


Informatics 1

Computation and Logic

Logic

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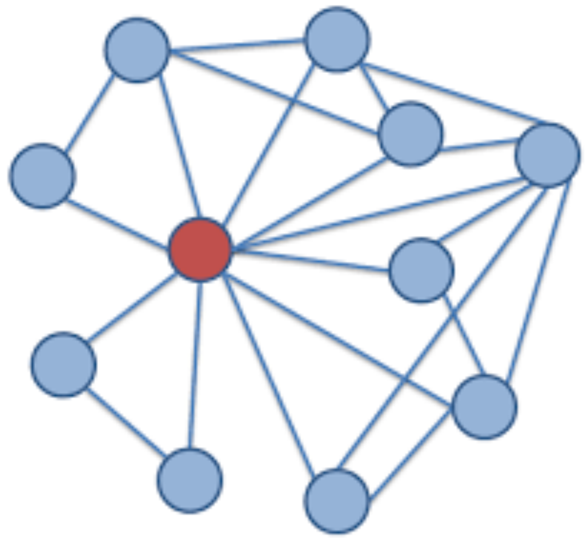
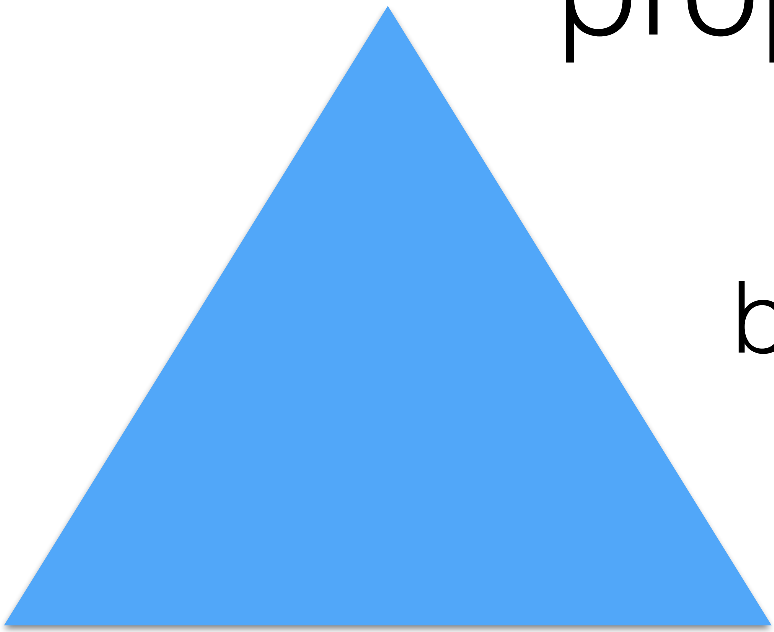
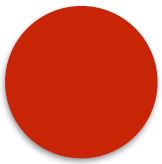
@mp4man

Propositional Logic concerns properties of things

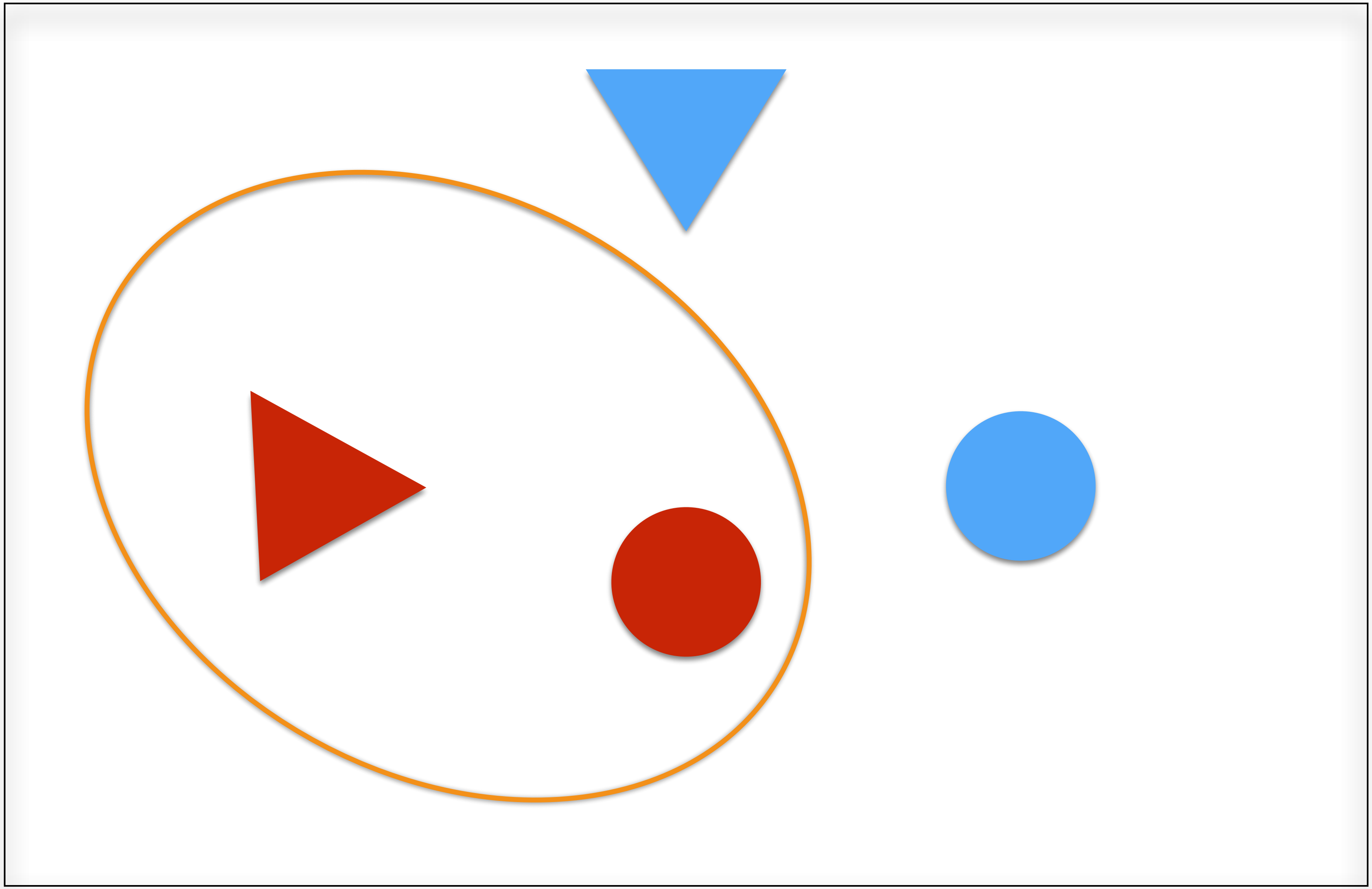


big blue triangle

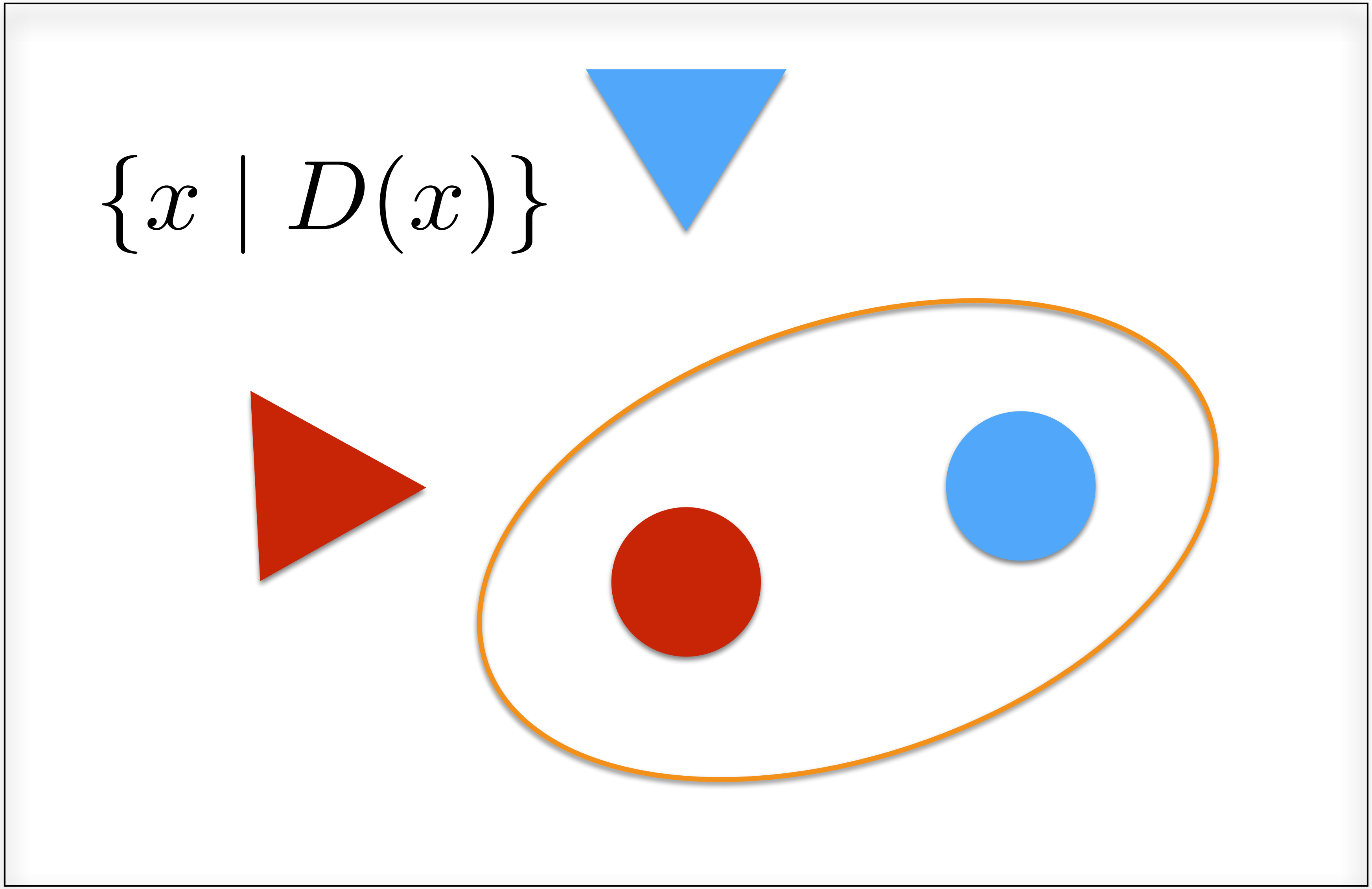


small red disc

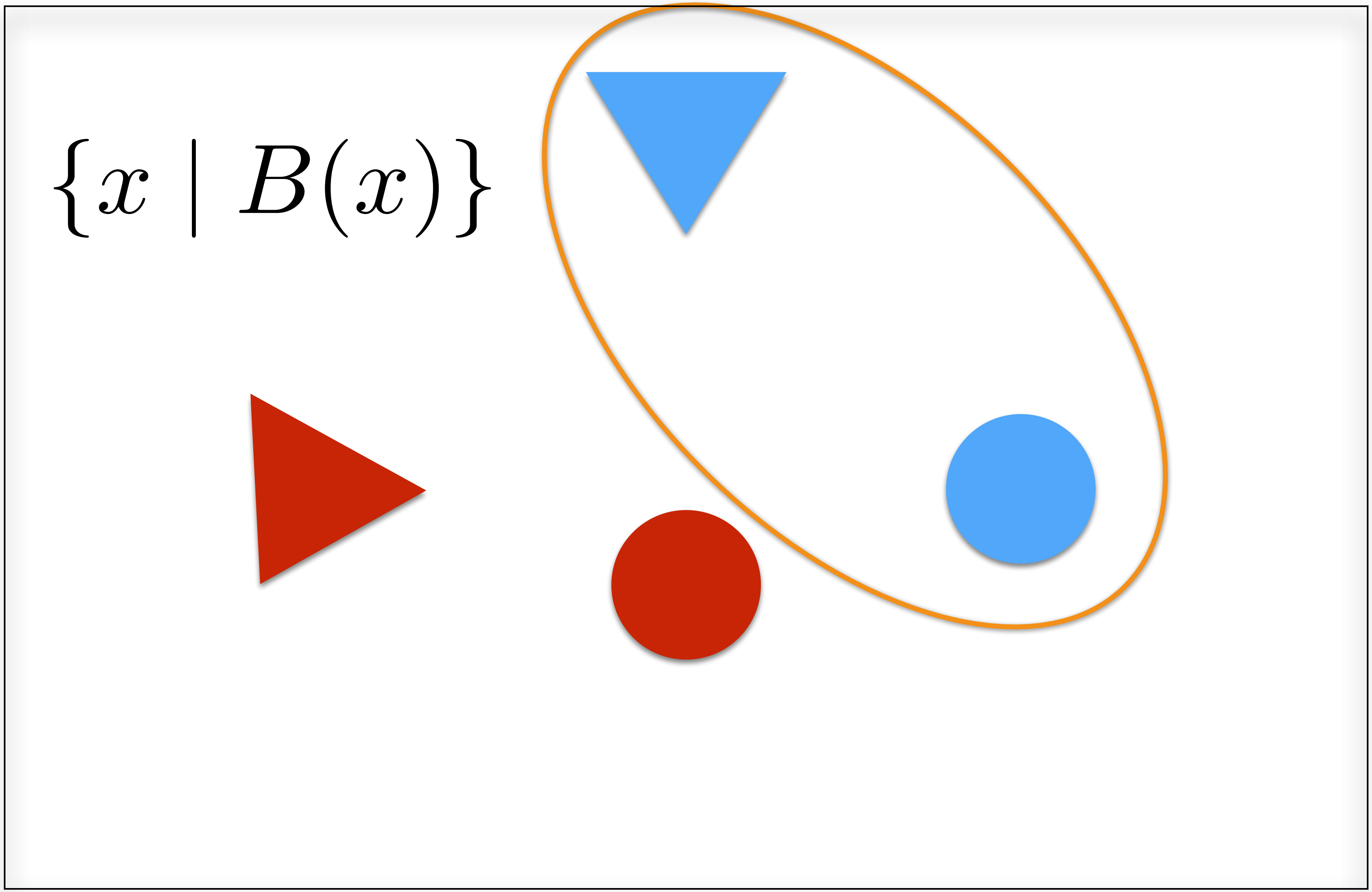
red	$R(x)$
blue	$B(x)$
large	$L(x)$
small	$S(x)$
disc	$D(x)$
triangle	$T(x)$



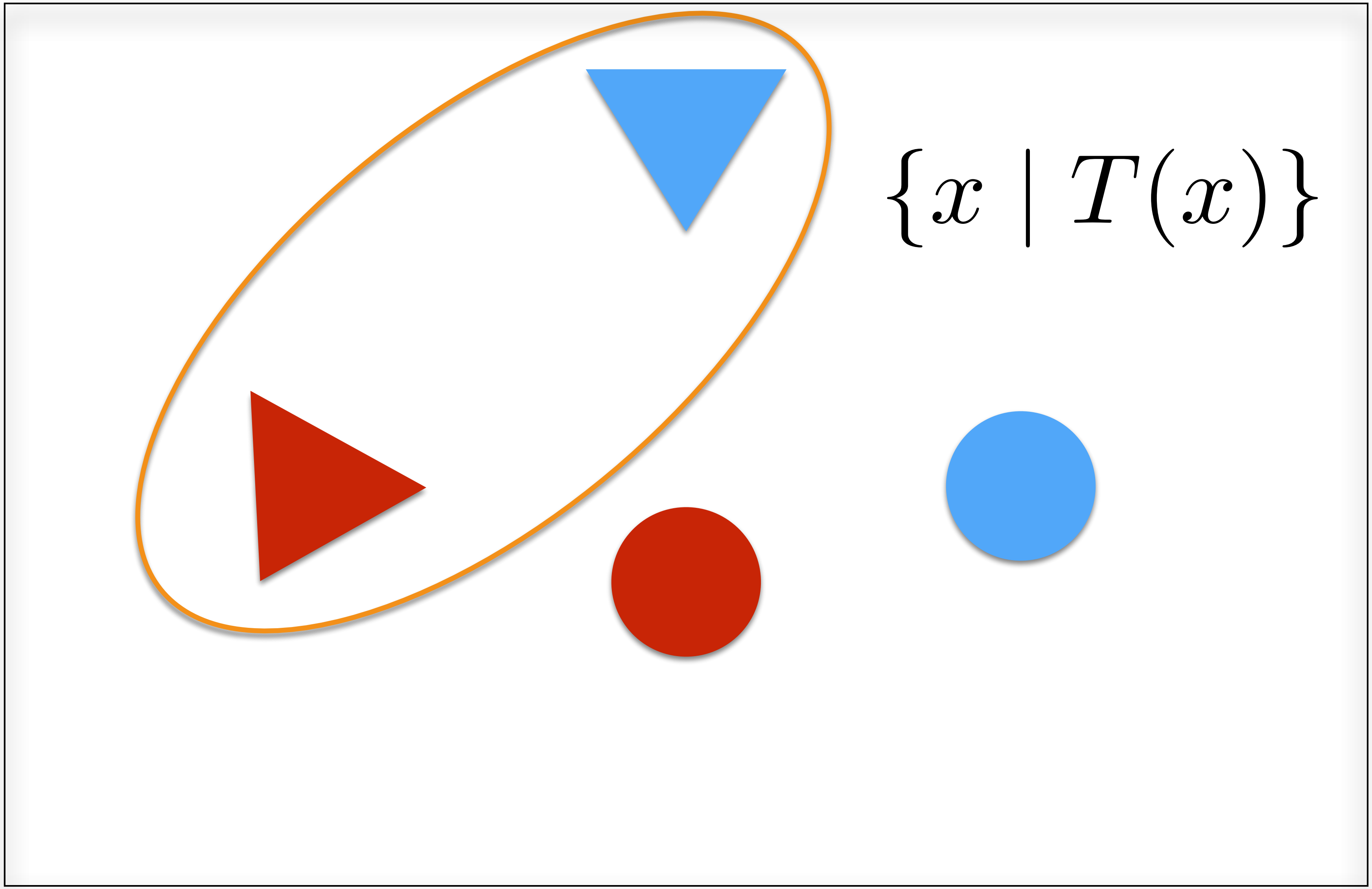
red $\{x \mid R(x)\}$



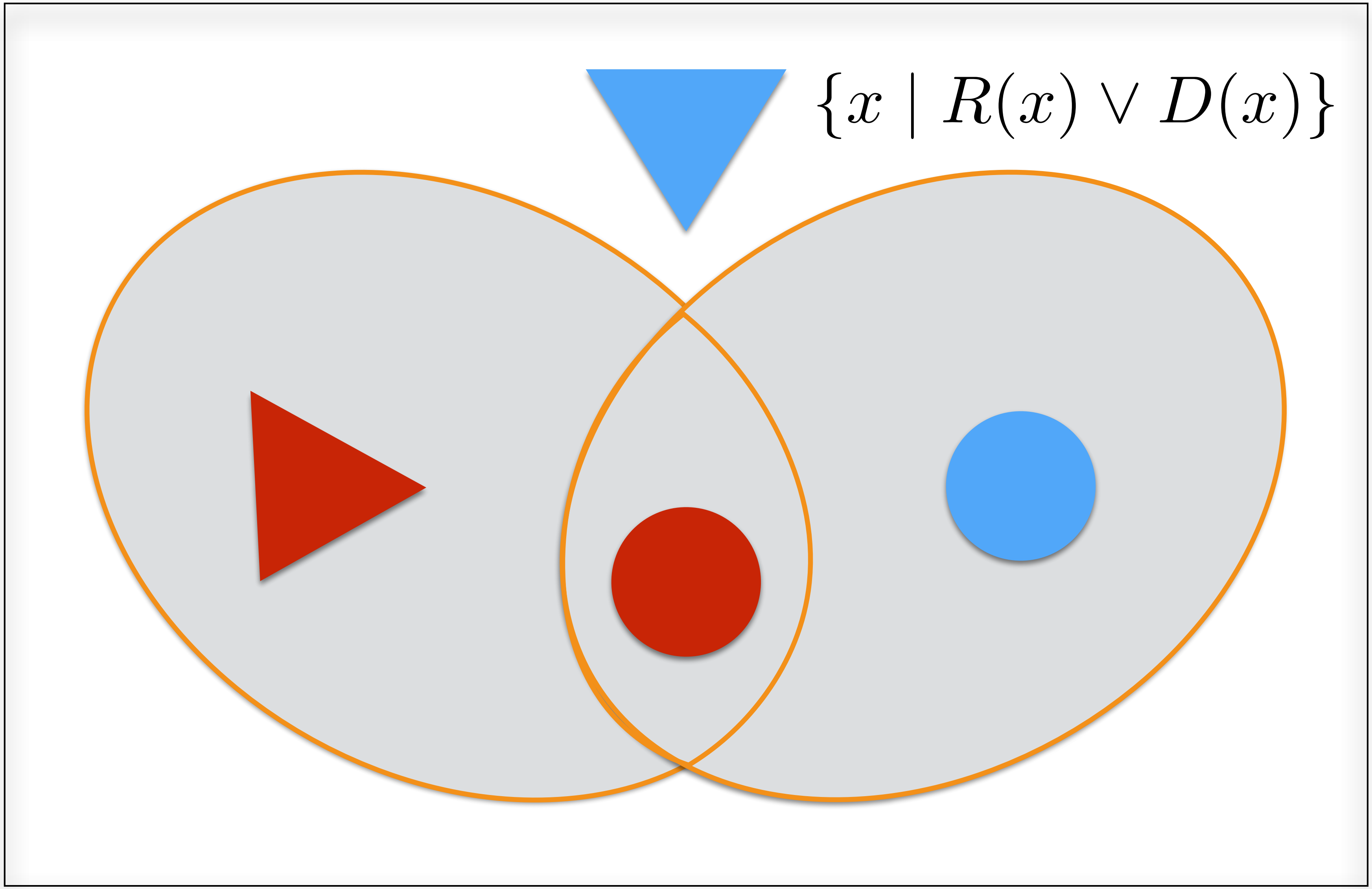
disc



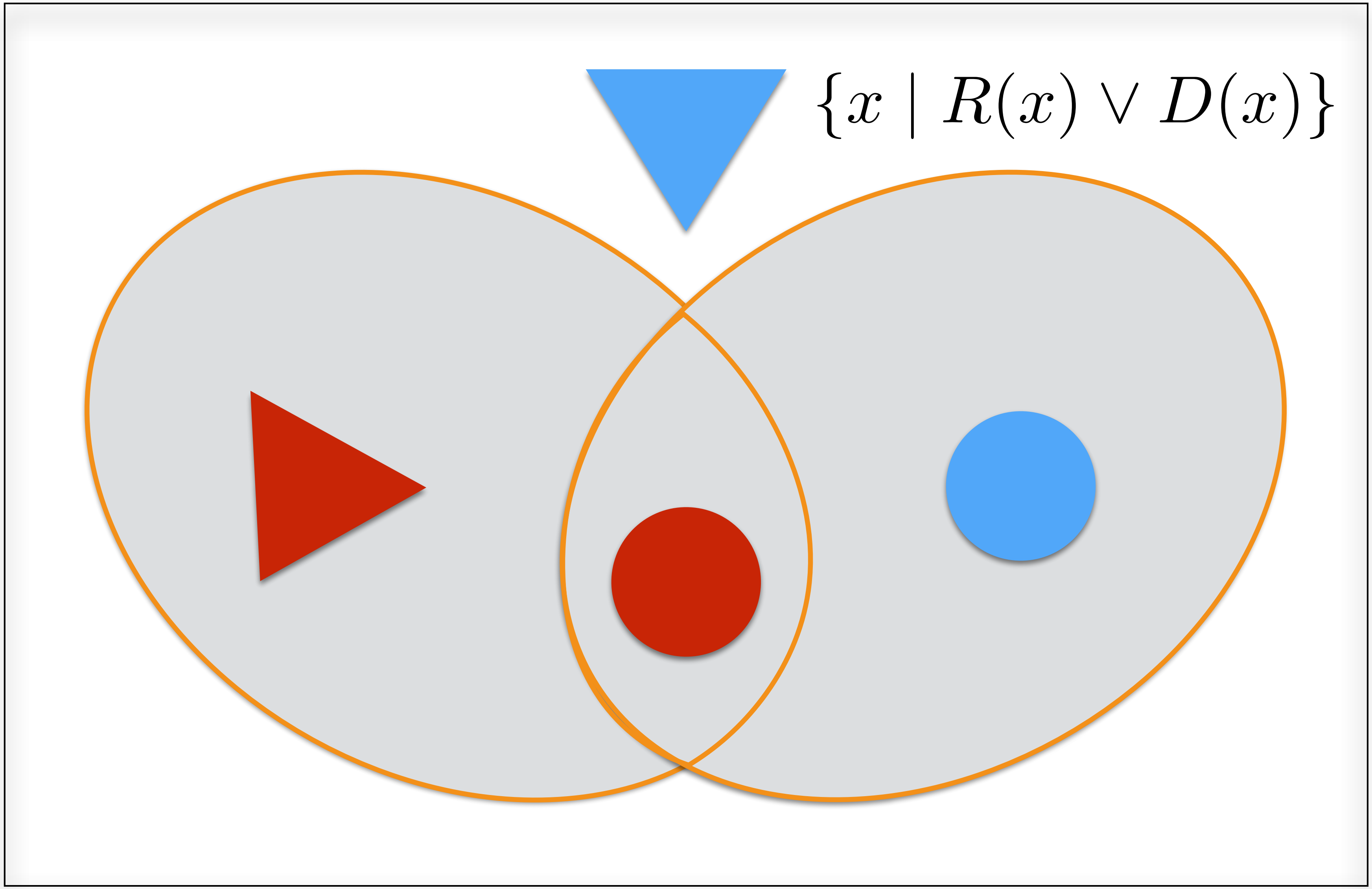
blue



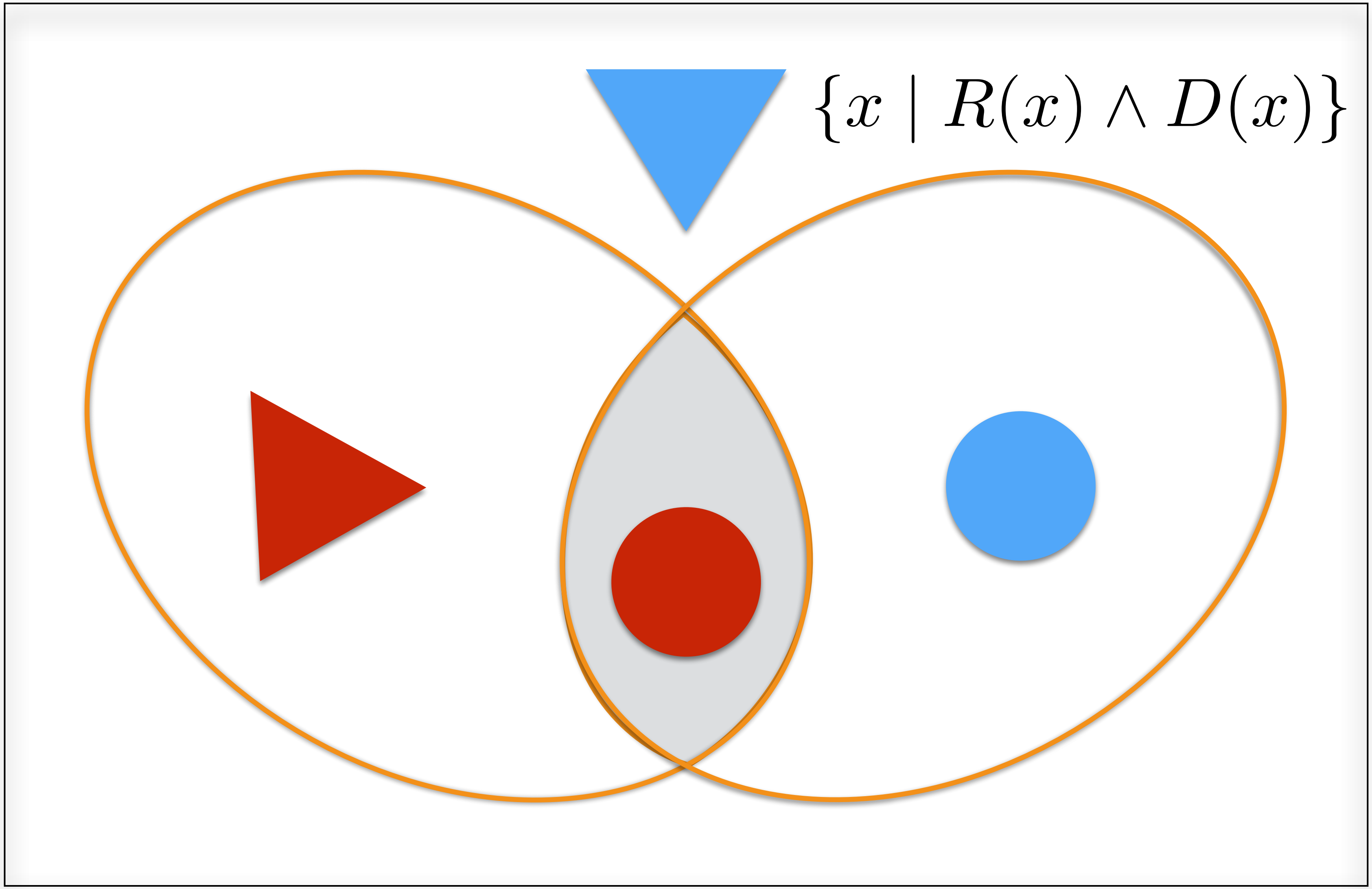
triangle



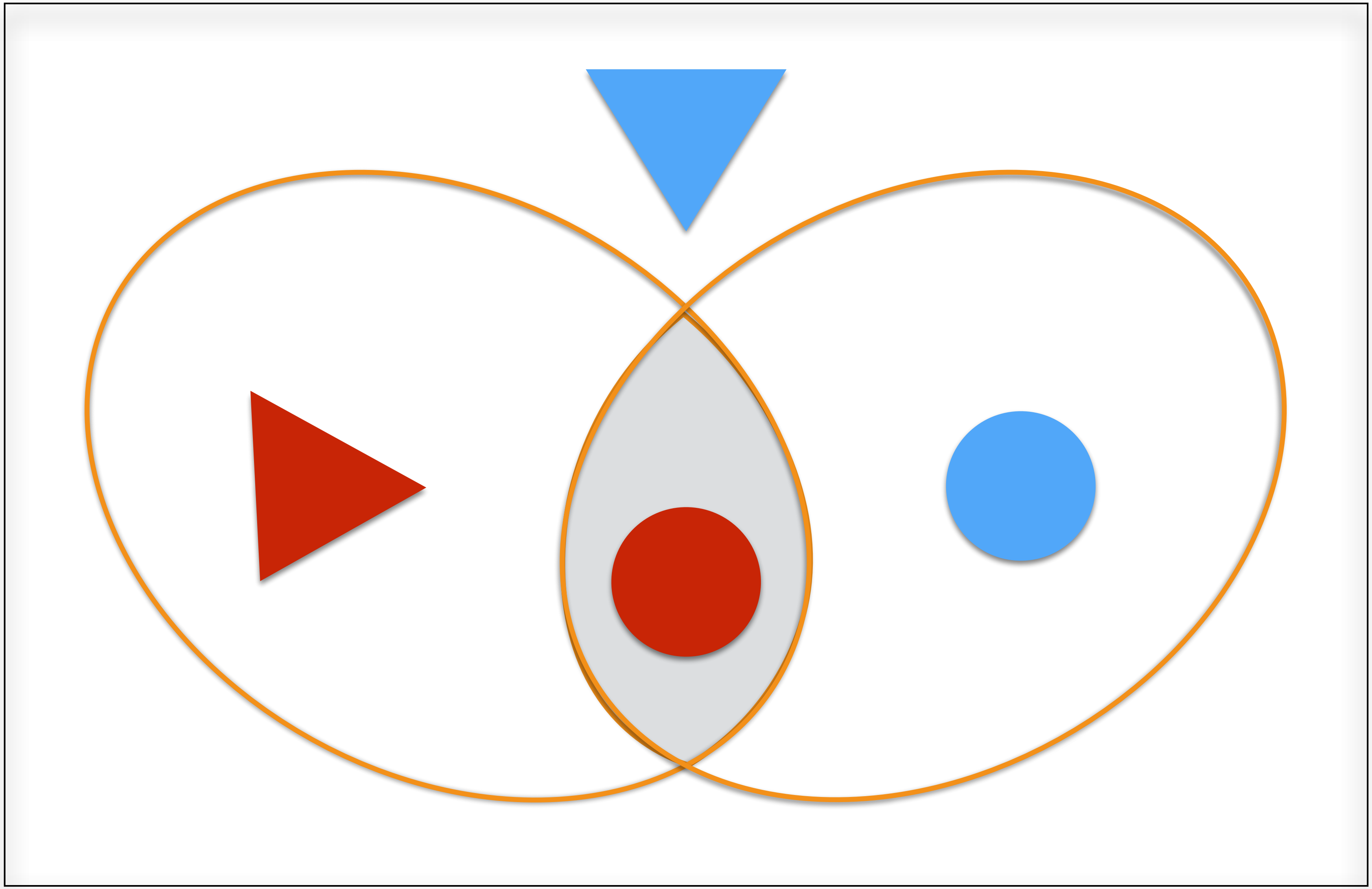
red **or** disc



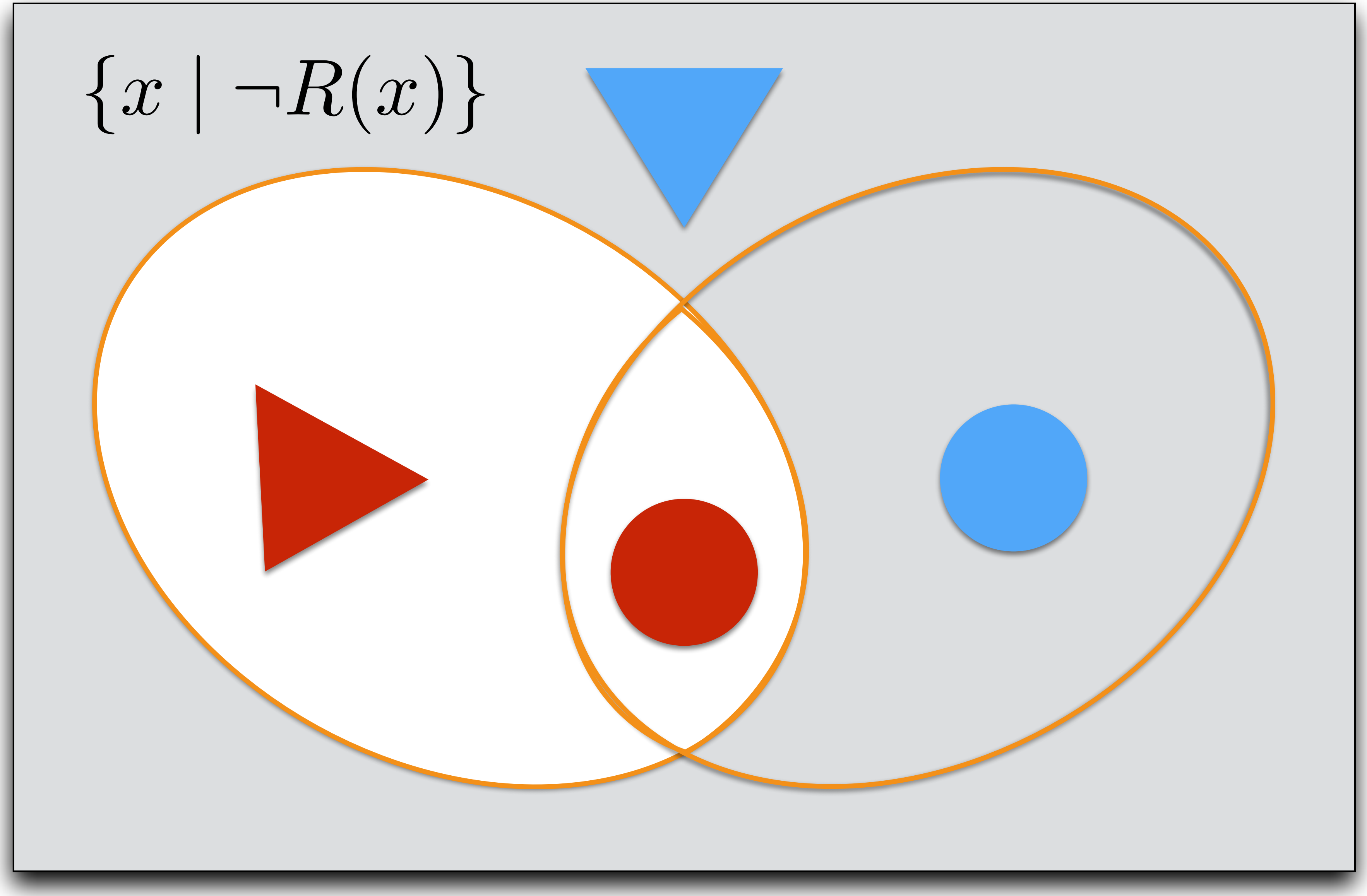
$$\{x \mid R(x) \lor D(x)\} = \{x \mid R(x)\} \cup \{x \mid D(x)\}$$



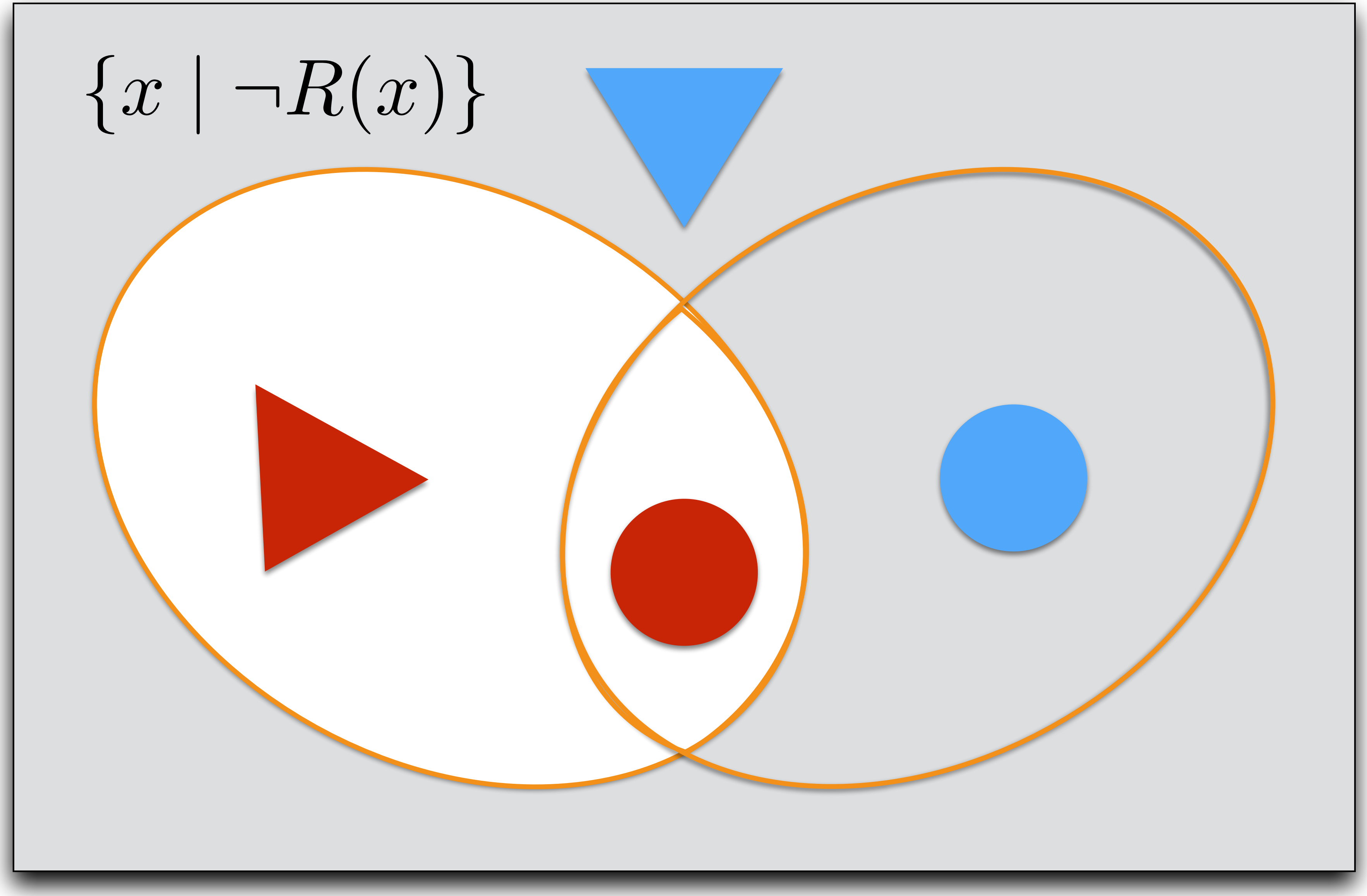
red **and** disc



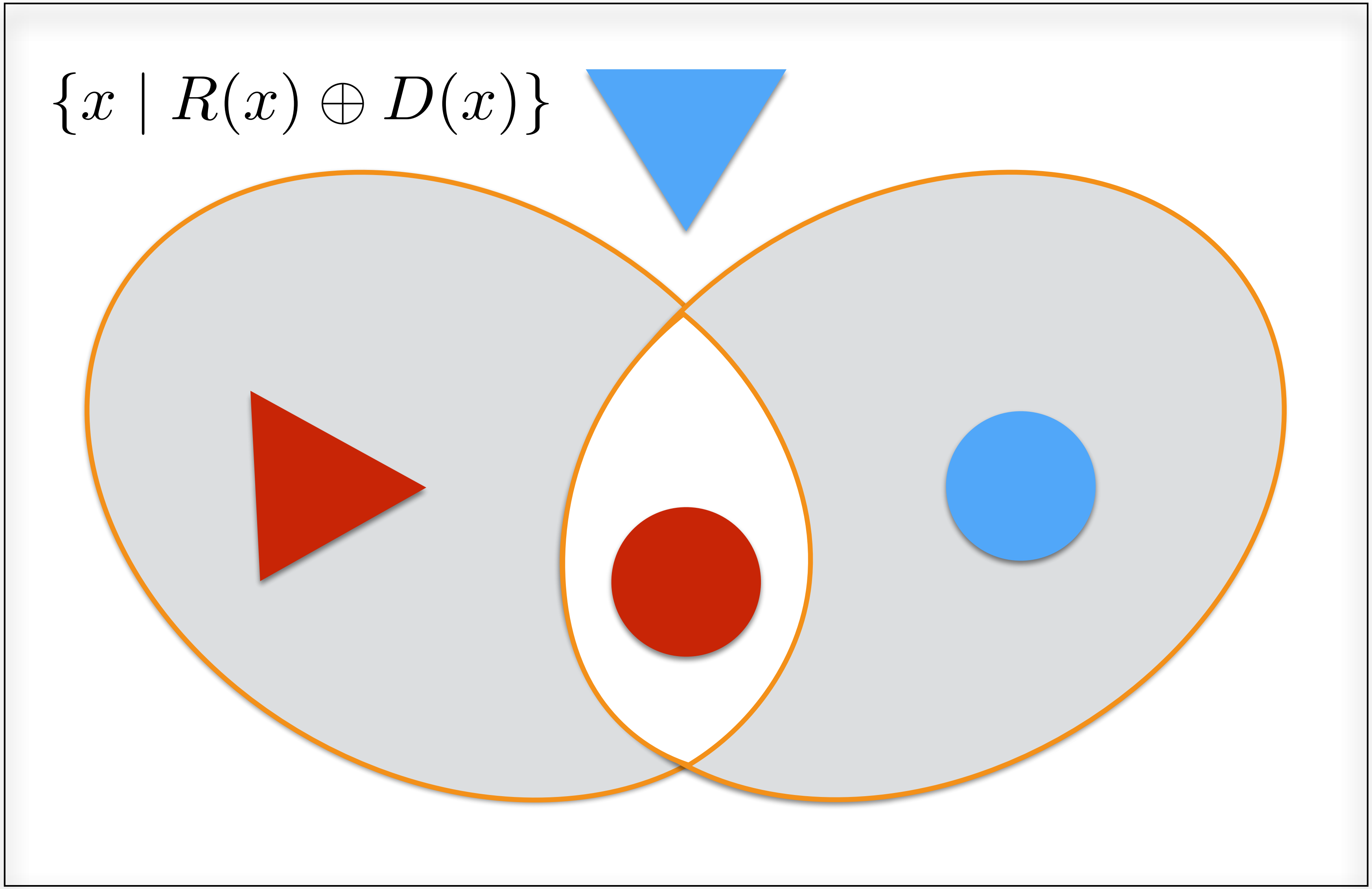
$$\{x \mid R(x) \wedge S(x)\} = \{x \mid R(x)\} \cap \{x \mid S(x)\}$$



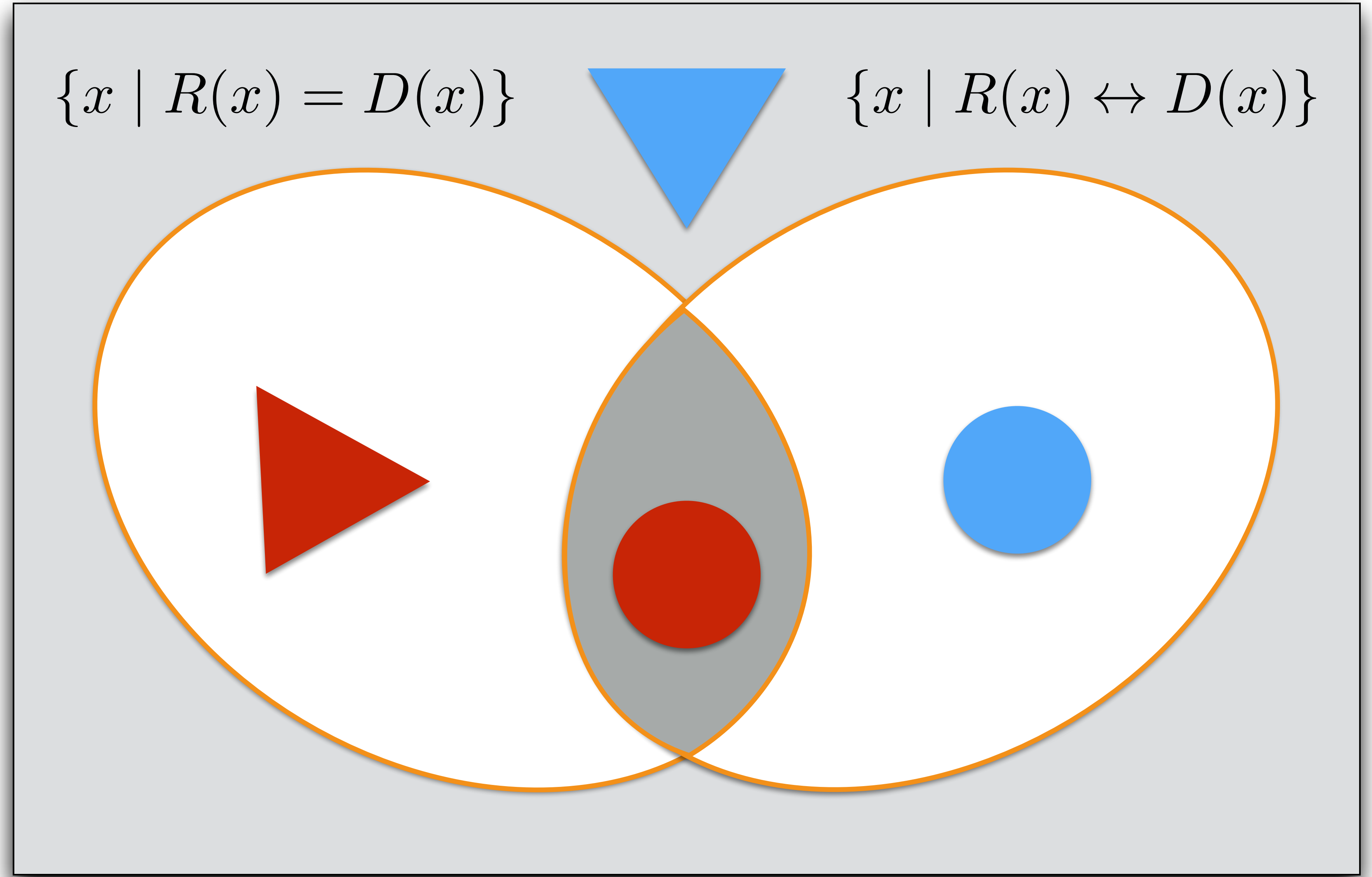
not red



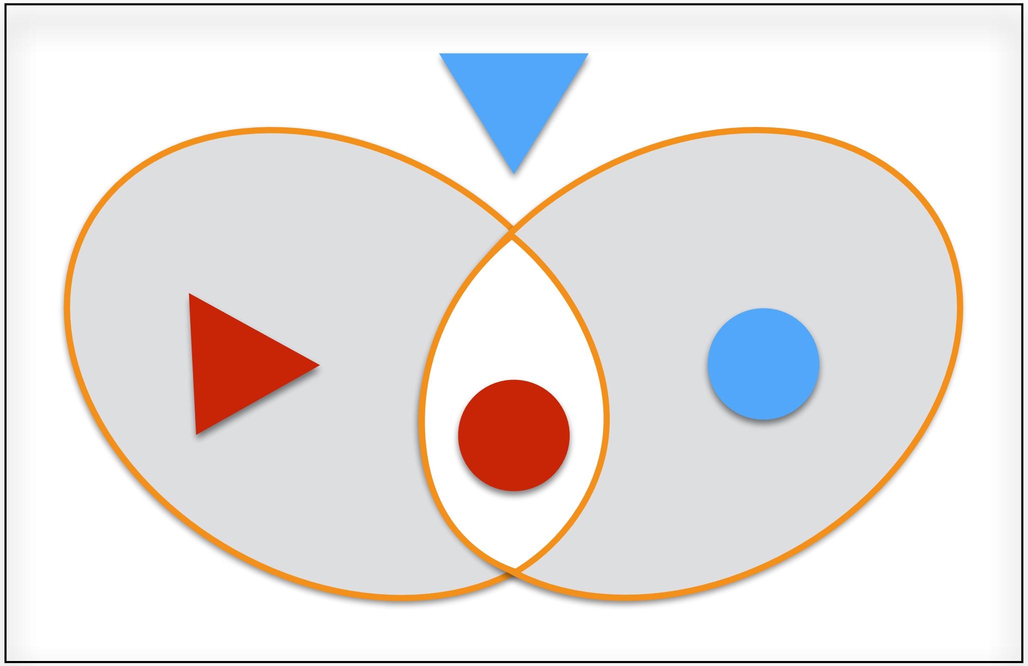
$$\{x \mid \neg R(x)\} = \{x \mid \top\} \setminus \{x \mid R(x)\}$$



red **xor** disc



red **iff** disc



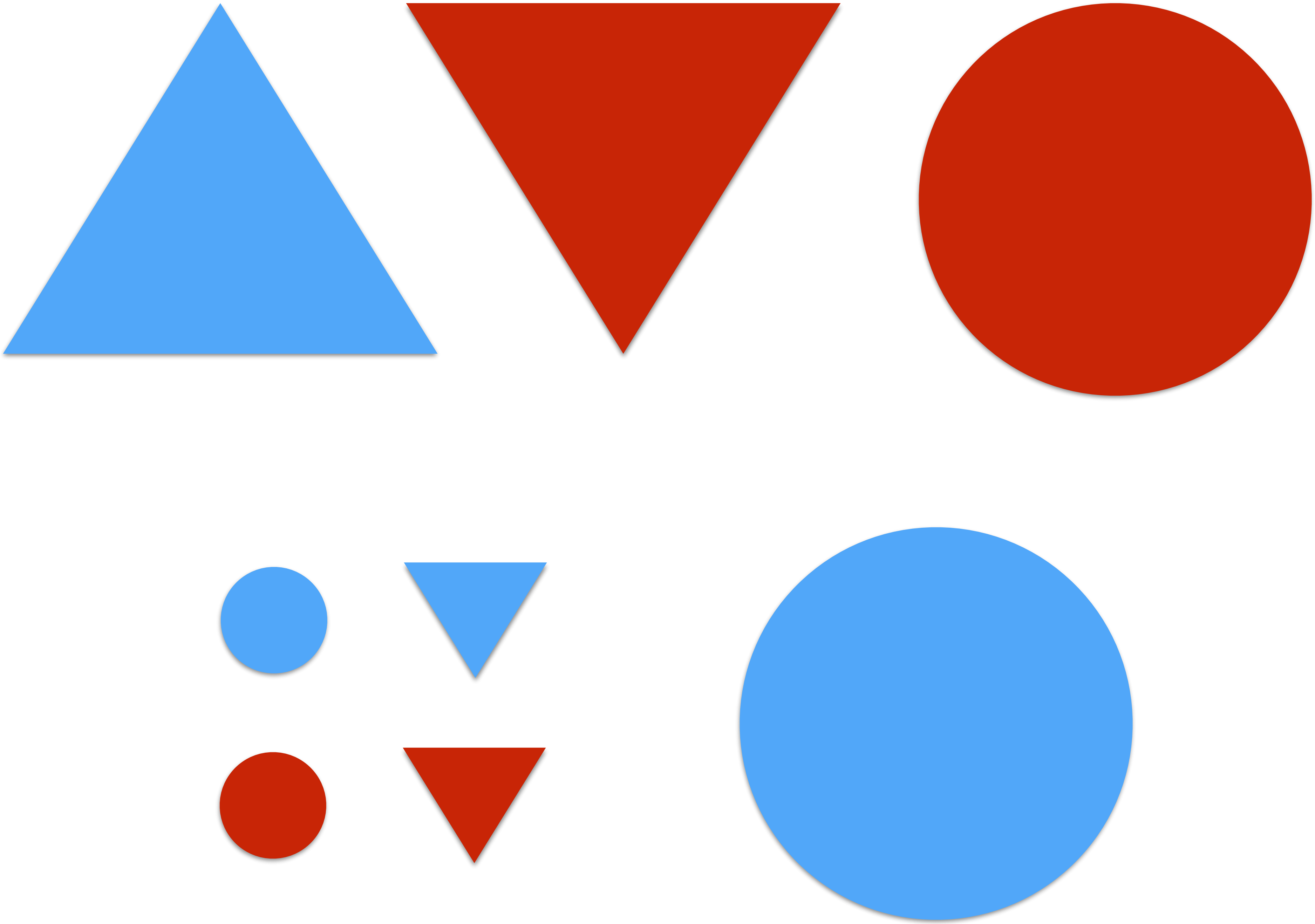
$$(R(x) \vee D(x)) \wedge \neg(R(x) \wedge D(x))$$

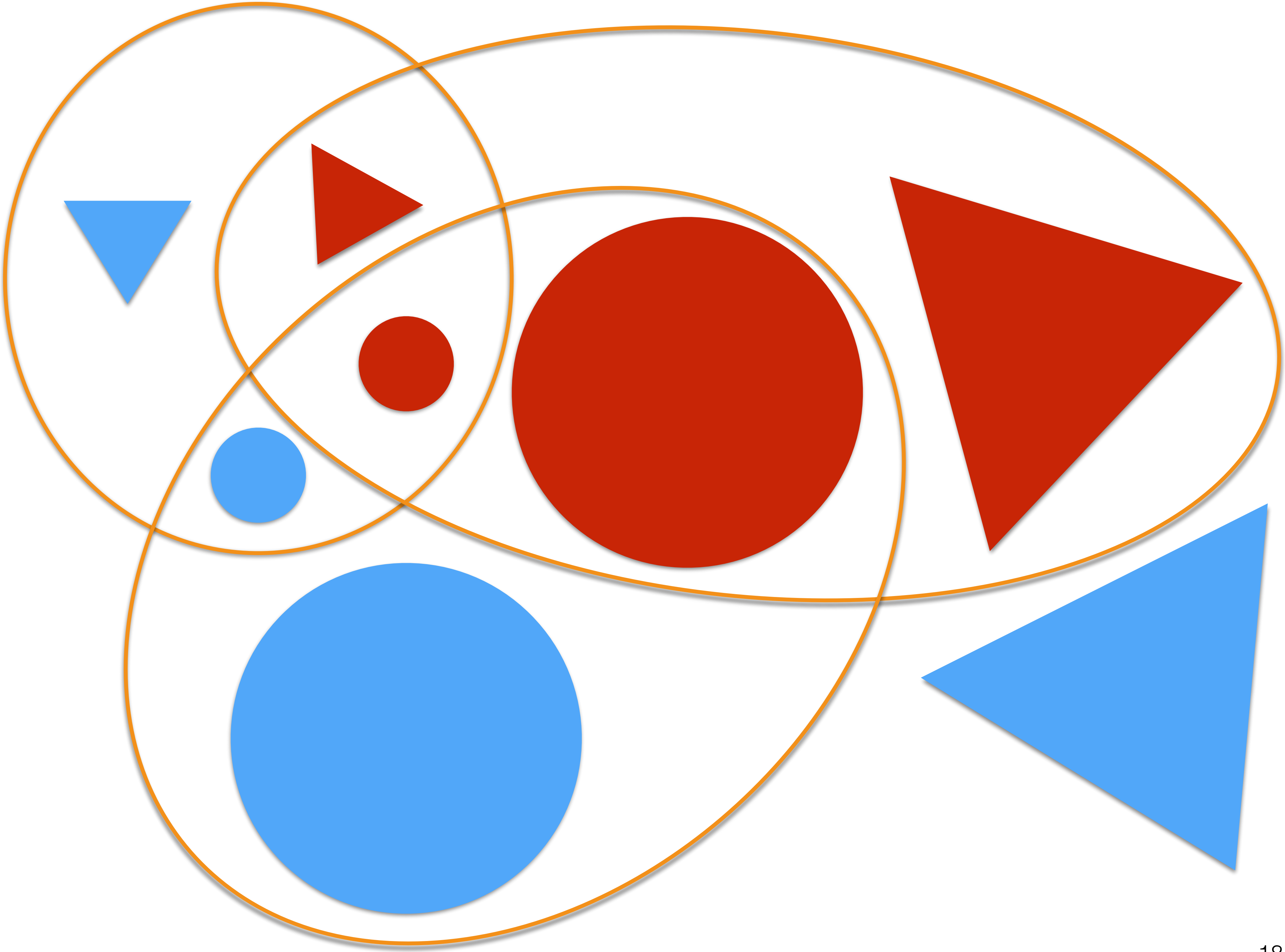
$$R(x) \oplus D(x)$$

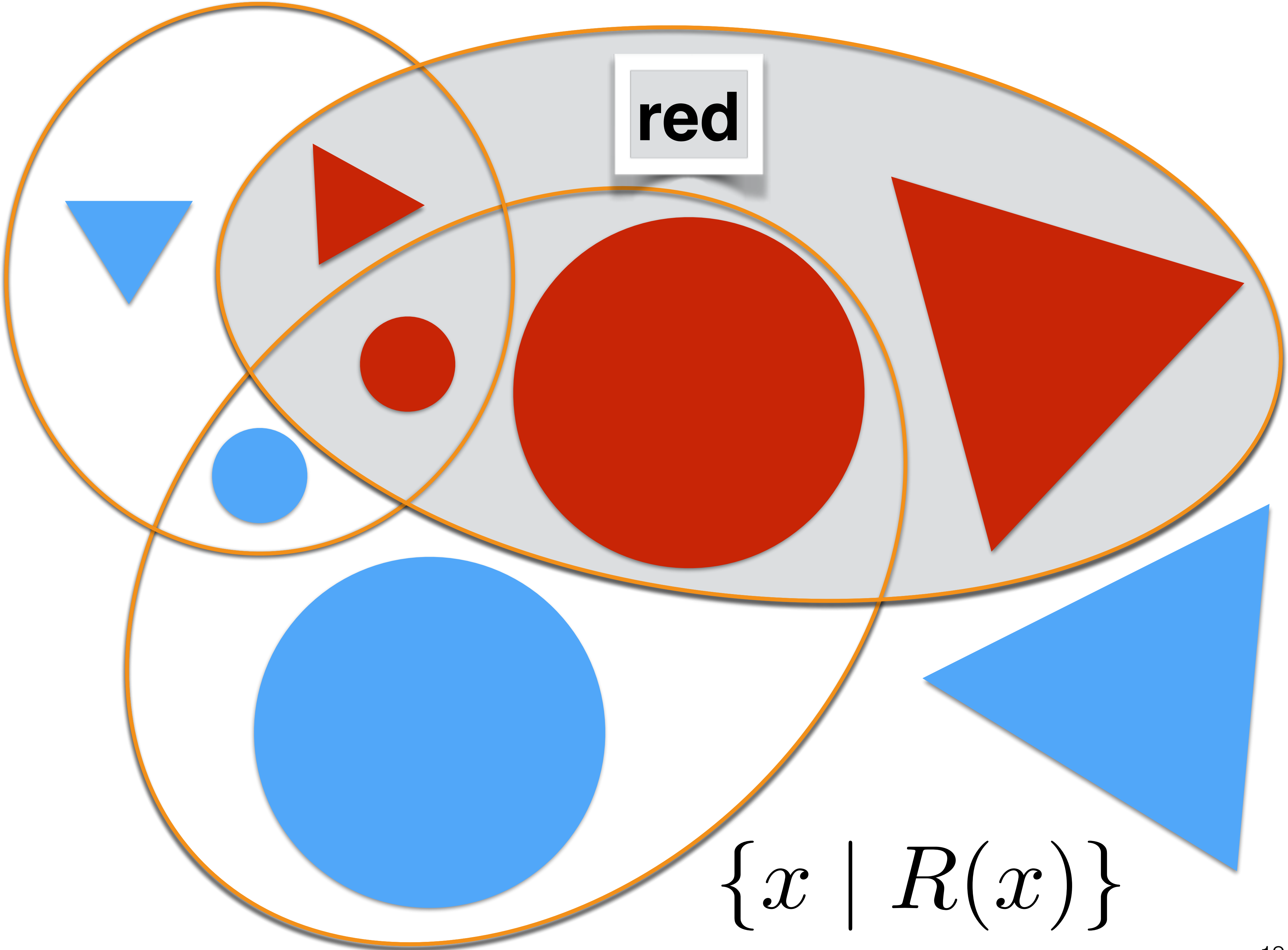
(red **or** disc) **and**
not (red **and** disc)

=

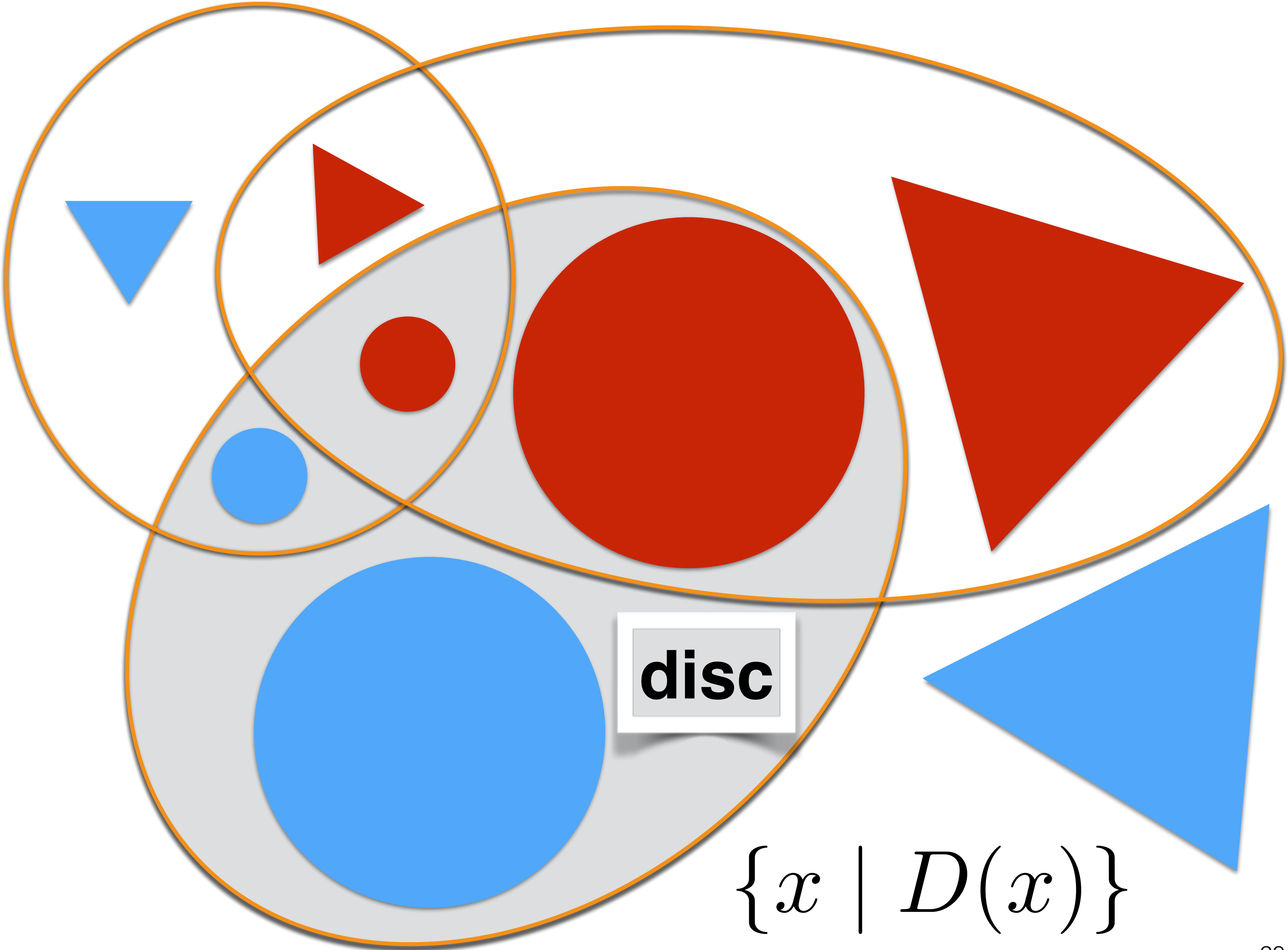
red **xor** disc



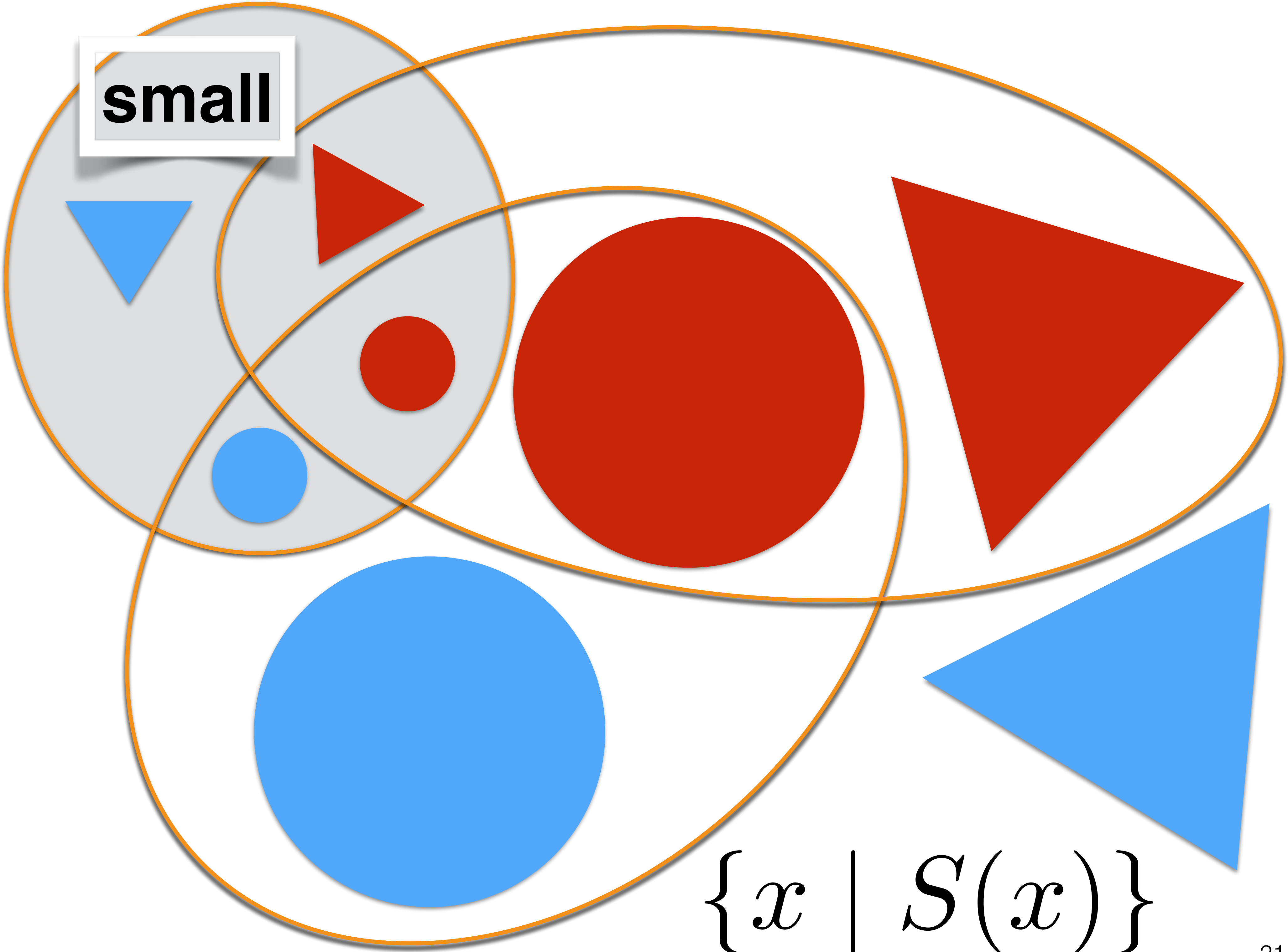




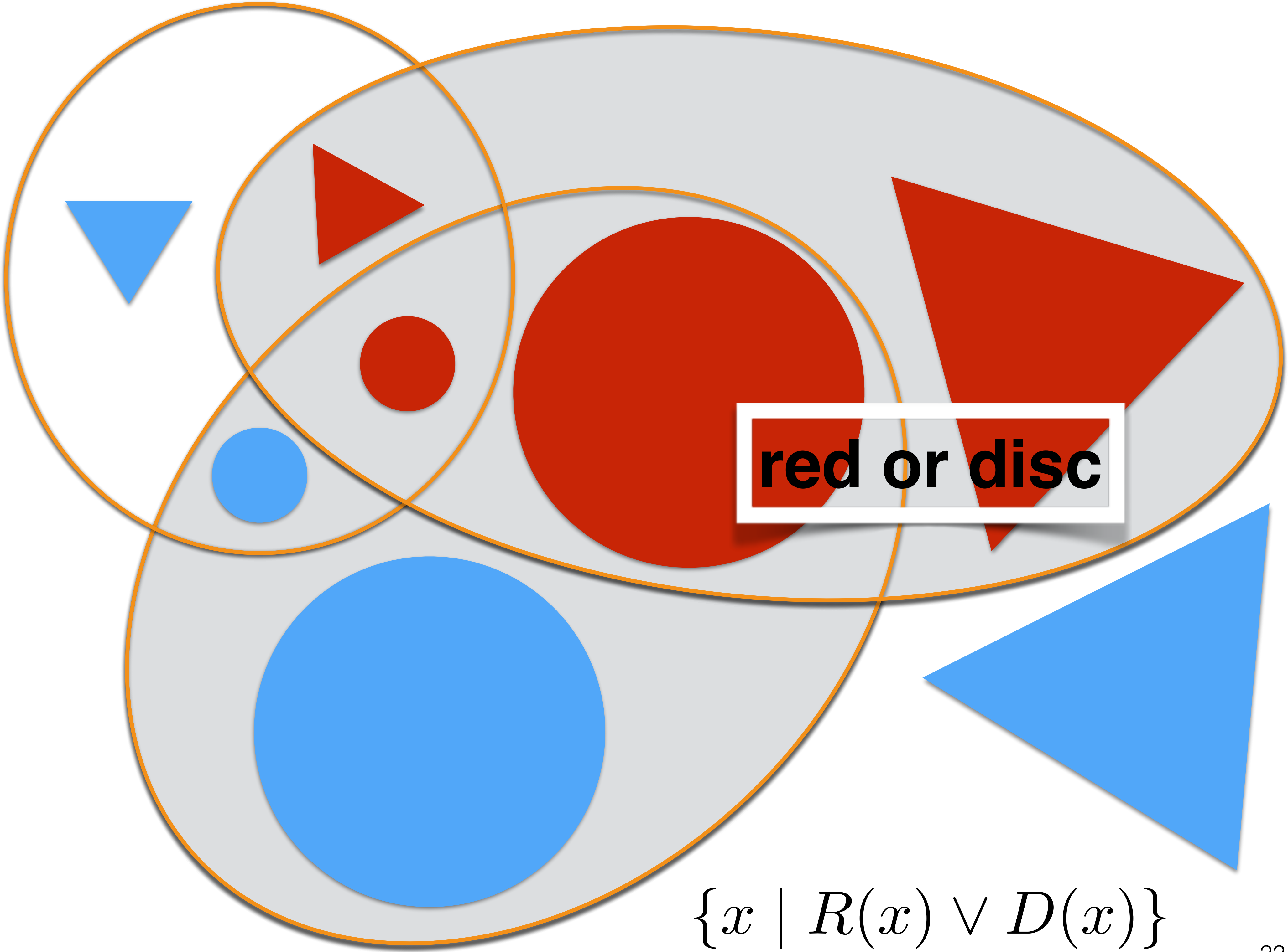
$$\{x \mid R(x)\}$$



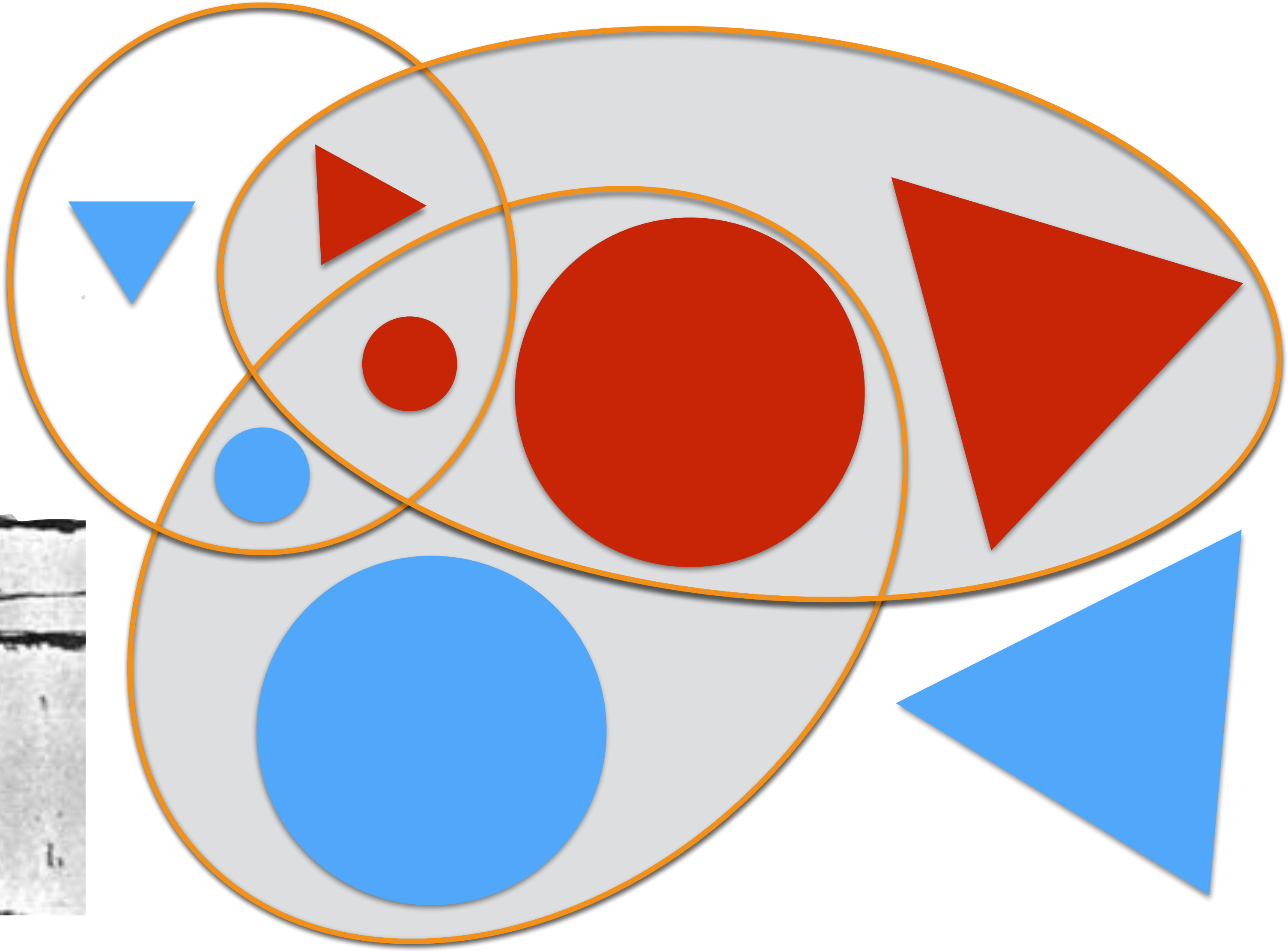
$$\{x \mid D(x)\}$$



$$\{x \mid S(x)\}$$



$$\{x \mid R(x) \vee D(x)\}$$

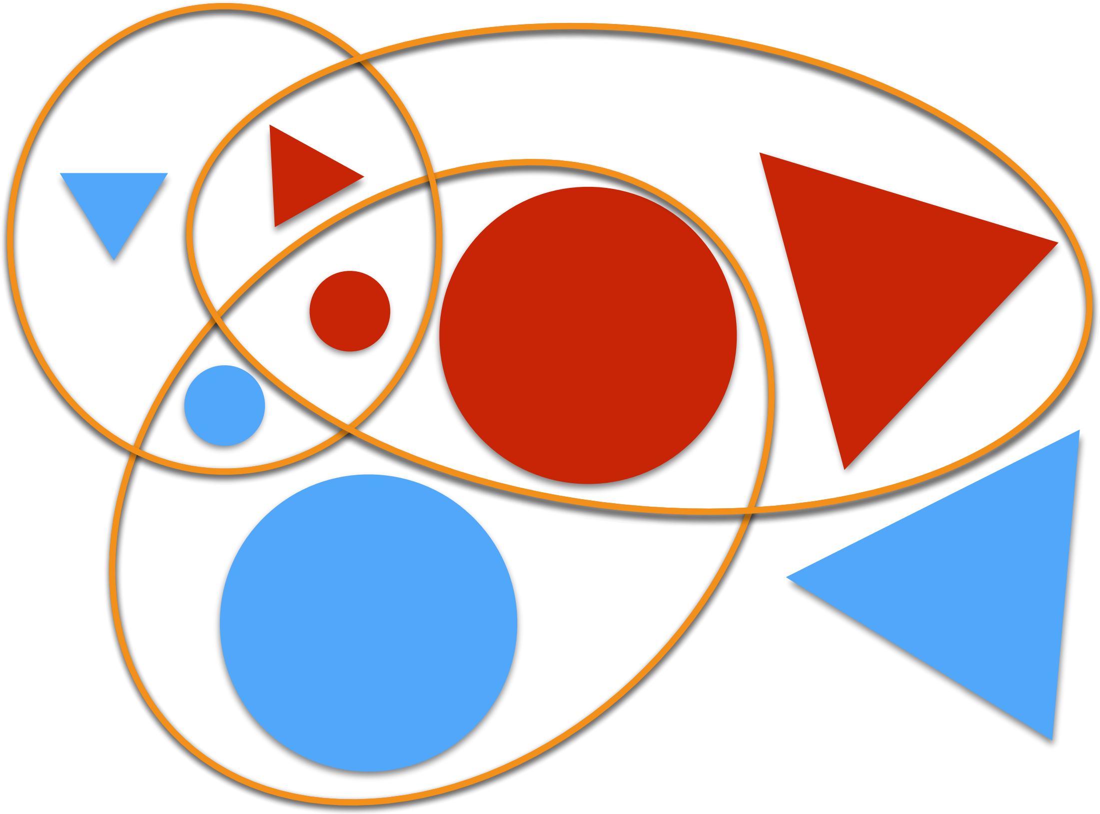


$$\neg(R(x) \vee D(x)) = (\neg R(x) \wedge \neg D(x))$$

not (red or disc) iff (not red and not disc)

Augustus de Morgan (1806 - 1871)

Exercise 1.1

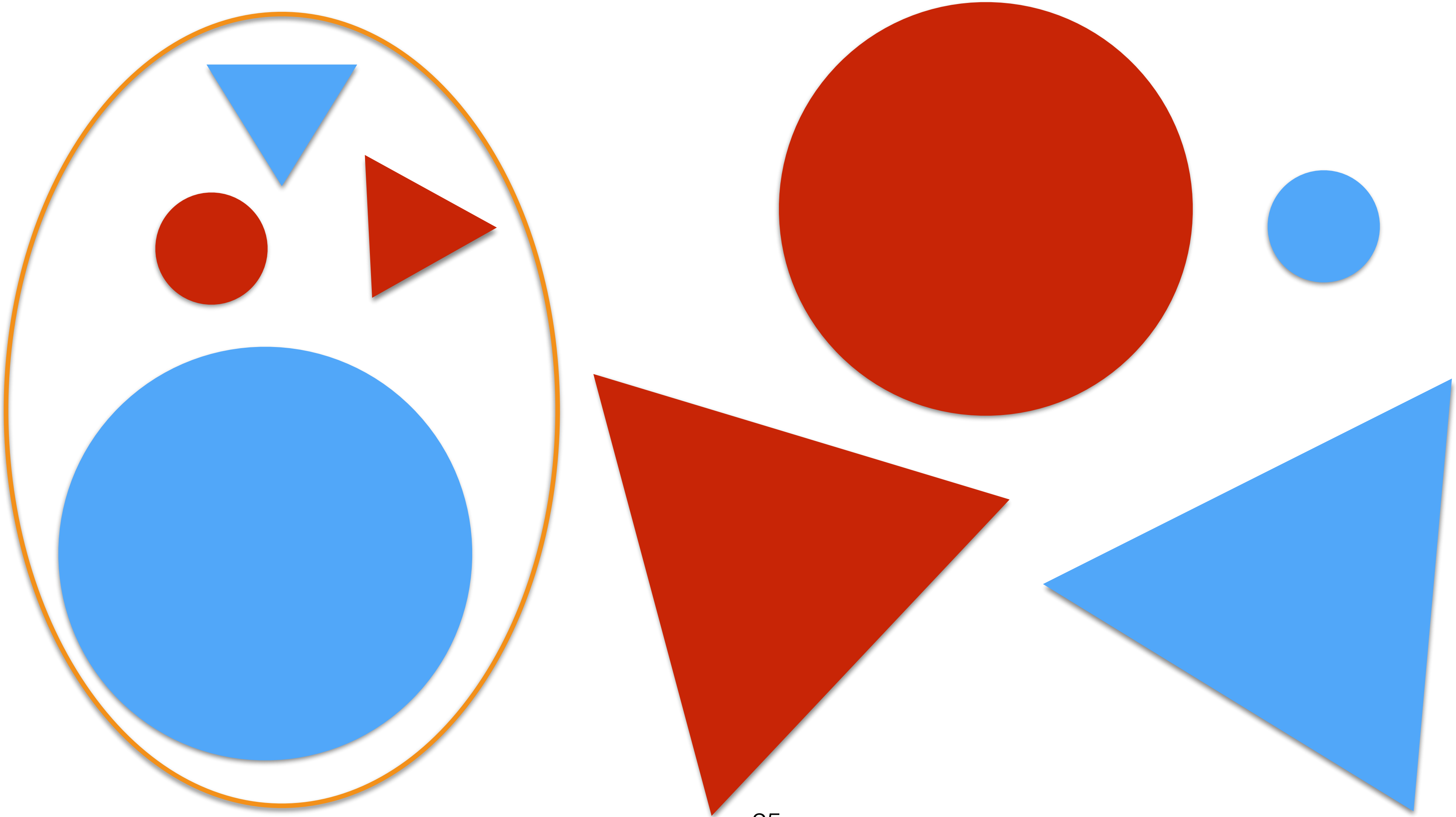


There are 8 shapes in the diagram. How many subsets of this set of 8 shapes are there?

Given any subset of the eight shapes can you write a complex proposition to which it corresponds, *using red, small and disc as primitives, and* **and**, **or**, *and* **not** *as connectives?*

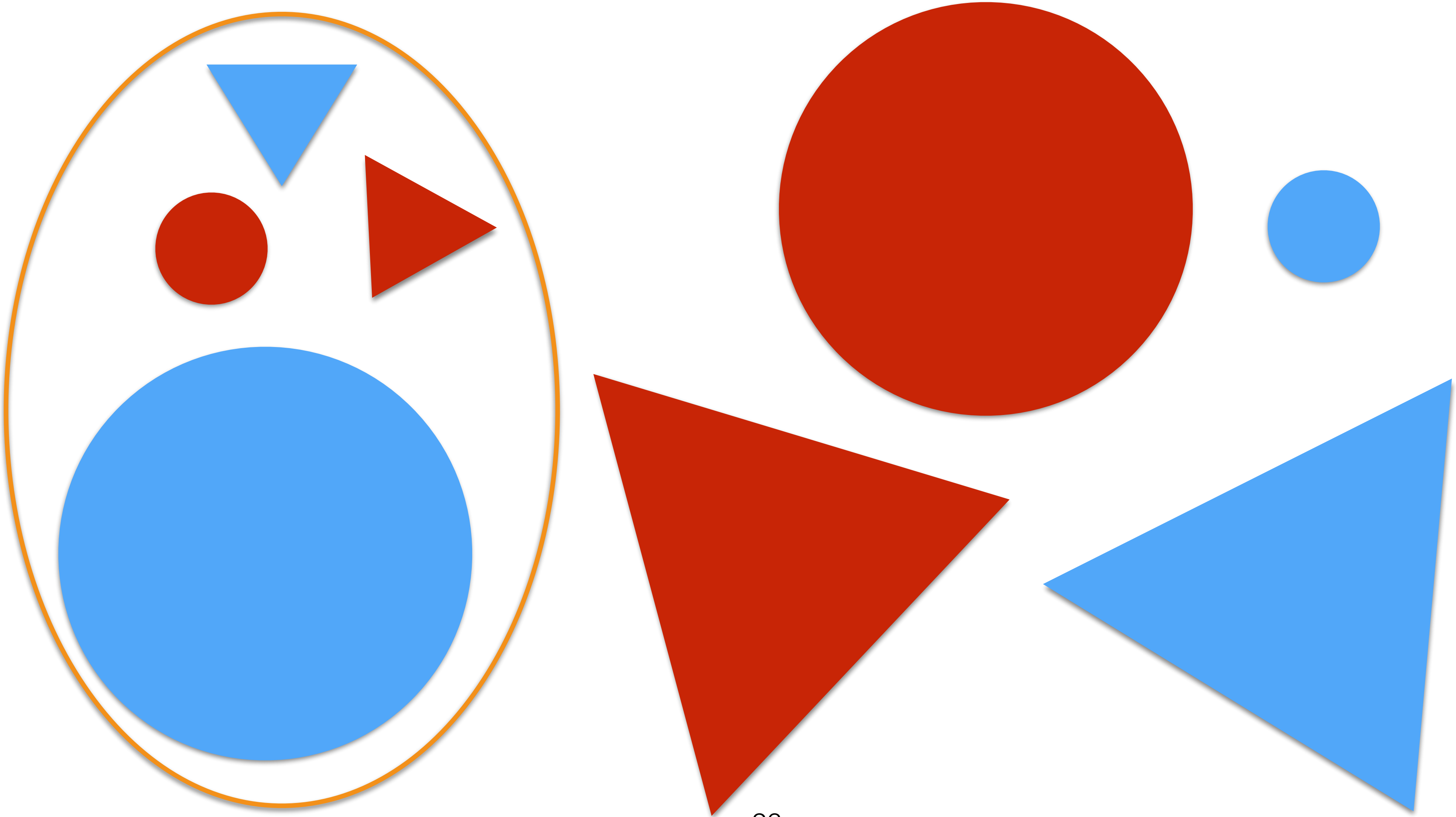
$\wedge$ $\vee$ $\neg$

Properties (yes-no questions) correspond to subsets



(red **and** small) **or** (blue **and** (disc **=** **not** small))

$$(R(x) \wedge S(x)) \vee (B(x) \wedge (D(x) \leftrightarrow \neg S(x)))$$

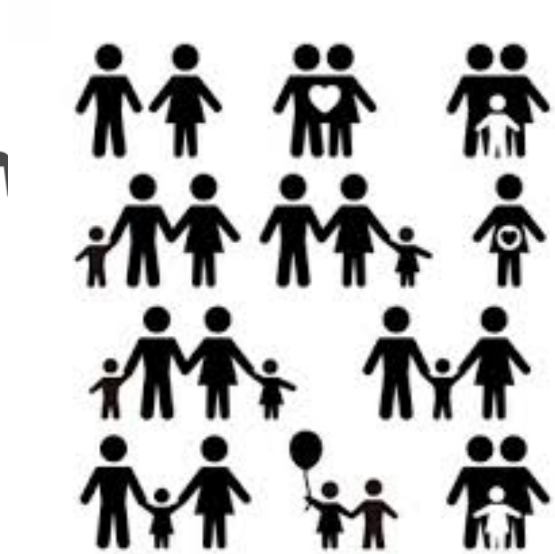


Sets of sets

An example:

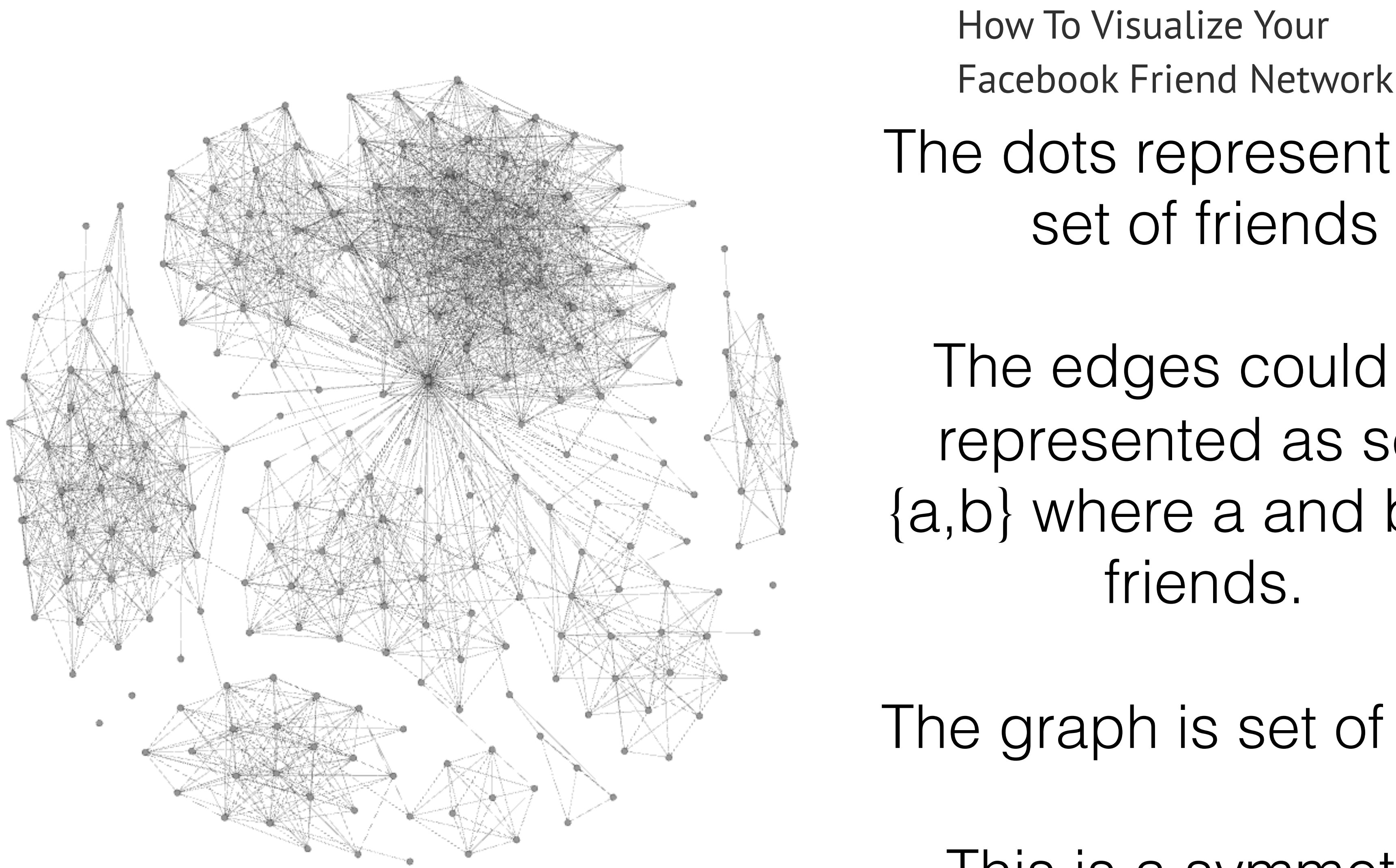
a family is a set of people

a set of families is
a set of sets of people



http://allthingsgraphed.com/2014/08/28/facebook-friends-network/

relations

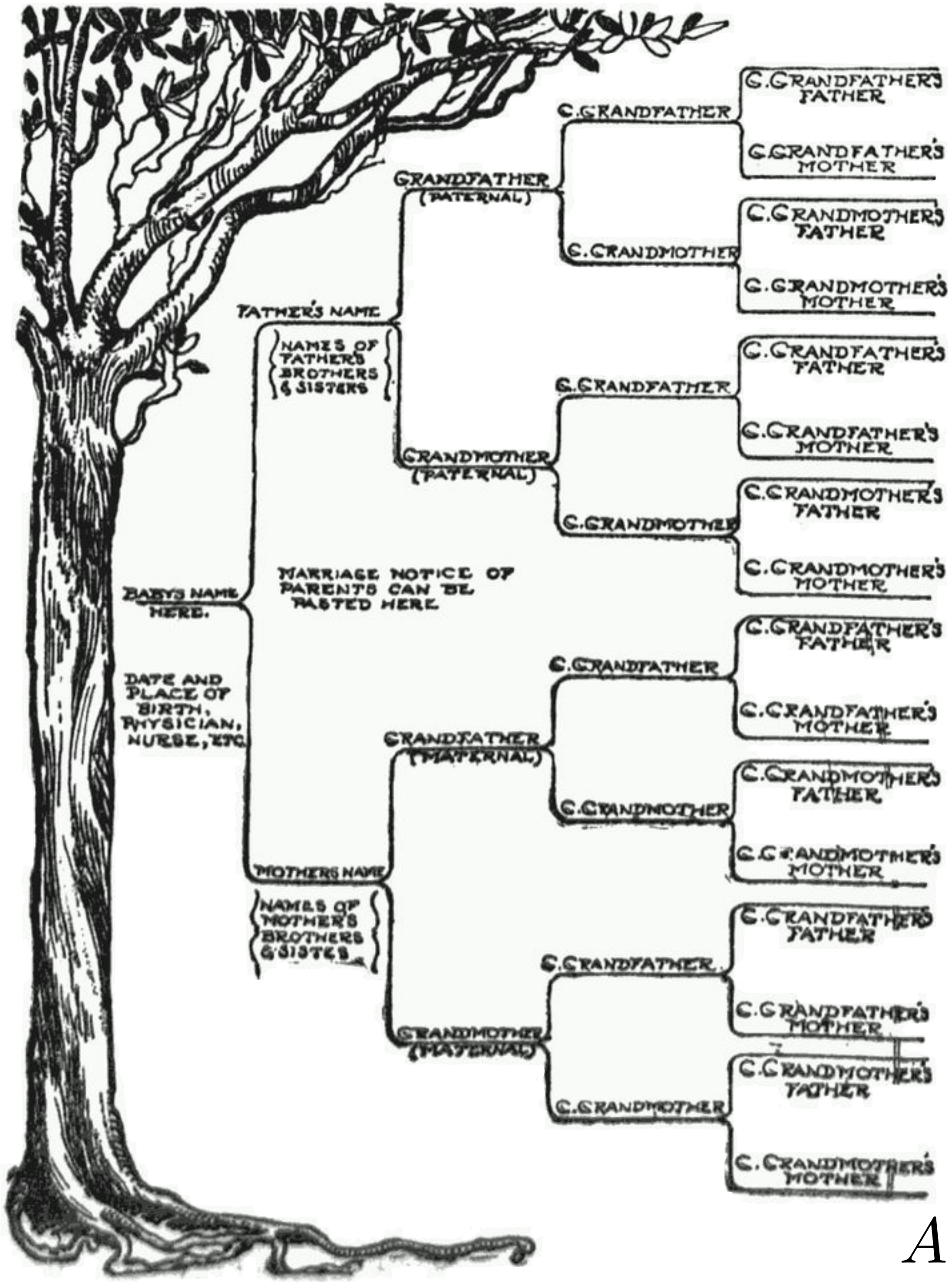


The dots represent your set of friends

The edges could be represented as sets {a,b} where a and b are friends.

The graph is set of sets.

This is a symmetric relation



A family tree

here the edges represent
child-parent relation
this is not symmetric

we represent this relation
as a set of **ordered** pairs

$$\langle a, b \rangle$$

If A and B are sets
the cartesian product

$$A \times B$$

is the set of ordered pairs

$$A \times B = \{\langle a, b \rangle \mid a \in A \wedge b \in B\}$$

(See MML Ch. 2)