Informatics 1 CL

Lecture 11 Entailment

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Entailment

In algebra, we consider expressions with variables, and write equations to express relationships between different expressions.

LHS = RHS

Boolean algebra, with equalities between expressions, gives us one way to express relationships between different logical expressions.

If we want to study logical arguments it is more natural to consider entailments.

$$\text{LHS} \vdash \text{RHS}$$

Entailment

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$$\text{LHS} \vdash \text{RHS}$$

The entailment is **valid** if any valuation that makes everything on the LHS true, makes the RHS true

$$\vdash \text{RHS}$$

an entailment with empty LHS is **valid** iff RHS is a **tautology**

i.e. every valuation makes it true

Is this a valid argument?

- Assumptions:

  If the races are fixed or the gambling houses are crooked, then the tourist trade will decline.

  If the tourist trade declines then the police force will be happy.

  The police force is never happy.

- Conclusion:

  The races are not fixed

- Assumptions:
  If the races are fixed or the gambling houses are crooked, then the tourist trade will decline.
  If the tourist trade declines then the police force will be happy.
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- Conclusion:
  The races are not fixed

a deduction

$$\dfrac{(RF \lor GC) \to TT \qquad \dfrac{TT \to PH \qquad \neg PH}{\neg TT}}{\dfrac{\dfrac{\neg(RF \lor GC)}{\neg RF \land \neg GC}}{\neg RF}}$$

$$RF \lor GC \to TT,\ TT \to PH,\ \neg PH \vdash \neg RF$$

the deduction is summarised in an entailment

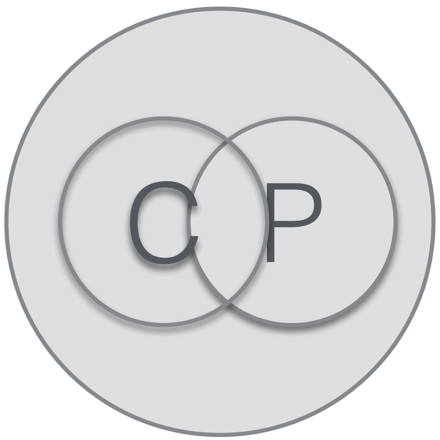
Is this a valid argument?

- Assumptions:
  If I am clever then I will pass
  If I will pass then I am clever,
  Either I am clever or I will pass

- Conclusion:
  I am clever and I will pass

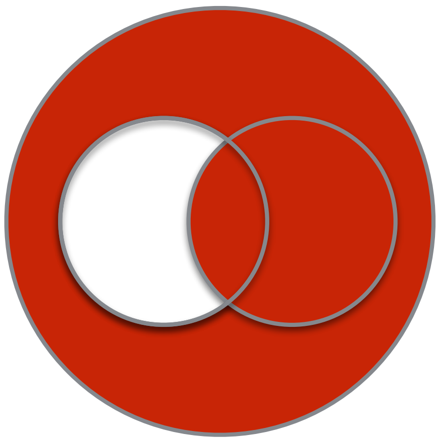
¿is this valid?

$$C \rightarrow P, P \rightarrow C, C \vee P \vdash C \wedge P$$

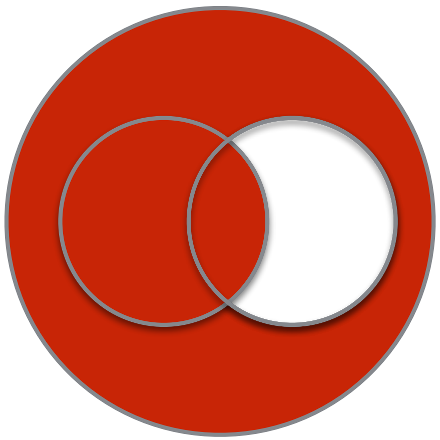


$$C \to P,\ P \to C,\ C \lor P \vdash C \land P$$

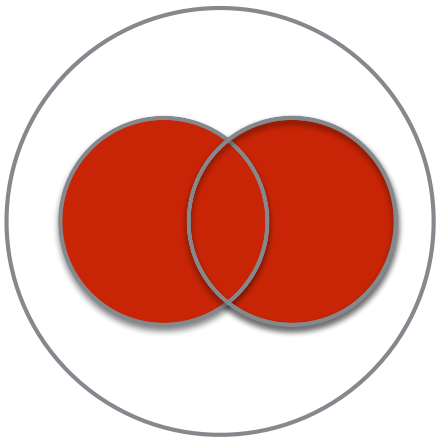
$C \to P$



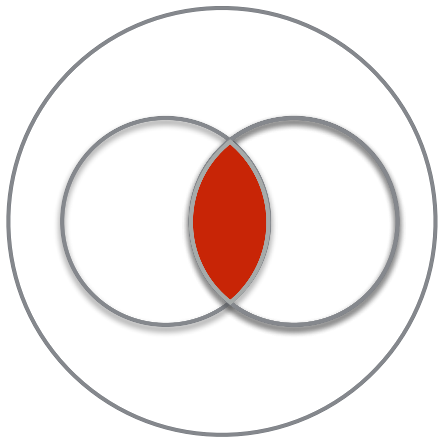
$P \to C$



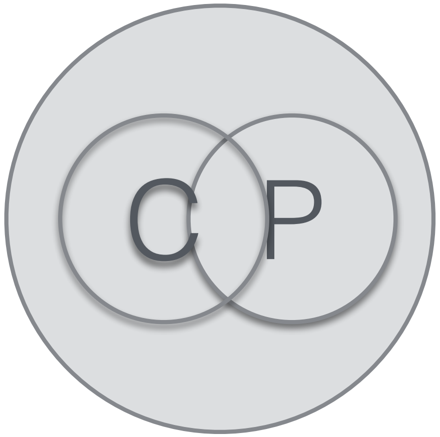
$C \lor P$



$C \land P$



Everything excluded by $C \land P$ is already excluded by one of the assumptions



$$C \rightarrow P, P \rightarrow C, C \vee P \vdash C \wedge P$$

Everything excluded by $C \wedge P$ is already excluded by one of the assumptions.

$\equiv$

Nothing excluded by $C \wedge P$ is allowed by all of the assumptions

States excluded by $C \wedge P$ satisfy $\neg(C \wedge P)$

So we show that

$$C \rightarrow P, P \rightarrow C, C \vee P, \neg(C \wedge P) \vdash$$

these constraints are inconsistent

Entailment

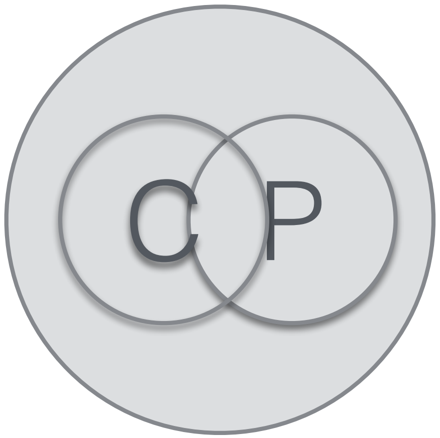
$$\text{LHS} \vdash \text{RHS}$$

The entailment is **valid** if any valuation that makes **everything** on the LHS true, makes **something** on the RHS true

$$\vdash \text{RHS}$$

an entailment with empty LHS is **valid** iff RHS is a **tautology**

an entailment with empty RHS is **valid** iff LHS is a **contradiction**



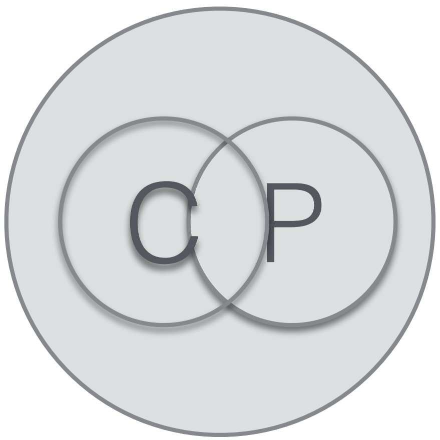
$$C \rightarrow P, P \rightarrow C, C \vee P \vdash C \wedge P$$

So we show that

$$C \rightarrow P, P \rightarrow C, C \vee P, \neg(C \wedge P) \vdash$$

these constraints are inconsistent

	C	P
$\neg C \vee P$	$\neg P \vee P$	
$\neg P \vee C$	P	$\{\}$
$C \vee P$	$\neg P$	
$\neg C \vee \neg P$	$\neg P \vee P$	



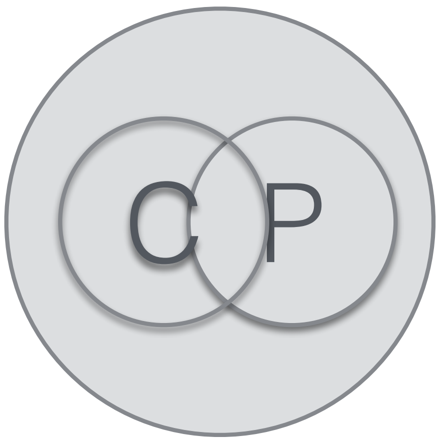
$$C \rightarrow P,\ P \rightarrow C,\ C \vee P \vdash C \wedge P$$

So we show that

$$C \rightarrow P,\ P \rightarrow C,\ C \vee P,\ \neg(C \wedge P) \vdash$$

these constraints are inconsistent

	C	P
$\neg C \vee P$	$\neg P \vee P$	
$\neg P \vee C$	P	$\{\}$
$C \vee P$	$\neg P$	
$\neg C \vee \neg P$	$\neg P \vee P$	

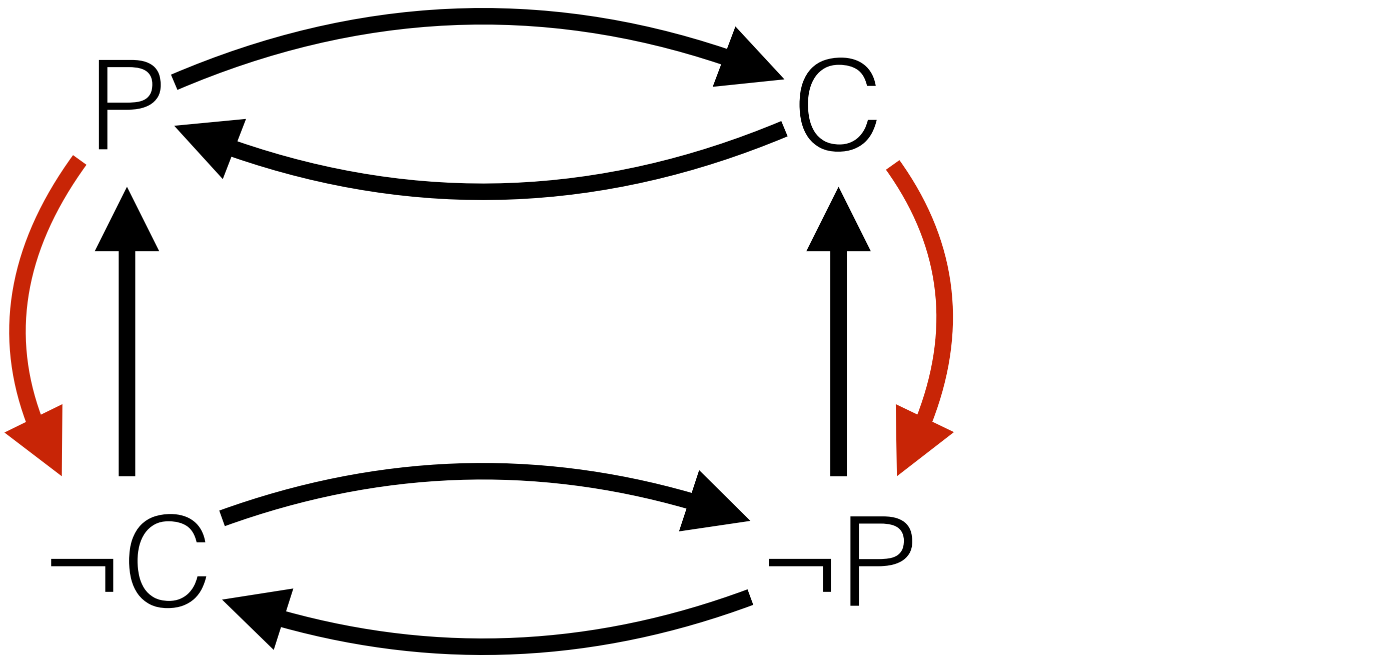


$$C \rightarrow P, P \rightarrow C, C \vee P \vdash C \wedge P$$

So we show that

$$C \rightarrow P, P \rightarrow C, C \vee P, \neg(C \wedge P)$$

is inconsistent



- Assumptions:

  If the races are fixed or the gambling houses are crooked, then the tourist trade will decline.

  If the tourist trade declines then the police force will be happy.

  The police force is never happy.

- Conclusion:

  The races are not fixed

a deduction

$$
\dfrac{(RF \vee GC) \to TT \qquad \dfrac{TT \to PH \qquad \neg PH}{\neg TT}}{\dfrac{\dfrac{\neg(RF \vee GC)}{\neg RF \wedge \neg GC}}{\neg RF}}
$$

RF ∨ GC → TT, TT → PH, ¬PH ⊢ ¬RF

the deduction is summarised in an entailment

$$\dfrac{\dfrac{\dfrac{(\mathrm{RF} \lor \mathrm{GC}) \to \mathrm{TT} \qquad \dfrac{\mathrm{TT} \to \mathrm{PH} \qquad \neg \mathrm{PH}}{\neg \mathrm{TT}}}{\neg(\mathrm{RF} \lor \mathrm{GC})}}{\neg \mathrm{RF} \land \neg \mathrm{GC}}}{\neg \mathrm{RF}}$$

$$\boxed{\mathrm{RF} \lor \mathrm{GC} \to \mathrm{TT},\ \mathrm{TT} \to \mathrm{PH},\ \neg \mathrm{PH} \vdash \neg \mathrm{RF}}$$

$\mathrm{RF} \lor \mathrm{GC} \to \mathrm{TT},\ \mathrm{TT} \to \mathrm{PH},\ \neg \mathrm{PH},\ \mathrm{RF} \vdash$

$\mathrm{RF} \to \mathrm{TT},\ \mathrm{GC} \to \mathrm{TT},\ \mathrm{TT} \to \mathrm{PH},\ \neg \mathrm{PH},\ \mathrm{RF} \vdash$

$\neg \mathrm{RF} \lor \mathrm{TT},\ \neg \mathrm{GC} \lor \mathrm{TT},\ \neg \mathrm{TT} \lor \mathrm{PH},\ \neg \mathrm{PH},\ \mathrm{RF} \vdash$

$\neg RF \lor TT, \neg GC \lor TT, \neg TT \lor PH, \neg PH , RF$

$\neg RF \lor TT$
$\neg GC \lor TT$
$\neg TT \lor PH$
$\neg PH$
RF

$$\neg RF \vee TT,\ \neg GC \vee TT,\ \neg TT \vee PH,\ \neg PH\ ,\ RF$$

PH

$\neg RF \vee TT$	$\neg TT$
$\neg GC \vee TT$	
$\cancel{\neg TT \vee PH}$	
$\cancel{\neg PH}$	
RF	

$\neg RF \vee TT,\ \neg GC \vee TT,\ \neg TT \vee PH,\ \neg PH\ ,\ RF$

PH	TT
$\neg \mathbf{RF} \vee \mathbf{TT}$	$\neg \mathbf{TT}$
$\neg \mathbf{GC} \vee \mathbf{TT}$	
$\cancel{\neg TT \vee PH}$	
$\cancel{\neg PH}$	
RF	

¬RF ∨ TT, ¬GC ∨ TT, ¬TT ∨ PH, ¬PH , RF

PH TT

¬RF ∨ **TT**	¬**TT**	**¬RF**
¬GC ∨ **TT**		**¬GC**
~~¬TT ∨ PH~~		
~~¬PH~~		
RF		

¬RF ∨ TT, ¬GC ∨ TT, ¬TT ∨ PH, ¬PH , RF

PH TT

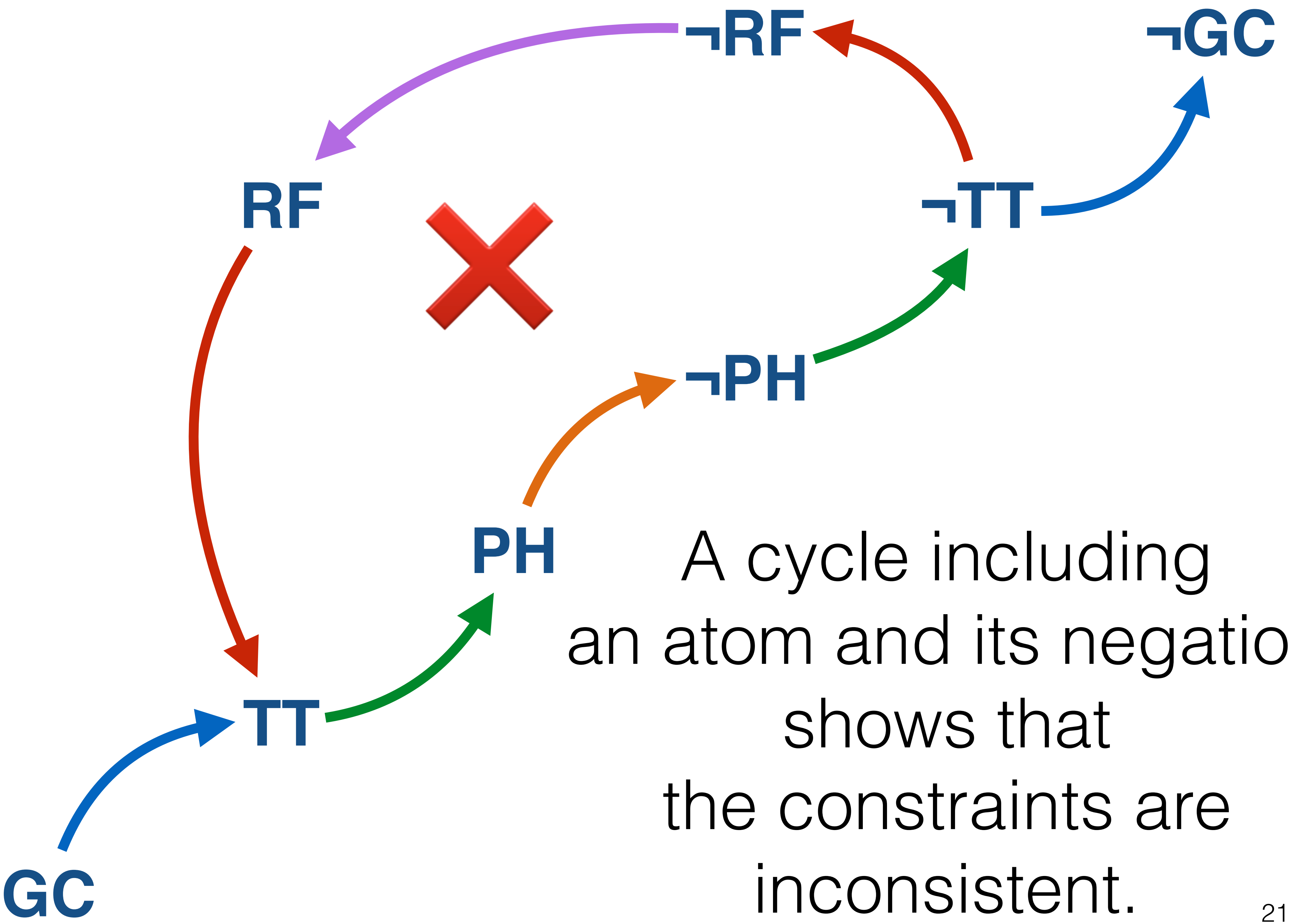
~~¬RF ∨ TT~~	~~¬TT~~	¬RF
~~¬GC ∨ TT~~		¬GC
~~¬TT ∨ PH~~		
~~¬PH~~		
RF		

$\neg RF \lor TT, \neg GC \lor TT, \neg TT \lor PH, \neg PH, RF$

PH	TT	RF	
~~¬RF ∨ TT~~	~~¬TT~~	**¬RF**	
~~¬GC ∨ TT~~		¬GC	{ }
~~¬TT ∨ PH~~			
~~¬PH~~			
RF			

A resolution proof shows that these constraints are inconsistent

$\neg RF \lor TT, \neg GC \lor TT, \neg TT \lor PH, \neg PH, RF$



A cycle including
an atom and its negation
shows that
the constraints are
inconsistent.