Computation and Logic
Traffic Lights
Michael Fourman
@mp4man
Traffic Light Signals

RED means 'Stop'. Wait behind the stop line on the carriageway.

RED AND AMBER also means ‘Stop’. Do not pass through or start until GREEN shows.

GREEN means you may go on if the way is clear. Take special care if you intend to turn left or right and give way to pedestrians who are crossing.

AMBER means ‘Stop’ at the stop line. You may go on only if the AMBER appears after you have crossed the stop line or are so close to it that to pull up might cause an accident.
logic & computation

red iff A or B
amber iff B or D
green iff C
current
A B C D
next
B C D A
\( R' = R \text{ xor } A = R \oplus A \)

\( A' = \text{ not } A = \neg A \)

\( G' = R \text{ and } A = R \land A \)

<table>
<thead>
<tr>
<th>( R )</th>
<th>( A )</th>
<th>( R \land A )</th>
<th>( R \oplus A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( A )</th>
<th>( \neg A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
\[ R' = R \text{ xor } A \]
\[ A' = \text{ not } A \]
\[ G' = R \text{ and } A \]
\( R' = R \text{ xor } A \)
\( A' = \text{ not } A \)
\( G' = R \text{ and } A \)
Exercise 1.2

\[ R' = R \text{ xor } A \]
\[ A' = G \text{ or } (R \text{ and not } A) \]
\[ G' = R \text{ and } A \]
Counting trains

A axle sensor (detects passing wheels)

from-a-to-b: a↓; b↓; a↑; b↑

from-b-to-a: b↓; a↓; b↑; a↑
Finite-state machines

axle sensor

inputs:
- $a^\uparrow$, $a^\downarrow$, $b^\uparrow$, $b^\downarrow$

outputs:
- from-a-to-b
- from-b-to-a
Hierarchical FSMs

carriage counter

inputs:
- a2b, b2a

outputs:
- A2B, B2A

\[ a2b = \text{from-a-to-b} \]
Application Fields

Industry
- real-time control, vending machines, cash dispensers, etc.

Electronic circuits
- data path / control path
- memory / cache handling
- protocols, USB, etc.

Communication protocols
- initiation and maintenance of communication links
- error detection and handling, packet retransmission

Language analysis
- natural languages
- programming languages
- search engines
A Decimal Number

![Diagram of a decimal number parser with nodes labeled 'S', 'd', '+', '−', 'ε', 'd', '·', 'd', and 'end'. The diagram shows alternative paths and repetition, with a 'skip' option.]