Propositional connectives, satisfying values,
Get expression from venn diagram

Definitions (tautology, satisfiability, etc) and basic entailment and arrow rule

3-sat to 2-sat,
turning a cnf into arrow rule to count how many satisfying states there are,

Arrow rule, 2-sat (arrow rule)

Using boolean algebra to rearrange expressions in preparation for resolution
 cnf Converting to cnf

Dec2015, q3

Let's cover ways of algebraically simplifying expressions. in particular expanding "r ? a : g" turns into "(r && a) || (!r && g)" which expands into "(r || g) && (a || g) && (a || !r)", one of these clauses is obsolete, because it is included in the others, but i cannot see any algebraic way to determine this.

Entailment & Inference

What's the difference between the entailment symbol with the equals vs the entailment symbol with one line?
Single turnstile meaning

resolution, Resolution, entailment!!! Resolution

Writing regex for fsm

Examples of questions using arden's rule, Arden's rule arden's rule

Arden's rule, arrow rule (2-sat, especially with the amount of satisfying states), conversion to cnf
Arrow rule - number of true valuations

There was a question that looked like this : |{f | f : a -> b }| . can you explain what this function means?

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Single turnstile meaning

- Today

  - Boolean basics, arrow rule

  - Entailment, inference.

- Friday 24th

  - Resolution

- Thursday 30th

  - Arden's rule

- Friday 1st

  - ???

1. (a) For each of the following propositional expressions, use a truth table to determine whether it is tautologous, contradictory, or contingent.

     i. $((\neg B \rightarrow \neg A) \rightarrow B) \rightarrow B$

    ii. $((A \vee B) \rightarrow A) \rightarrow (B \rightarrow A)$

Boolean Algebra

$$\neg(a \rightarrow b) = a \wedge \neg b \qquad\qquad a \leftrightarrow b = (a \rightarrow b) \wedge (b \rightarrow a) \qquad a \rightarrow b = \neg a \vee b$$

$$\neg(a \vee b) = \neg a \wedge \neg b \qquad\qquad\qquad\qquad\qquad\qquad \neg(a \wedge b) = \neg a \vee \neg b$$

$$\neg 0 = 1 \qquad\qquad\qquad \neg\neg a = a \qquad\qquad\qquad \neg 1 = 0$$

$$a \vee 1 = 1 \qquad\qquad a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \qquad\qquad a \wedge 0 = 0$$

$$a \vee 0 = a \qquad\qquad a \vee \neg a = 1 \qquad a \wedge \neg a = 0 \qquad\qquad a \wedge 1 = a$$

Derived Operations

Definitions:

$$x \rightarrow y \equiv \neg x \vee y \qquad\qquad\qquad \text{implication}$$

$$x \leftarrow y \equiv x \vee \neg y$$

$$x \leftrightarrow y \equiv (\neg x \wedge \neg y) \vee (x \wedge y) \qquad\qquad \text{equality (iff)}$$

$$x \oplus y \equiv (\neg x \wedge y) \vee (x \wedge \neg y) \qquad\qquad \text{inequality (xor)}$$

Some equations:

$$x \leftrightarrow y = (x \rightarrow y) \wedge (x \leftarrow y)$$

$$x \oplus y = \neg(x \leftrightarrow y)$$

$$x \oplus y = \neg x \oplus \neg y$$

$$x \leftrightarrow y = \neg(x \oplus y)$$

$$x \leftrightarrow y = \neg x \leftrightarrow \neg y$$

3. (a) Express each of the following expressions in clausal form. Write your answers in the table provided for Q3c. (Please order the literals in each clause alphabetically.) [4 marks]

    i. $P \rightarrow (Q \lor R)$

    ii. $(P \land Q) \rightarrow \neg R$

    iii. $\neg P \rightarrow (R \rightarrow Q)$

    iv. $(Q \land \neg P) \rightarrow R$

2. You are given the following inference rules, due to Gentzen:

$$\overline{\Gamma, A \vdash \Delta, A} \ (I)$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \ (\wedge L) \qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta} \ (\vee R)$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta} \ (\vee L) \qquad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta} \ (\wedge R)$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \to B \vdash \Delta} \ (\to L) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \to B, \Delta} \ (\to R)$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta} \ (\neg L) \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta} \ (\neg R)$$

Here, A and B are propositional expressions, Γ, Δ are finite sets of expressions, and Γ, A is shorthand for $\Gamma \cup \{A\}$.

An entailment $\Gamma \vdash \Delta$ is *valid* iff every valuation that makes each expression in Γ true makes some expression in Δ true.

(a)   i. Explain what it means to say that a valuation V is a **counterexample** for an entailment, $\Gamma \vdash \Delta$. [*2 marks*]

ii. Show that the rule $(\to L)$ has the property that a valuation V is a counterexample for the rule's conclusion iff it is a counterexample to at least one of its assumptions. [*4 marks*]

(b) Use the Gentzen rules to show that [*4 marks*]

$$(Q \to P) \to Q \vdash Q$$

(c) Use the Gentzen rules to build an attempted proof of [*4 marks*]

$$(P \wedge Q) \to R, P \vee Q \vdash R$$

(d) Derive a counter-example from your attempted proof. (Show your working, and briefly justify your answer in terms of the particular properties of this set of rules.) [*2 marks*]

3. (a) Express each of the following expressions in clausal form. Write your answers in the table provided for Q3c. (Please order the literals in each clause alphabetically.) [4 marks]

  i. $P \rightarrow (Q \vee R)$
  ii. $(P \wedge Q) \rightarrow \neg R$
  iii. $\neg P \rightarrow (R \rightarrow Q)$
  iv. $(Q \wedge \neg P) \rightarrow R$

Use the Gentzen rules to determine whether there is a valuation satisfying the conjunction of these four expressions.