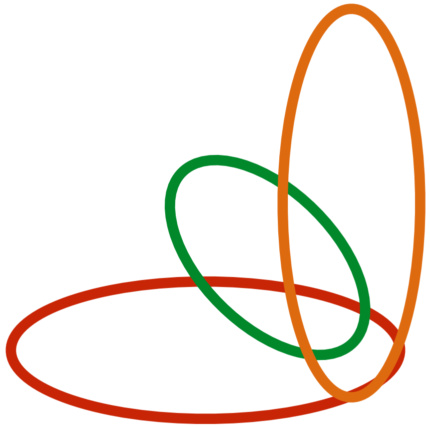
G

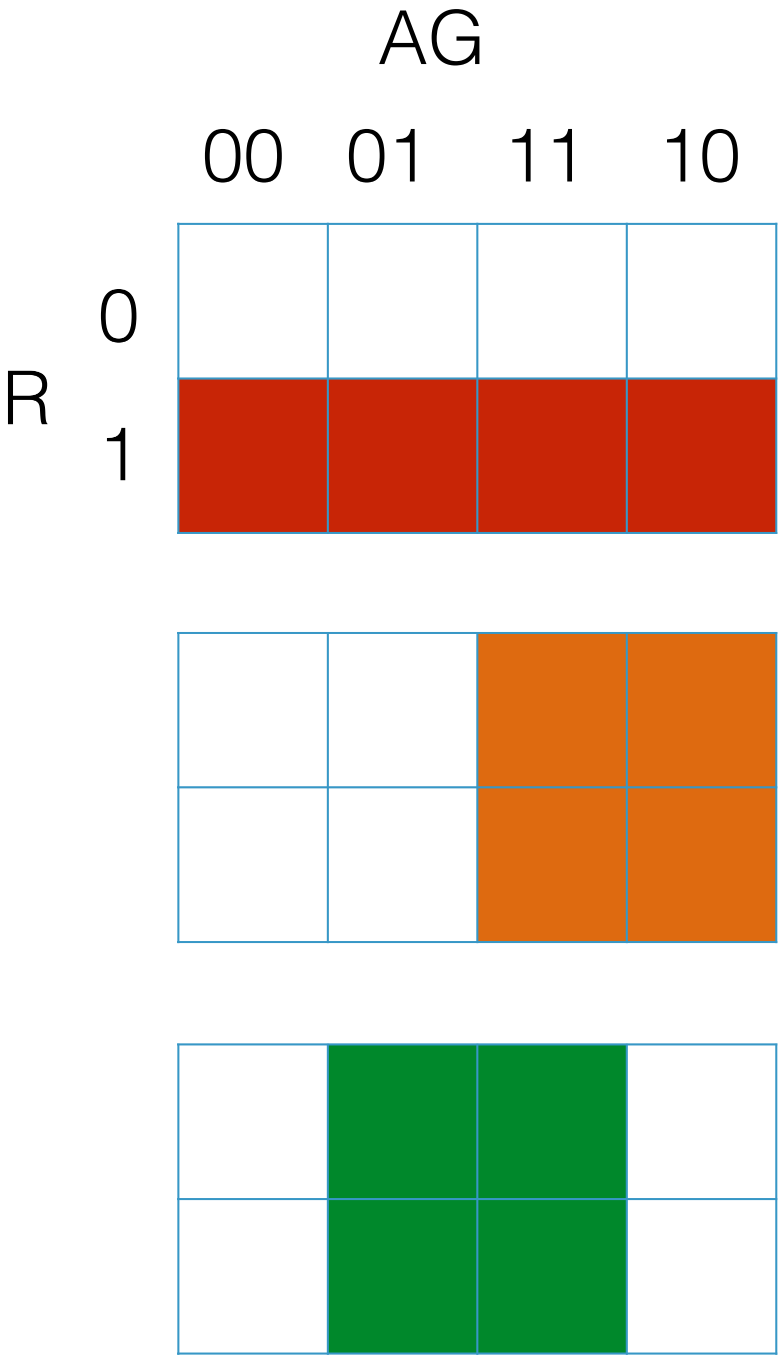


$$\{x \mid G(x) \leftrightarrow R(x) \leftrightarrow A(x)\}$$

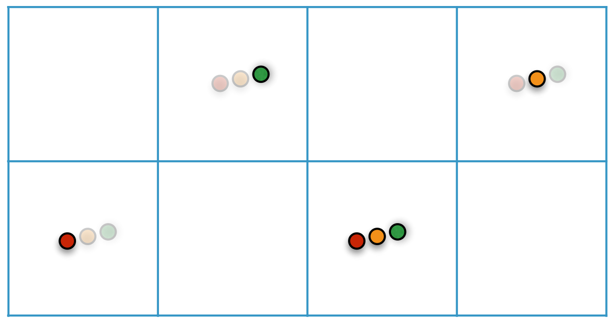


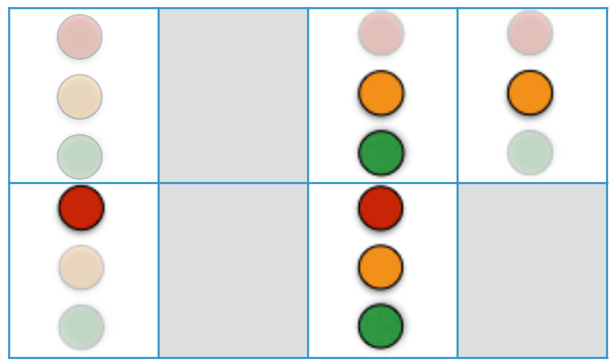


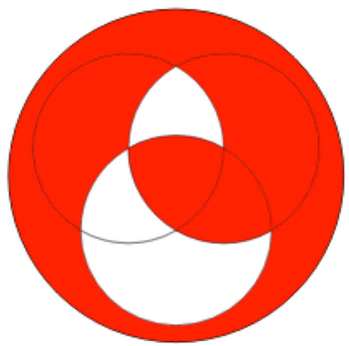


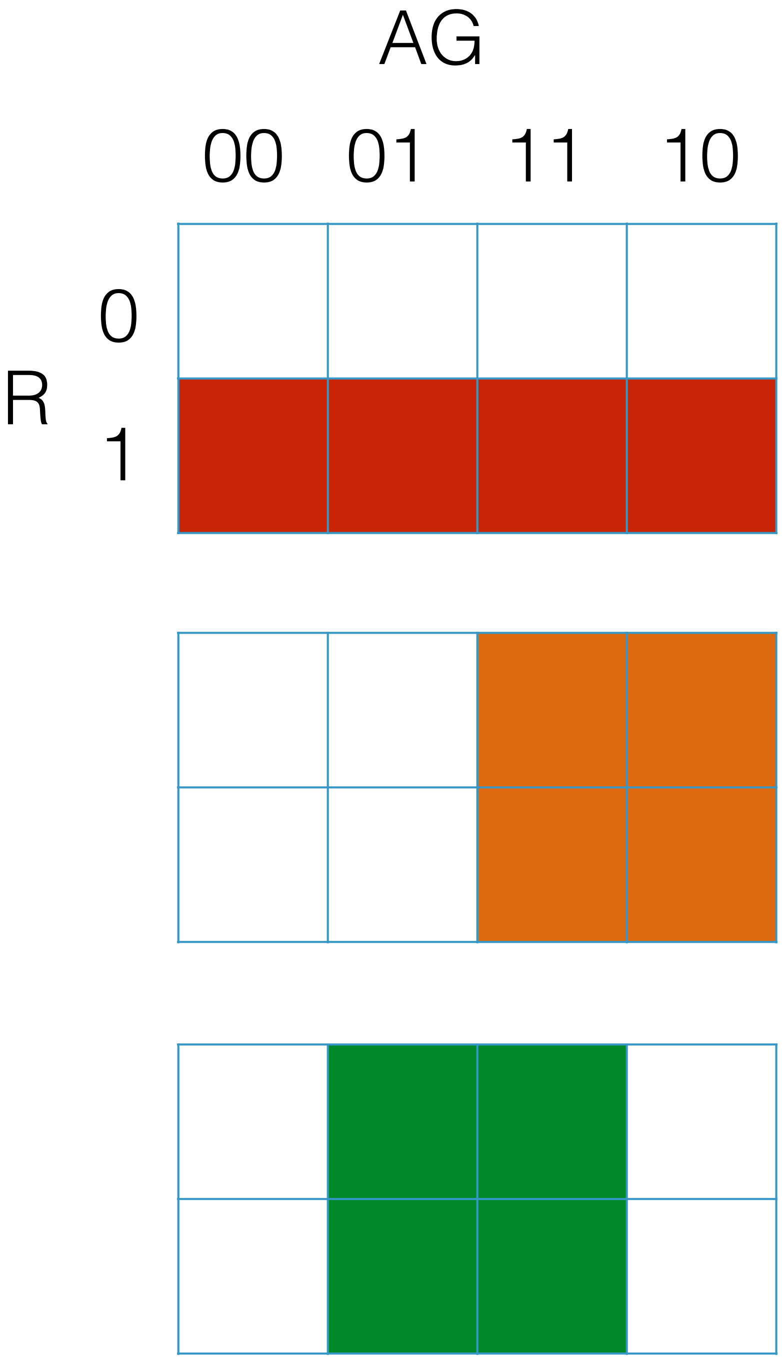


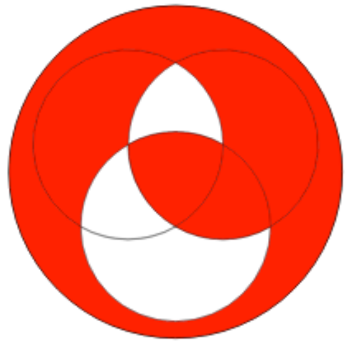
$$\{x \mid G(x) \leftrightarrow R(x) \leftrightarrow A(x)\}$$

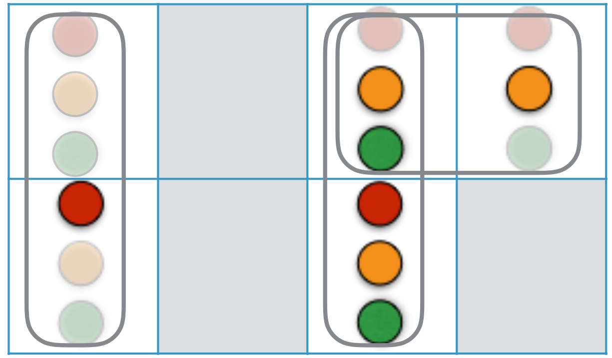




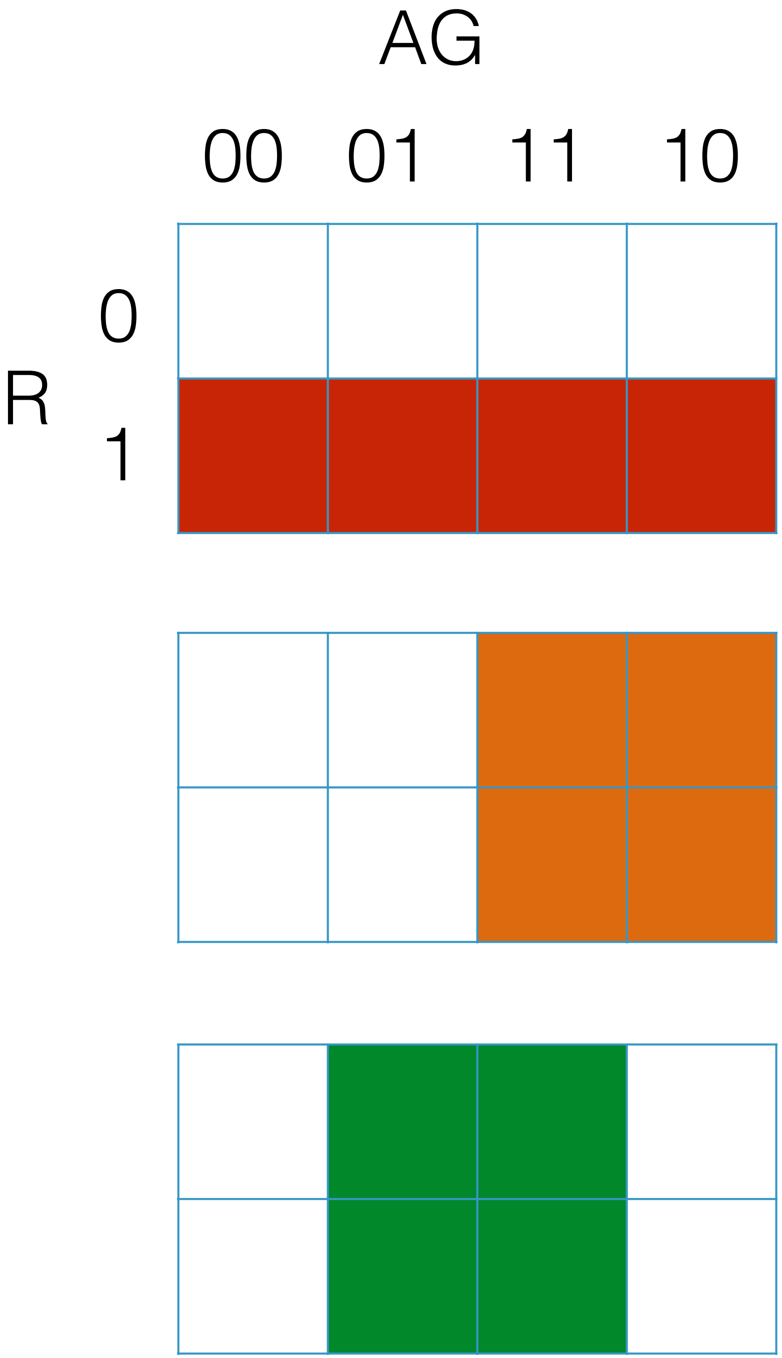


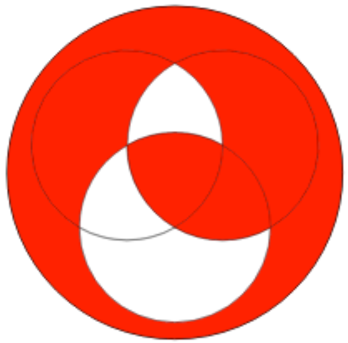


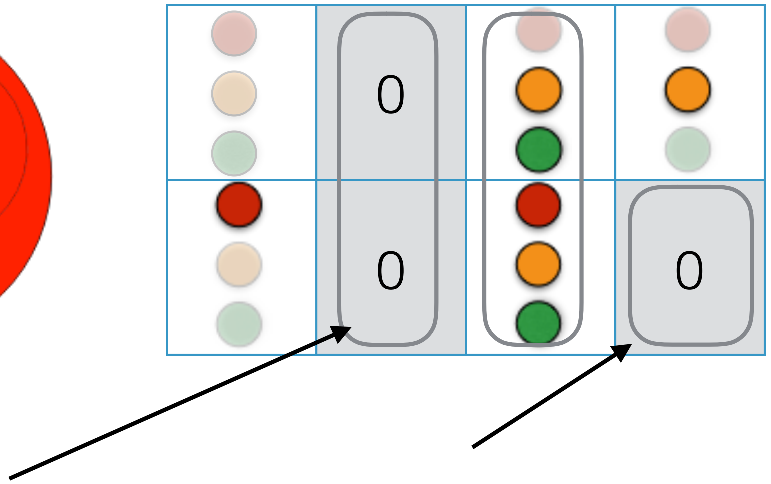




$$(\neg A \wedge \neg G) \vee (A \wedge G) \vee (A \wedge \neg R)$$

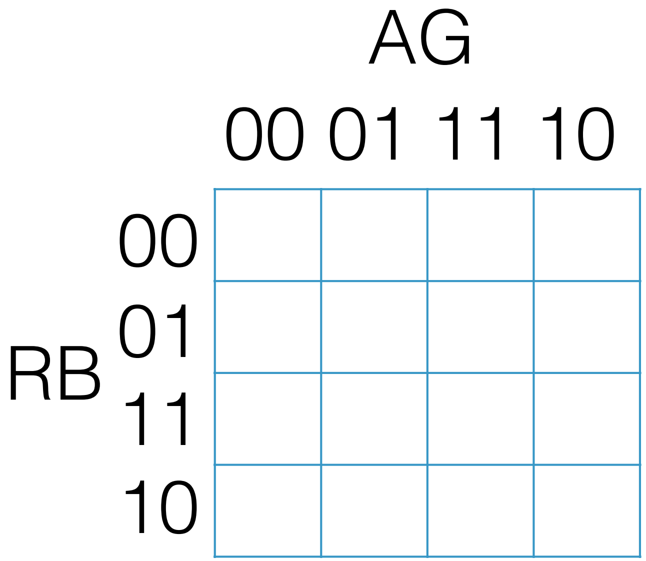


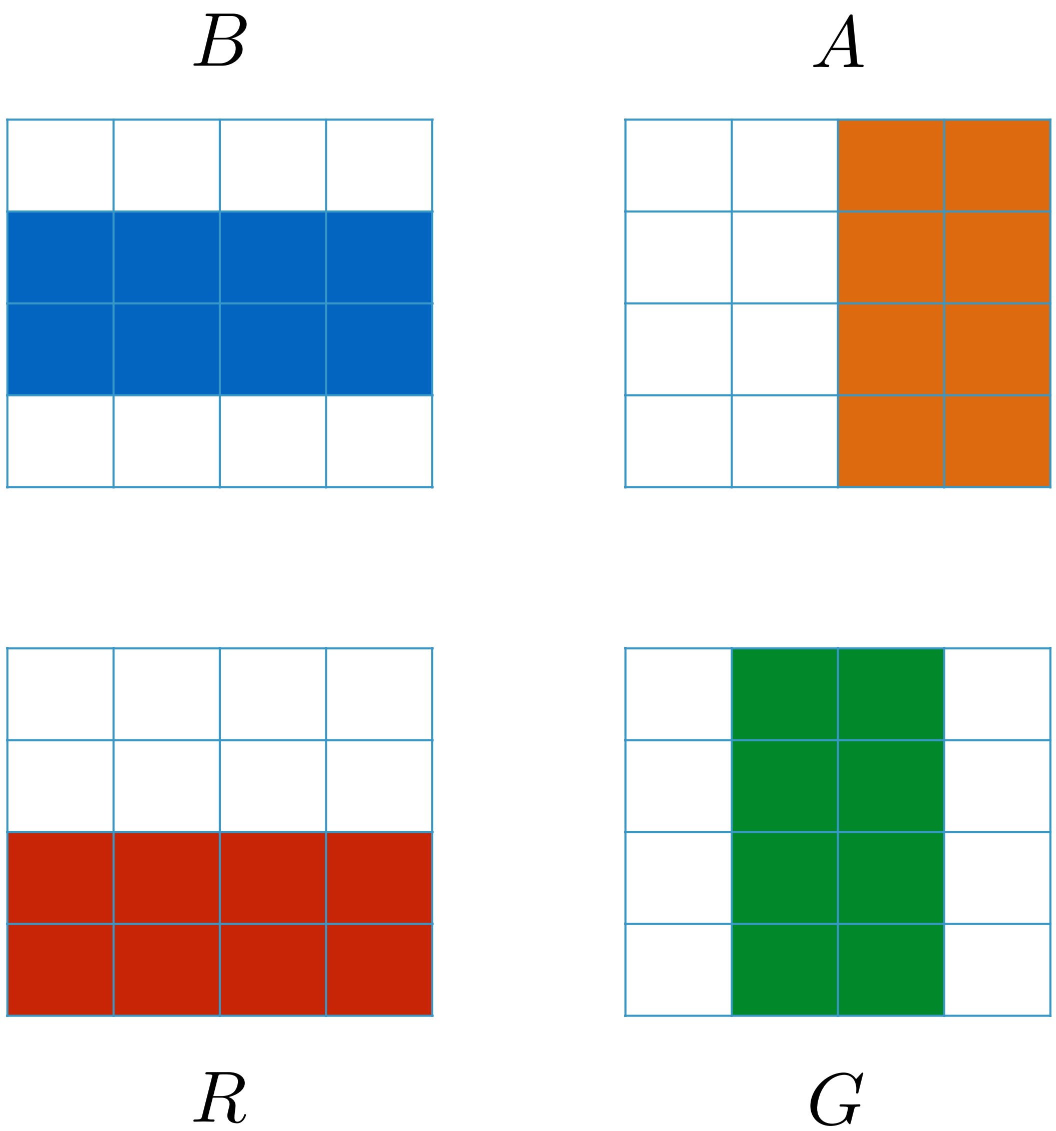




$$(\neg G \vee A) \wedge (\neg R \vee G \vee \neg A)$$

Karnaugh Maps



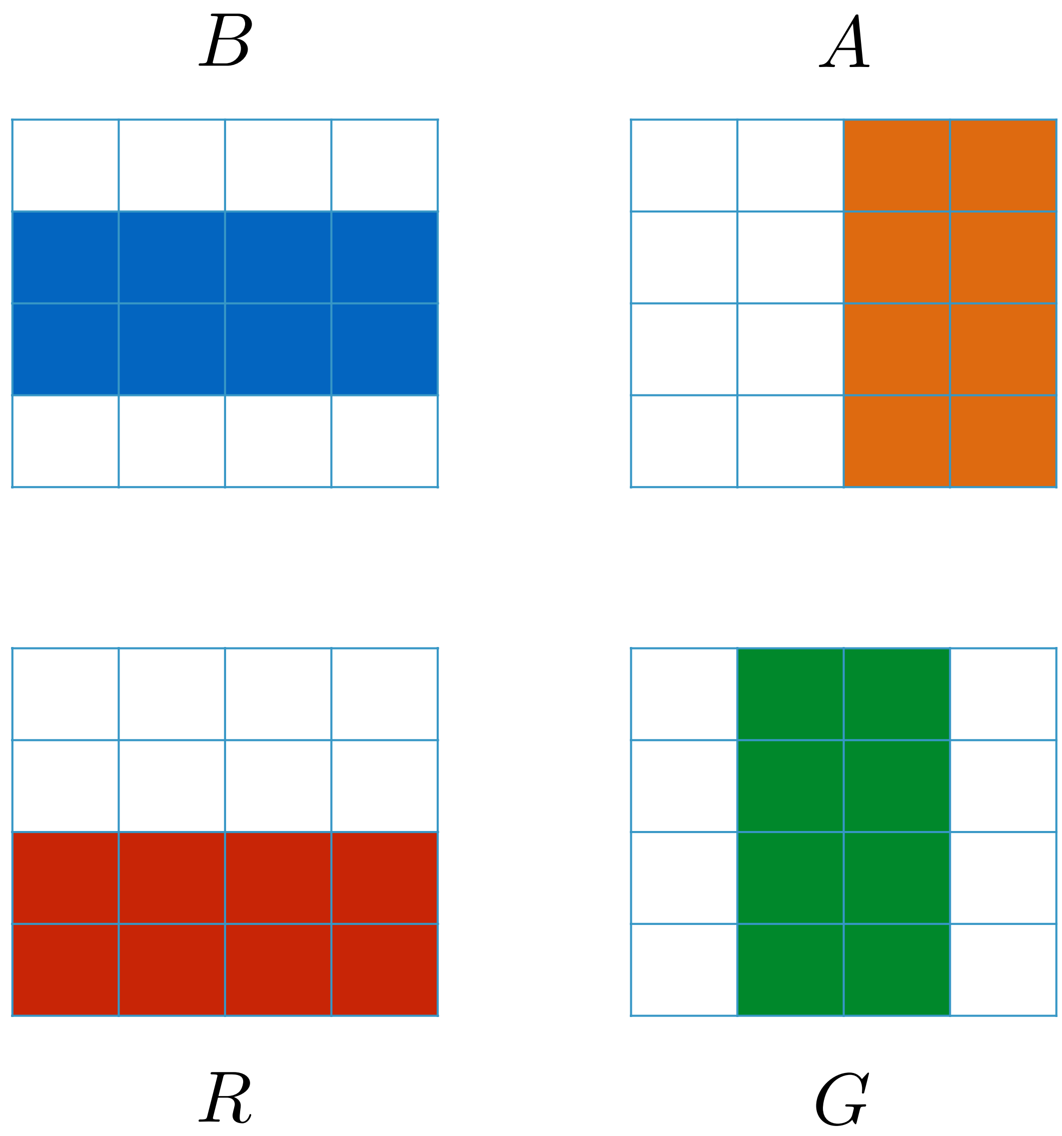


4 atoms:16 states: 64K subsets

Karnaugh Maps

RB \ AG	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	0	0	0

$$G \wedge B$$

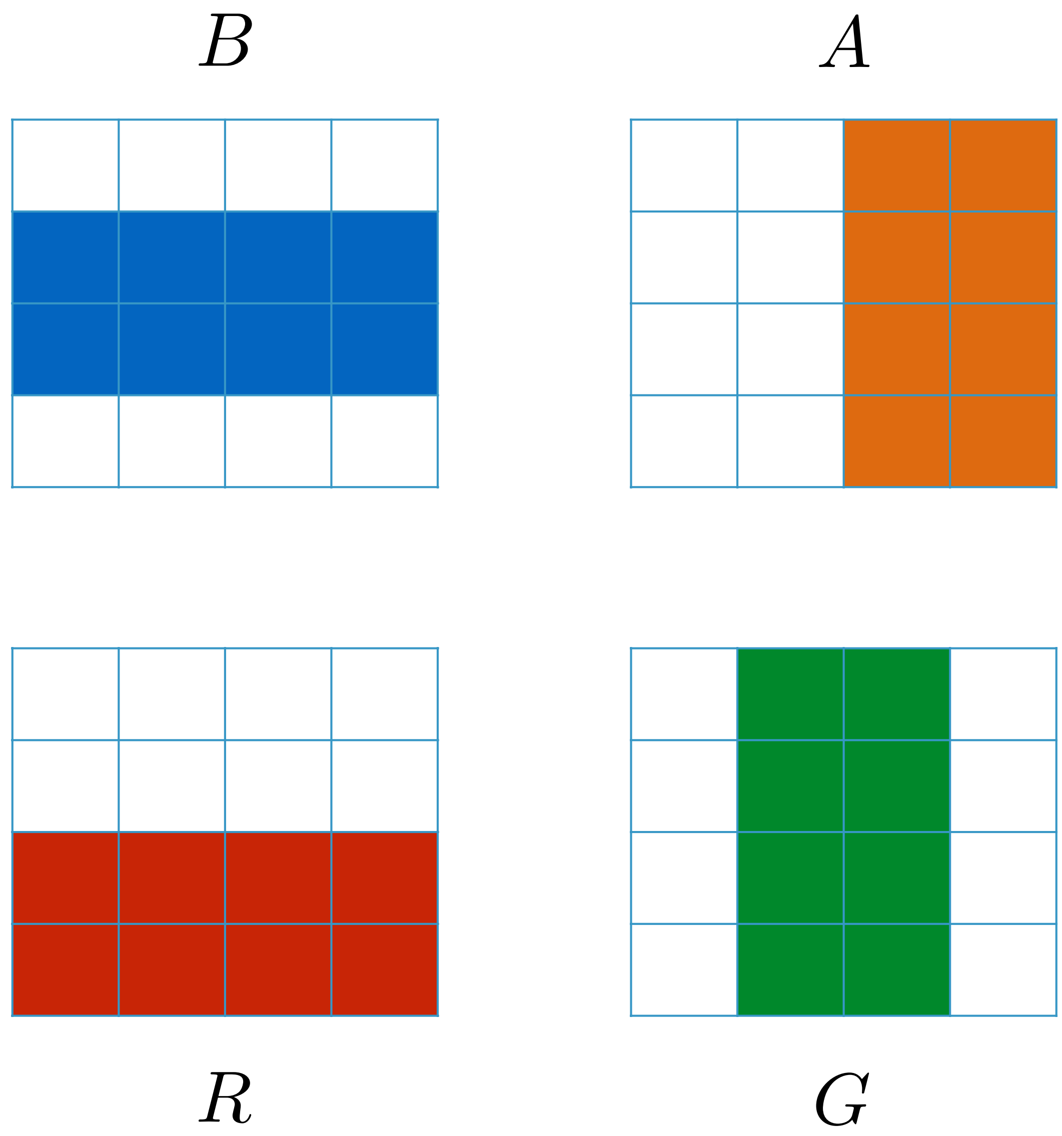


4 atoms:16 states: 64K subsets

Karnaugh Maps

		AG			
		00	01	11	10
RB	00	0	0	0	0
	01	1	1	0	0
	11	1	1	0	0
	10	0	0	0	0

$$\neg A \wedge B$$

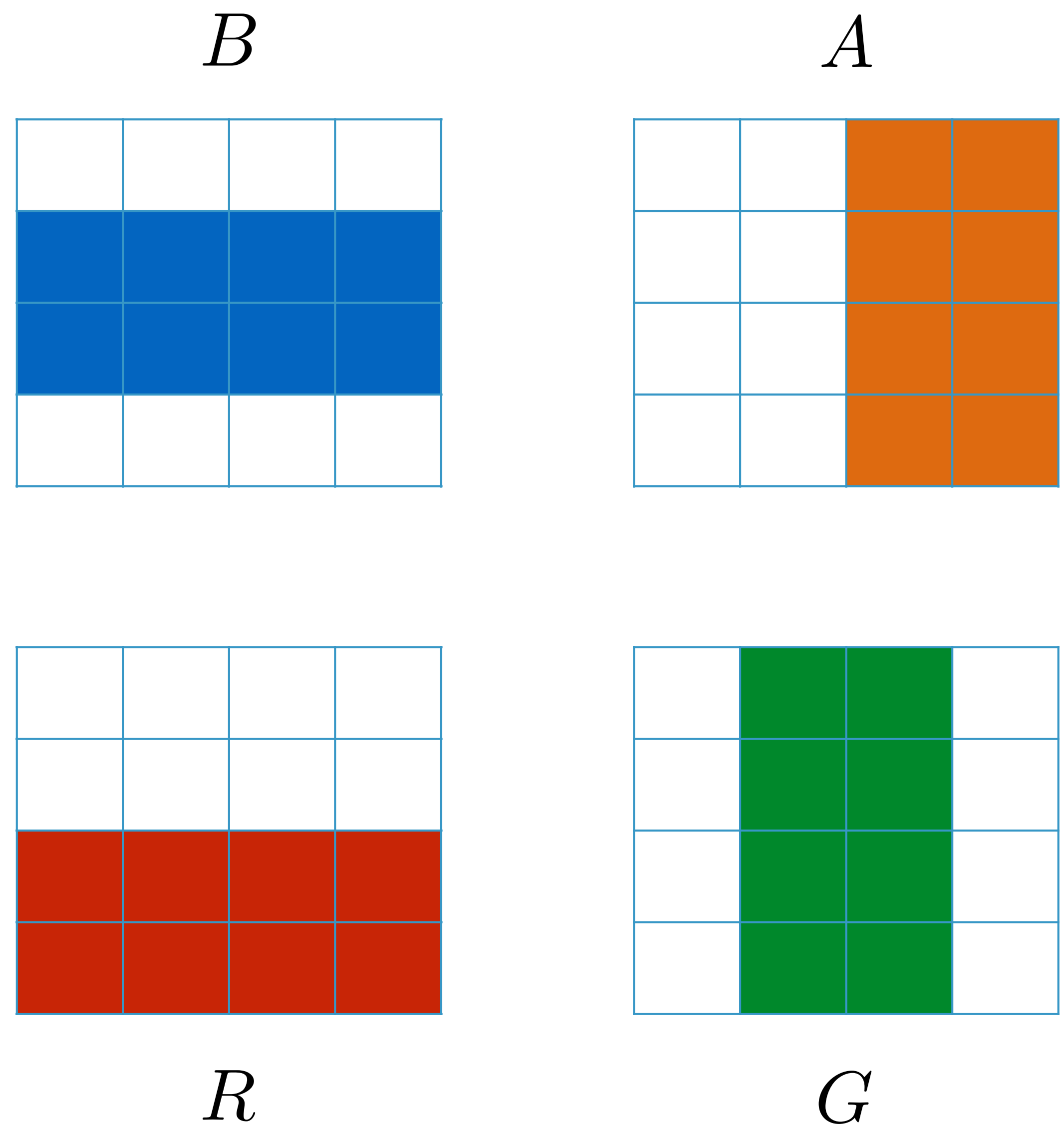


4 atoms:16 states: 64K subsets

Karnaugh Maps

RB \ AG	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	0	0	0	0
10	0	0	0	0

$$\neg A \wedge \neg R$$

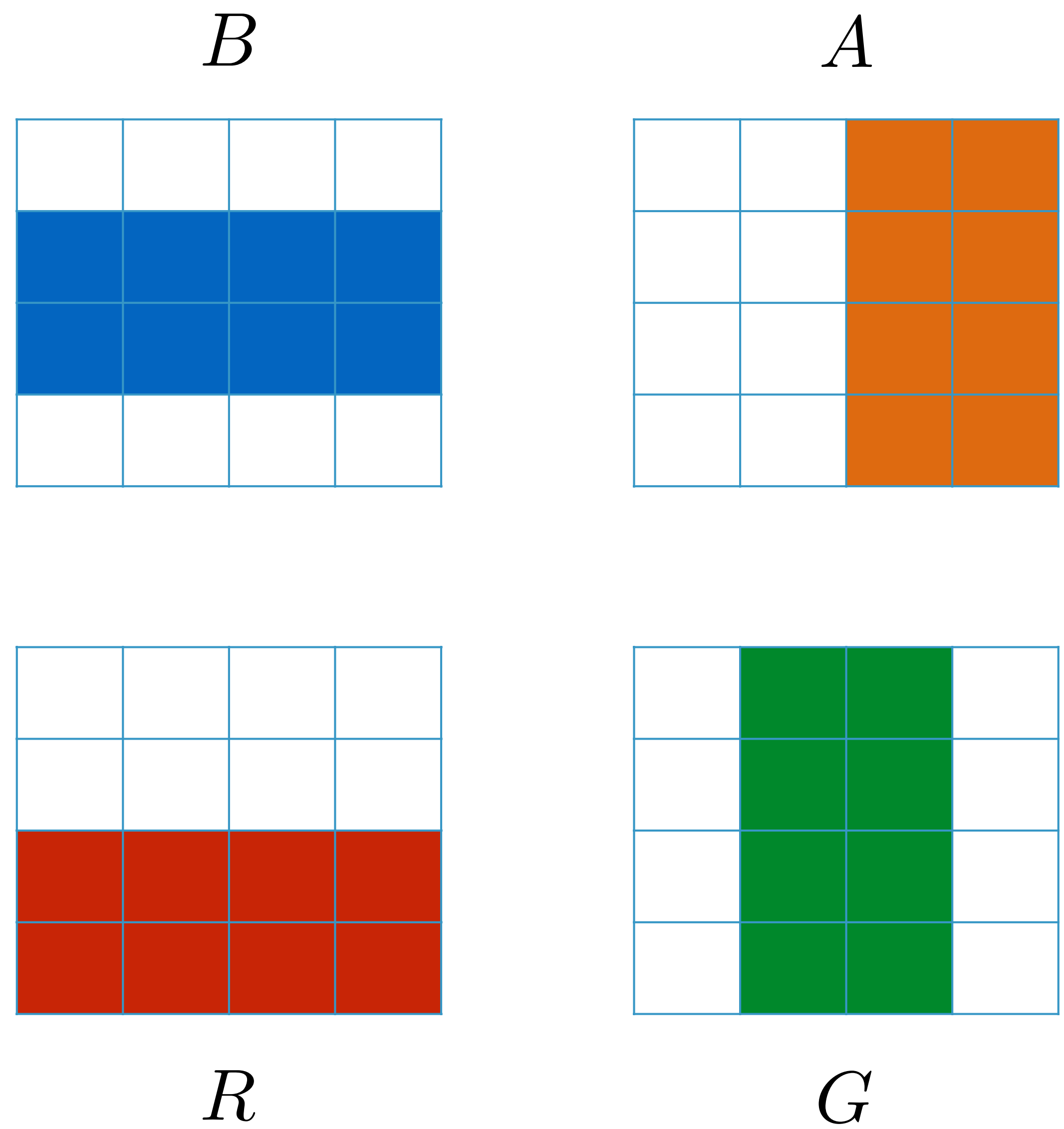


4 atoms:16 states: 64K subsets

Karnaugh Maps

		AG			
		00	01	11	10
RB	00	1	1	1	1
	01	0	0	0	0
	11	0	0	0	0
	10	0	0	0	0

$$\neg B \wedge \neg R$$

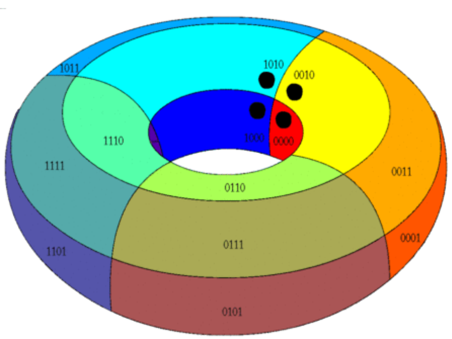


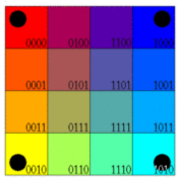
4 atoms:16 states: 64K subsets

Karnaugh Maps

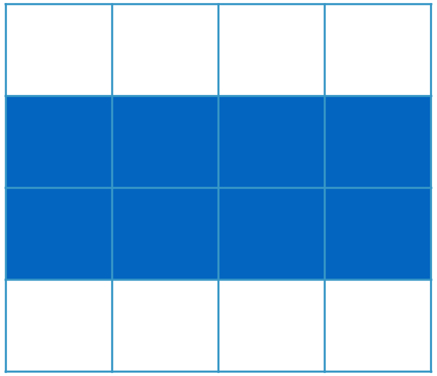
RB \ AG	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	0	0	0	0
10	0	0	0	0

$$\neg G \wedge \neg R$$

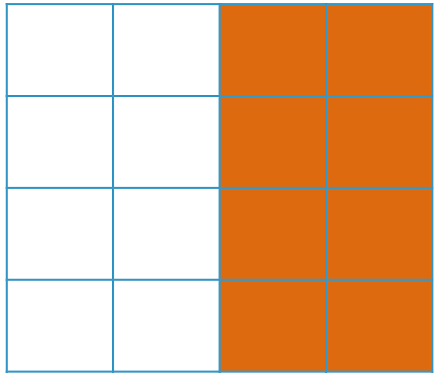


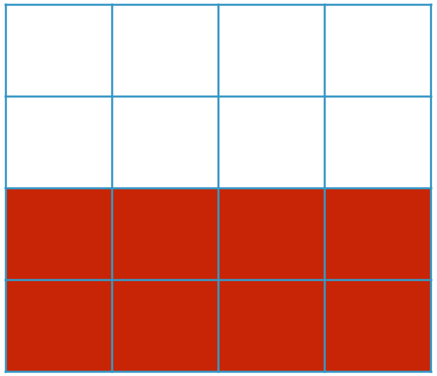


B

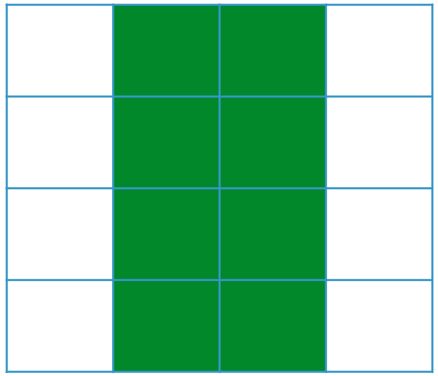


A





R

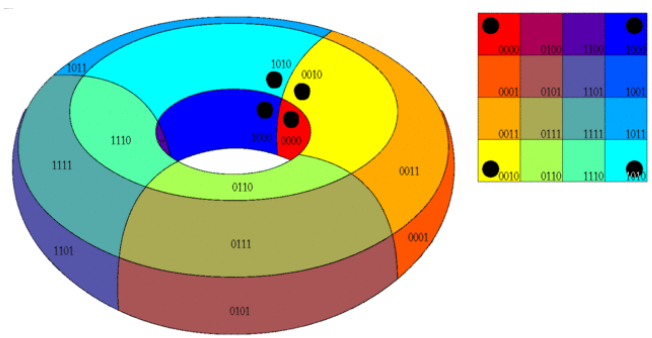


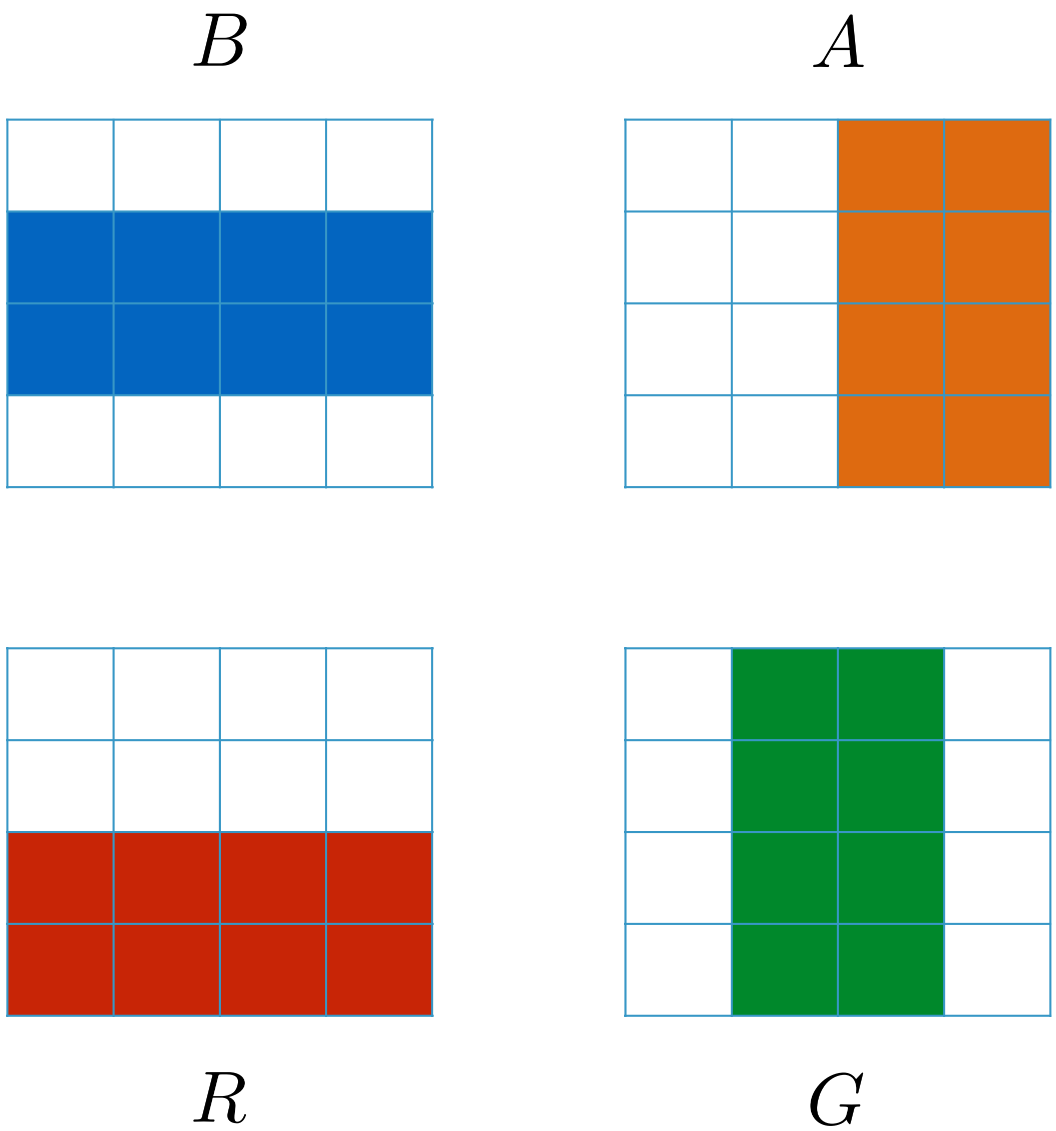
G

Karnaugh Maps

RB \ AG	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$$\neg G \wedge \neg B$$

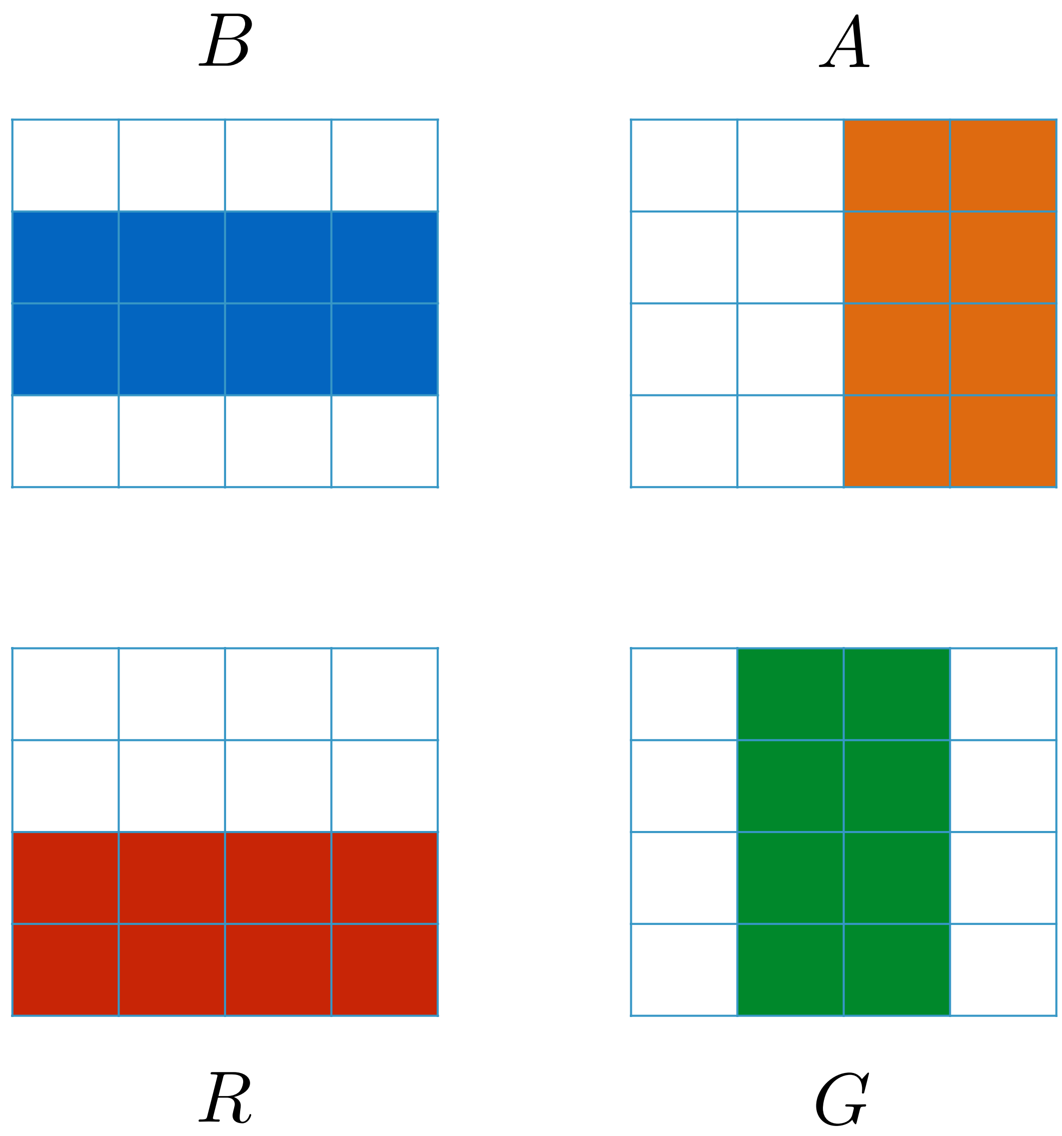




Karnaugh Maps

RB \ AG	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	0	0	0

$$G \wedge B$$

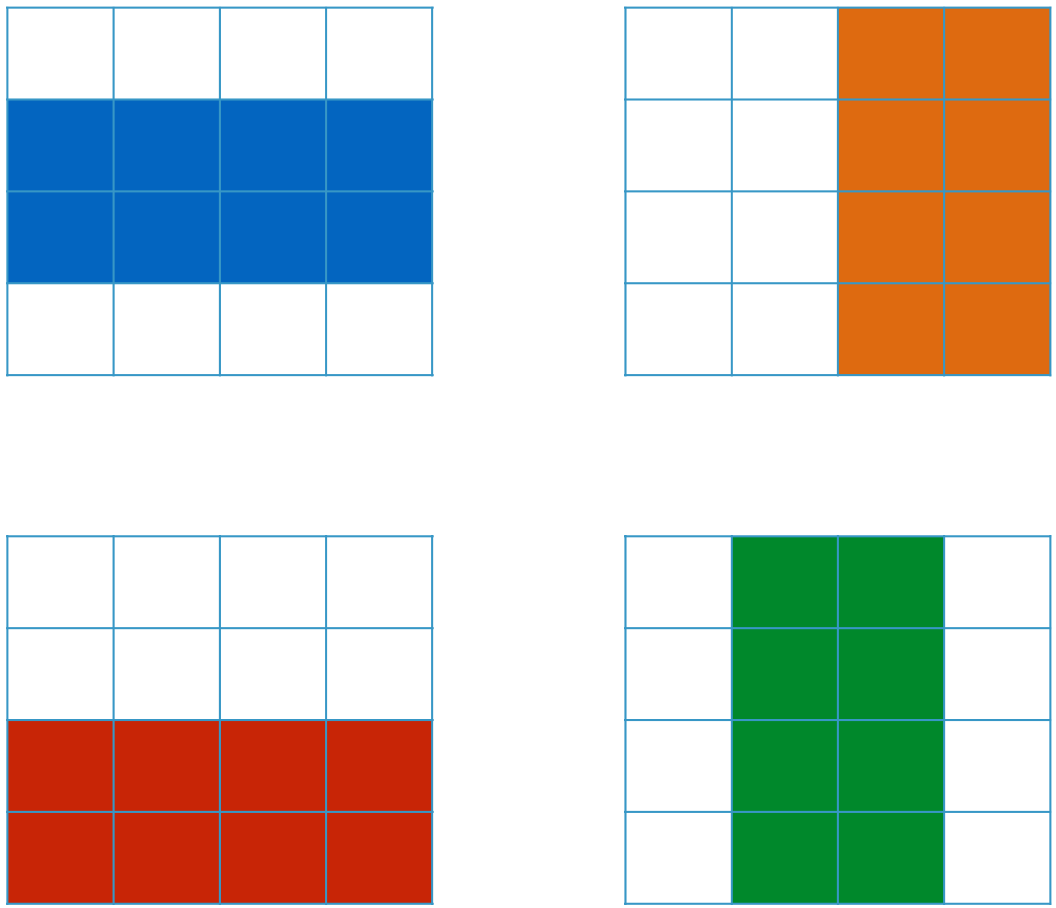


4 atoms:16 states: 64K subsets

Karnaugh Maps

	AG	00	01	11	10
RB	00	1	1	1	1
	01	1	0	0	1
	11	1	0	0	1
	10	1	1	1	1

$$\neg B \vee \neg G$$

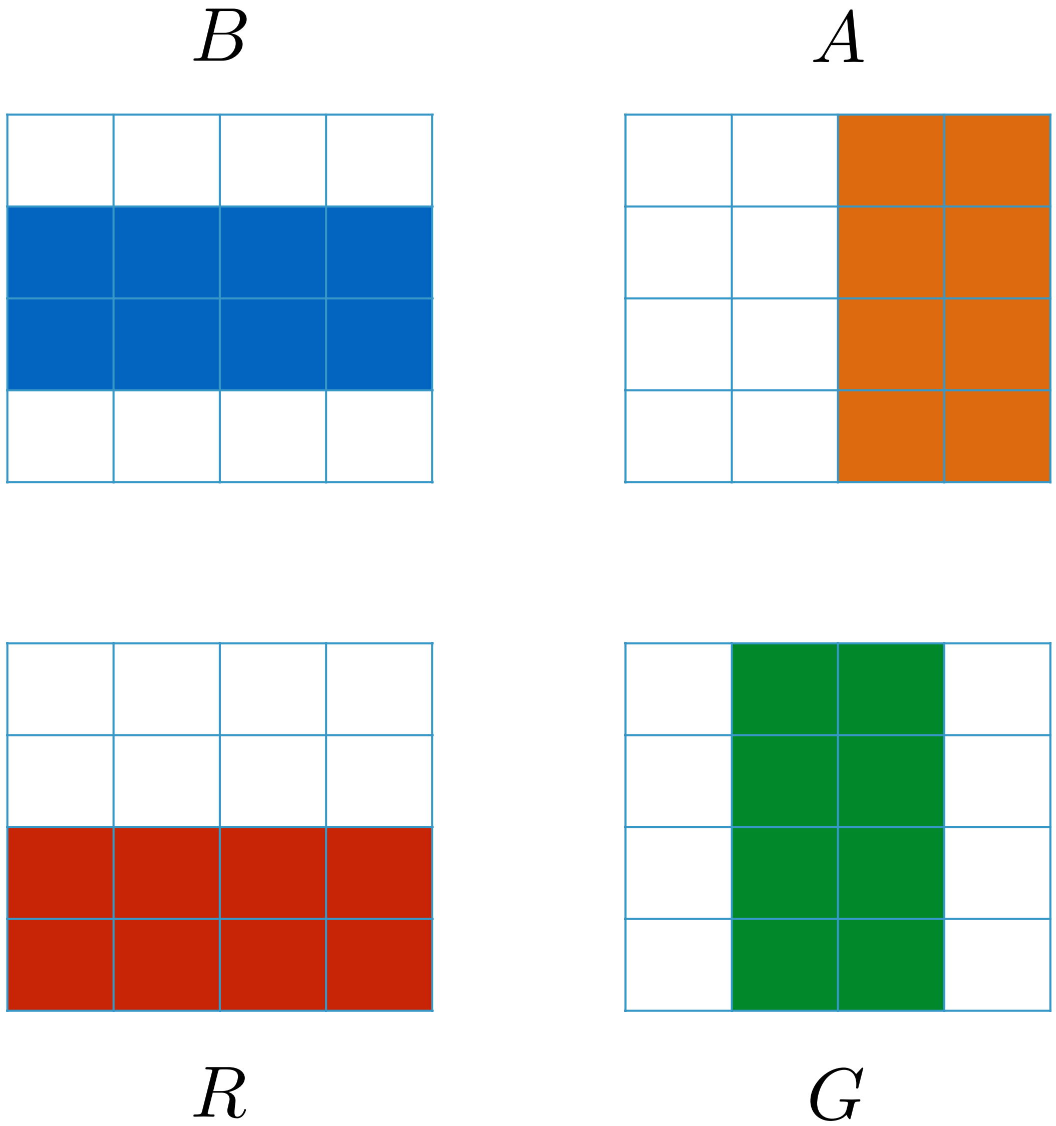


4 atoms:16 states: 64K subsets

Karnaugh Maps

RB \ AG	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	0	0	1	1
10	0	0	1	1

$$(\neg A \wedge \neg R) \vee (A \wedge R)$$

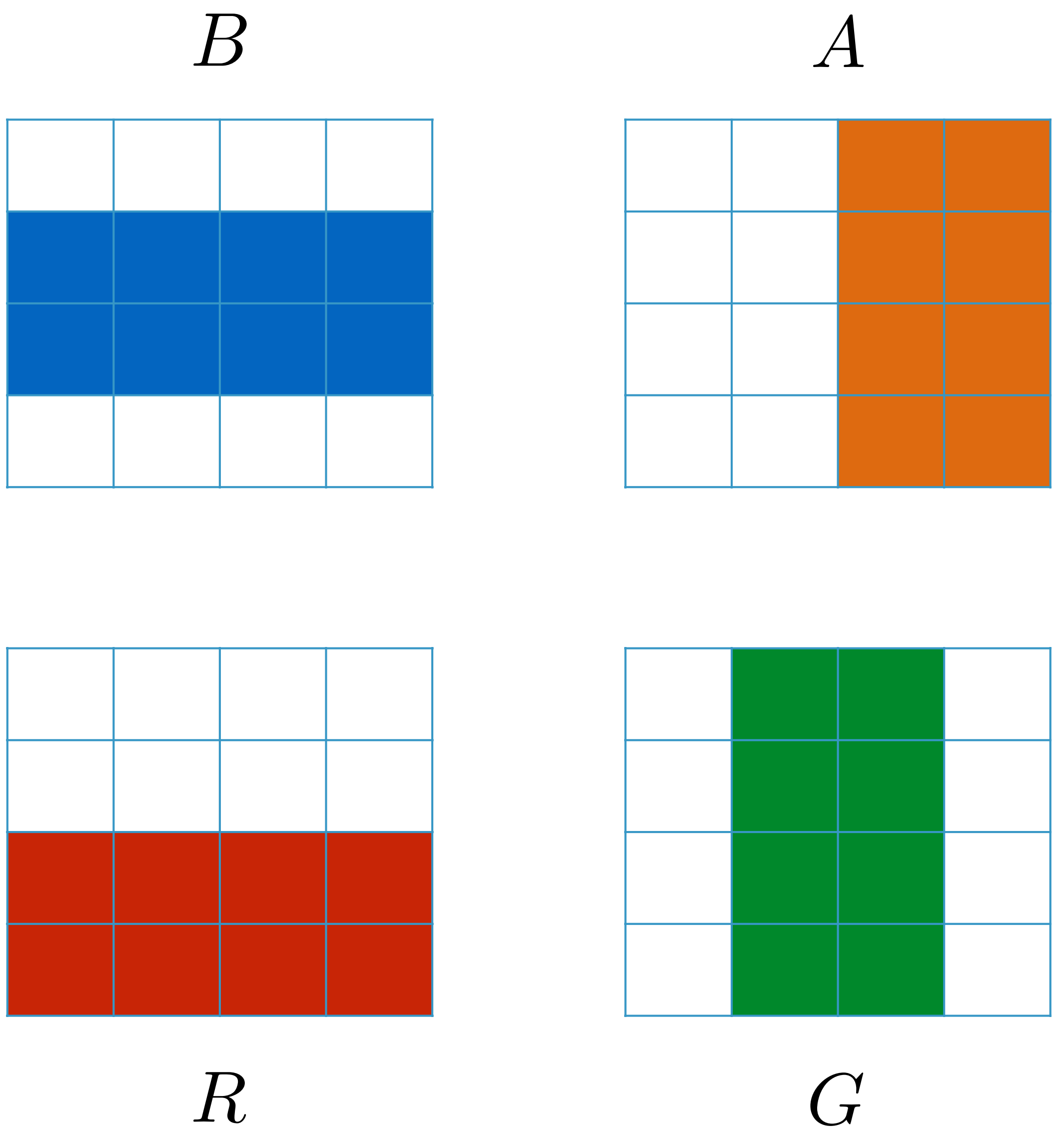


4 atoms:16 states: 64K subsets

Karnaugh Maps

RB \ AG	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	0	0	1	1
10	0	0	1	1

$$(\neg R \vee A) \wedge (\neg A \vee R)$$

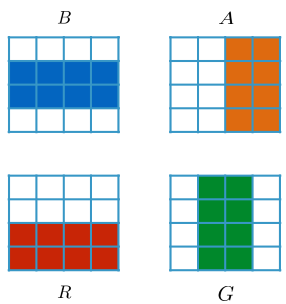


4 atoms:16 states: 64K subsets

Karnaugh Maps

to produce a DNF / SOP
identify blocks of 1s
and write a product for each

RB \ AG	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	1	1	1
10	0	0	1	1



4 atoms:16 states: 64K subsets

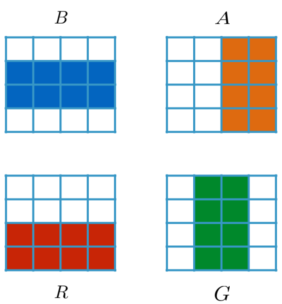
Karnaugh Maps

to produce a DNF / SOP
identify blocks of 1s
and write a product for each

RB \ AG	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	1	1	1
10	0	0	1	1

RA + BA′G + R′A′G′

$$(R \wedge A) \vee (B \wedge \neg A \wedge G) \vee (\neg R \wedge \neg A \wedge \neg G)$$

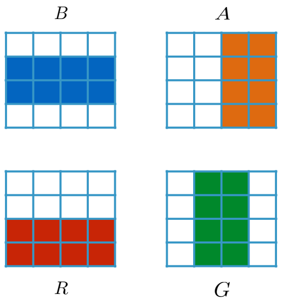


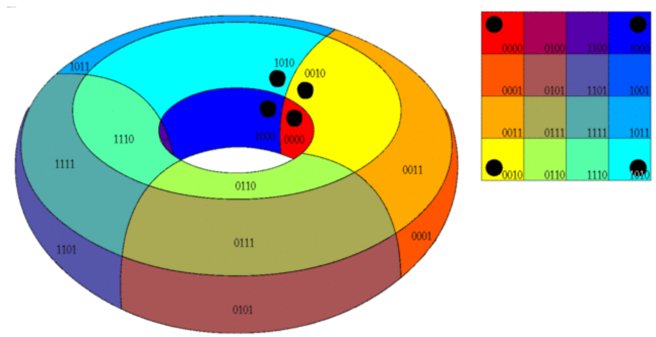
4 atoms:16 states: 64K subsets

Karnaugh Maps

to produce a CNF / POS
identify blocks of 0s
and write a sum for each

RB \ AG	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	1	1	1
10	0	0	1	1





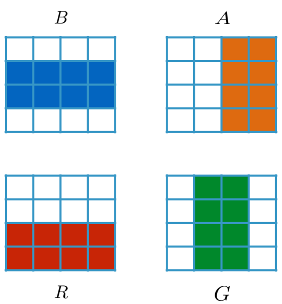
Karnaugh Maps

to produce a CNF / POS
identify blocks of 0s
and write a sum for each

AG

RB \ AG	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	1	1	1
10	0	0	1	1

(R′+A+G)(B+A+G′)(R+A′)



$$(\neg R \vee A \vee G) \wedge (B \vee A \vee \neg G) \wedge (R \vee \neg A)$$

4 atoms:16 states: 64K subsets