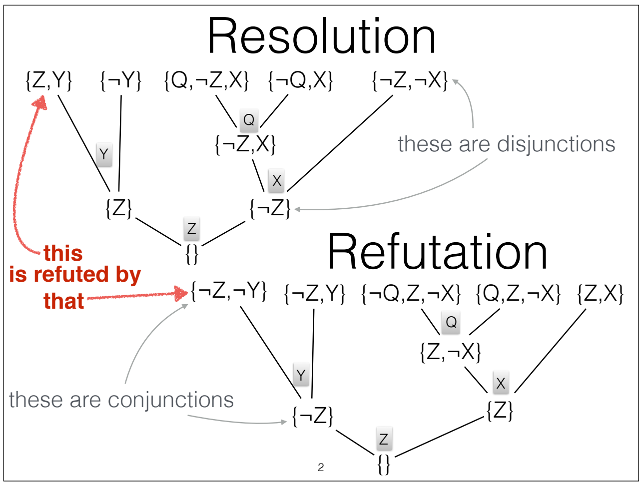
# Informatics 1

## Lecture 9 Searching for Satisfaction

Michael Fourman

In this lecture we consider formal descriptions of the relationships between a finite number of individuals. We may have different types of individual

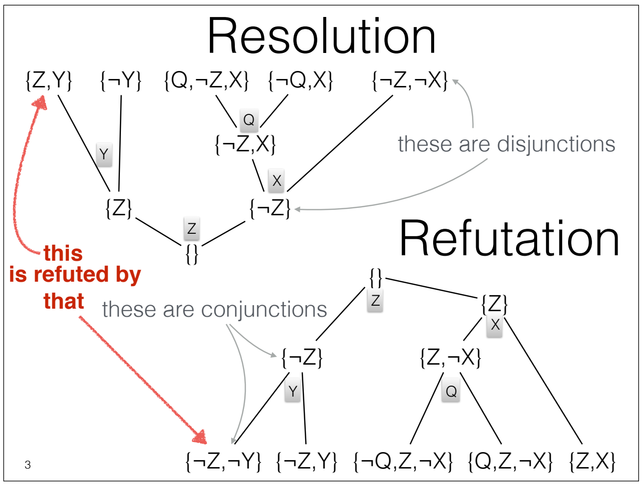



From the resolution proof we can derive a refutation.

The lower tree demonstrates the fact that whatever values we choose for the variables, we will arrive at a clause that is false for our chosen values.

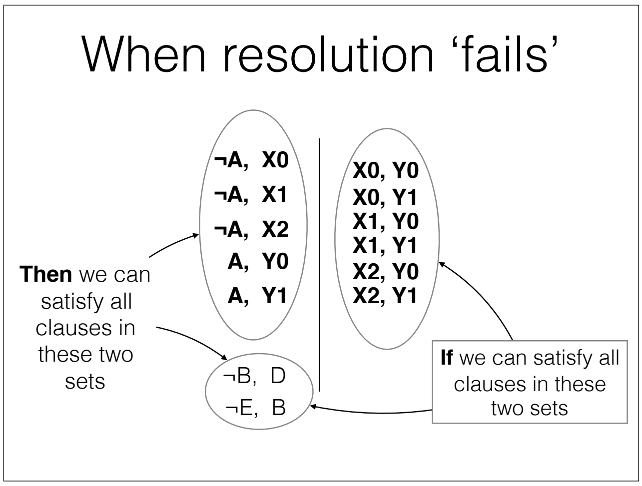
This suffices to show that, no matter what choice of values we make, the conjunction is false. The CNF is not satisfiable.

In the next lecture we will build the refutation tree directly, by searching for a satisfying valuation.

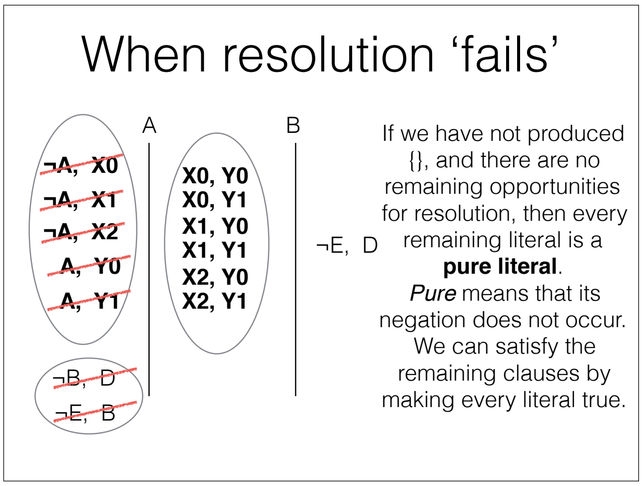



We normally grow refutation trees downwards.

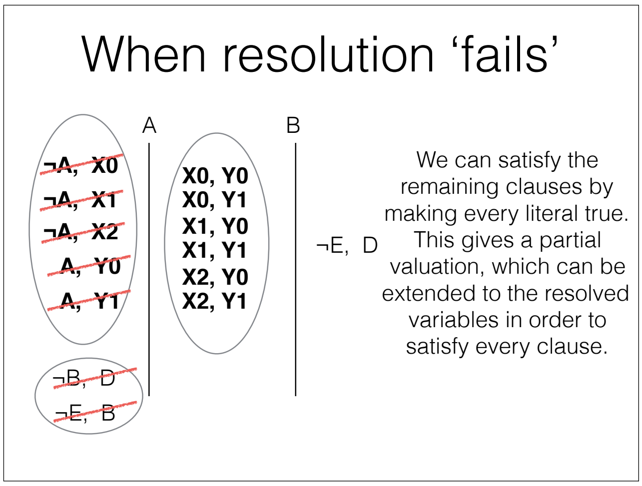
A refutation tree demonstrates the fact that whatever values we choose for the variables, we will arrive at a clause that is false for our chosen values. This suffices to show that, no matter what choice of values we make, the conjunction is false. The CNF is not satisfiable.




If we can satisfy all the Xs, then making A true will do the trick.
If we cannot satisfy Xi then we must be able to satisfy all the Ys, and so making A false will do the trick.




So, once we have resolved all the X, ¬X pairs, we can focus on clauses not mentioning A. Eventually we will either produce {}, or have a set of clauses with no complementary pairs.

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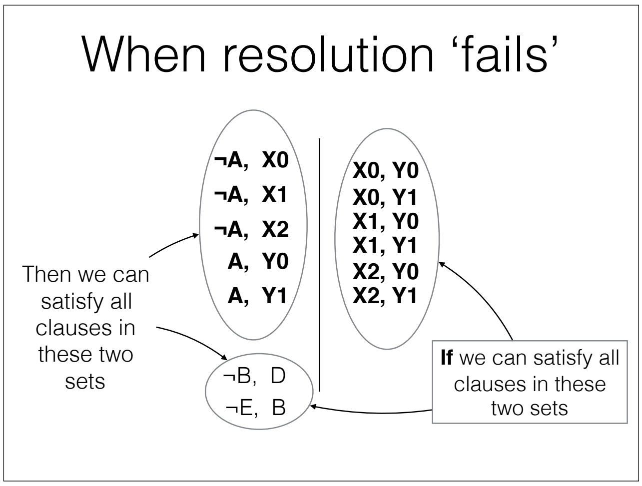
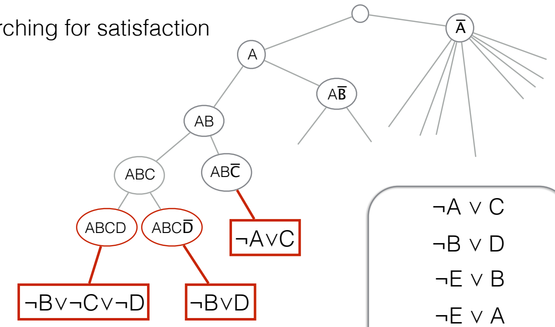



If we can satisfy all the Xs, then making A true will do the trick.
If we cannot satisfy Xi then we must be able to satisfy all the Ys, and so making A false will do the trick.

Searching for satisfaction




Finding Obstacles

$\neg A \vee C$

$\neg B \vee D$

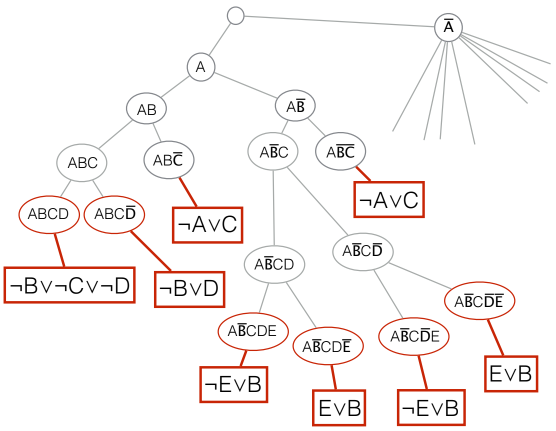
$\neg E \vee B$

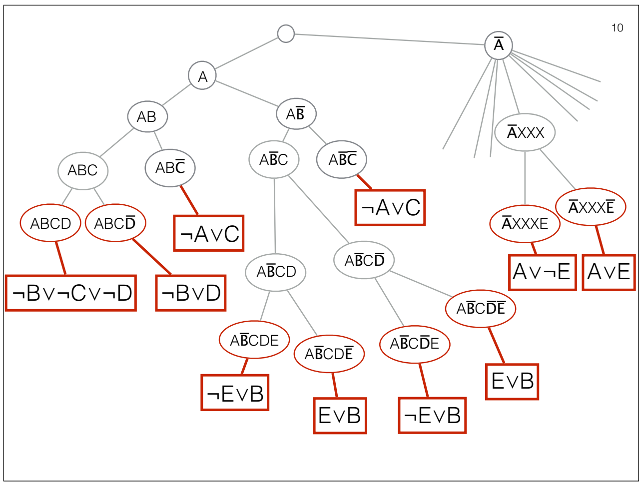
$\neg E \vee A$

$A \vee E$

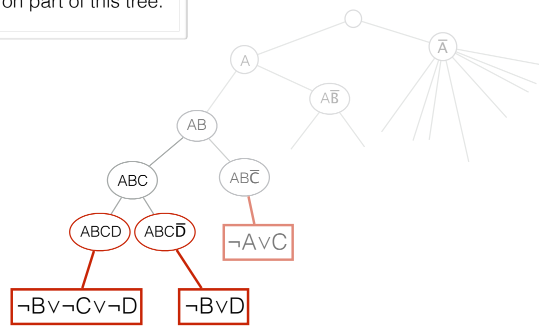
$E \vee B$

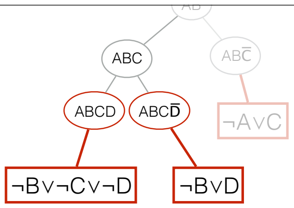
$\neg B \vee \neg C \vee \neg D$


A
Ā
AB
AB̄
ABC
ABC̄
AB̄C
AB̄C̄
ABCD
ABCD̄
¬A∨C
¬A∨C
AB̄CD
AB̄CD̄
¬B∨¬C∨¬D
¬B∨D
AB̄CDE
AB̄CDĒ
AB̄CD̄E
AB̄CD̄Ē
¬E∨B
E∨B
¬E∨B
E∨B


A
Ā
AB
AB̄
ABC
ABC̄
AB̄C
AB̄C̄
ĀXXX
ABCD
ABCD̄
¬B∨¬C∨¬D
¬B∨D
¬A∨C
¬A∨C
ĀXXXE
ĀXXXĒ
A∨¬E
A∨E
AB̄CD
AB̄CD̄
AB̄CDE
AB̄CDĒ
AB̄CD̄E
AB̄CD̄Ē
¬E∨B
E∨B
¬E∨B
E∨B

focus on part of this tree.





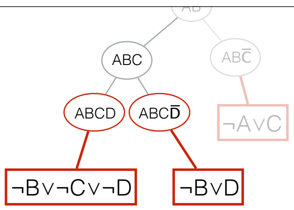

**Premises**

$$\frac{\neg B \lor \neg C \lor \mathbf{\neg D} \qquad \neg B \lor \mathbf{D}}{\mathbf{\neg B \lor \neg C}}$$

**Conclusion**

| | |
|---|---|
| *A valid inference* | Any assignment of truth values that makes all the premises true will make the conclusion true. |

The conclusion follows from the premises




**Premises**

¬B∨¬C∨**¬D** ¬B∨**D**

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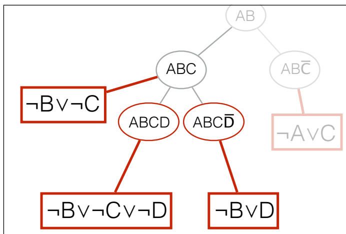
**¬B∨¬C**

**Conclusion**

| | |
|---|---|
| *For any valid inference* | Any assignment of truth values that makes the conclusion false will make at least one of the premises false. |




**Premises**

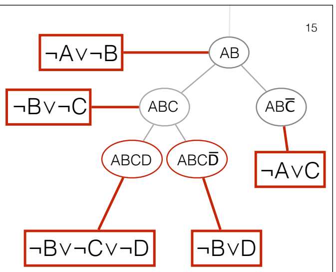
$$\frac{\neg B \vee \neg C \vee \mathbf{\neg D} \qquad \neg B \vee \mathbf{D}}{\mathbf{\neg B \vee \neg C}}$$

**Conclusion**

*A **special property** of this inference*

If some assignment XYZ of values for ABC makes the conclusion false then the assignments XYZ**D** and XYZ**D̄** each make one or other of the two premises false.

# Resolution




$$\dfrac{\neg A \lor \mathbf{C} \qquad \dfrac{\neg B \lor \neg C \lor \neg D \qquad \neg B \lor D}{\neg B \lor \mathbf{\neg C}}}{\neg B \lor \neg A}$$

# Resolution

$$\frac{U \lor \mathbf{V} \lor \mathbf{W} \lor \mathbf{X} \lor \neg\mathbf{C} \qquad X \lor \mathbf{Y} \lor \mathbf{Z} \lor \mathbf{C}}{U \lor \mathbf{V} \lor \mathbf{W} \lor X \lor \mathbf{Y} \lor \mathbf{Z}}$$

# Resolution

¬B∨¬C∨**¬D**   ¬B∨**D**

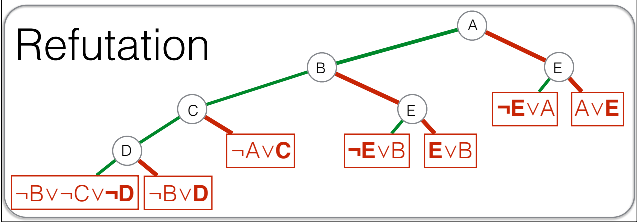
¬B∨**¬C**   ¬A∨**C**   **¬E**∨B   **E**∨B

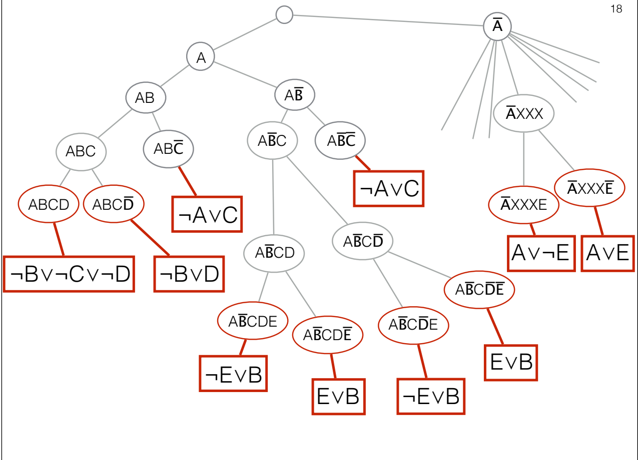
**¬B**∨¬A   **B**   **¬E**∨A   A∨**E**

**¬A**   **A**

⊥

# Refutation





A
$\bar{A}$
AB
A$\bar{B}$
ABC
AB$\bar{C}$
A$\bar{B}$C
A$\bar{B}\bar{C}$
$\bar{A}$XXX
ABCD
ABC$\bar{D}$
¬A∨C
¬A∨C
$\bar{A}$XXXE
$\bar{A}$XXX$\bar{E}$
¬B∨¬C∨¬D
¬B∨D
A$\bar{B}$CD
A$\bar{B}$C$\bar{D}$
A∨¬E
A∨E
A$\bar{B}$C$\bar{D}\bar{E}$
A$\bar{B}$CDE
A$\bar{B}$CD$\bar{E}$
A$\bar{B}$C$\bar{D}$E
¬E∨B
E∨B
¬E∨B
E∨B

Idea! Use the problem to simplify the search