Informatics 1

Lecture 8 Resolution

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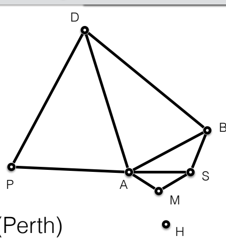
In this lecture we consider formal descriptions of the relationships between a finite number of individuals. We may have different types of individual

21 atoms	**M**elbourne	**S**ydney	**H**obart	**D**arwin	**P**erth	**A**delaide	**B**risbane
red	Mr	Sr	Hr	Dr	Pr	Ar	Br
green	Mg	Sg	Hg	Dg	Pg	Ag	Bg
amber	Ma	Sa	Ha	Da	Pa	Aa	Ba



eg:
Pr ≡ red(Perth)

34 clauses

1 for each node (eg D)

Dr ∨ Dg ∨Da

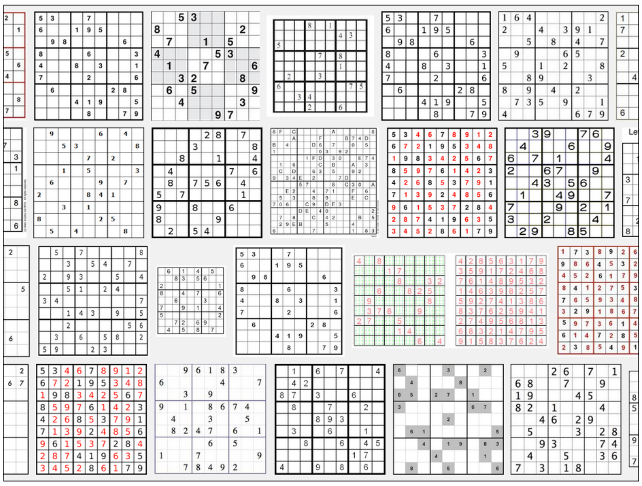
3 for each edge (eg D–B)

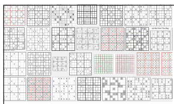
¬Dr ∨ ¬Br

¬Dg ∨ ¬Bg

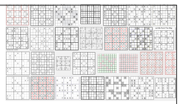
¬Da ∨ ¬Ba

We introduce atomic propositions Pr = red(Perth), and express the constraints





Sudoku



Squares i, j (i, j ∈ (1..9))
Numbers k (k ∈ (1..9))

729 (= 9^3) Atoms $\mathbf{p}_{i,j,k}$

$\mathbf{p}_{i,j,k}$
means
the number k is in square i,j

A sudoku problem is defined
by saying which numbers are in which squares

```
((p 1 2 3) and (p 1 6 1) and (p 2 3 6) and (p 2 8 5) and (p 3 1 5) and (p 3 7 9) and (p 3 8 8))
```

$(p_{1,2,3} \wedge p_{1,6,1} \wedge p_{2,3,6} \wedge p_{2,8,5} \wedge p_{3,1,5} \wedge p_{3,7,9} \wedge p_{3,8,8})$

```
((p 4 2 8) and (p 4 6 6) and (p 4 7 3) and (p 4 9 2) and (p 5 5 5) and (p 6 1 9) and (p 6 3 3) and (p 6 4 8) and (p 6 8 6))
```

$(p_{4,2,8} \wedge p_{4,6,6} \wedge p_{4,7,3} \wedge p_{4,9,2} \wedge p_{5,5,5} \wedge p_{6,1,9} \wedge p_{6,3,3} \wedge p_{6,4,8} \wedge p_{6,8,6})$

```
((p 7 1 7) and (p 7 2 1) and (p 7 3 4) and (p 7 9 9) and (p 8 2 2) and (p 8 7 8) and (p 9 4 4) and (p 9 8 3))
```

$(p_{7,1,7} \wedge p_{7,2,1} \wedge p_{7,3,4} \wedge p_{7,9,9} \wedge p_{8,2,2} \wedge p_{8,7,8} \wedge p_{9,4,4} \wedge p_{9,8,3})$

```
(bigand i (1 .. 9) (bigand j (1 .. 9) (bigand n (1 .. 9) (bigand m (1 .. 9) (m diff n) ((p i j n) imply (not (p i j m)))))))
```

$$\bigwedge_{i\in\{1,..,9\}} \bigwedge_{j\in\{1,..,9\}} \bigwedge_{n\in\{1,..,9\}} \bigwedge_{m\in\{1,..,9\}|(m\neq n)} (p_{i,j,n} \rightarrow \neg p_{i,j,m})$$

at most one number per square

```
(bigand n (1 .. 9) (bigand i (1 .. 9) (bigor j (1 .. 9) (p i j n))))
```

$$\bigwedge_{n\in\{1,..,9\}} \bigwedge_{i\in\{1,..,9\}} \bigvee_{j\in\{1,..,9\}} p_{i,j,n}$$

every number occurs in each row

```
(bigand n (1 .. 9) (bigand j (1 .. 9) (bigor i (1 .. 9) (p i j n))))
```

$$\bigwedge_{n\in\{1,..,9\}} \bigwedge_{j\in\{1,..,9\}} \bigvee_{i\in\{1,..,9\}} p_{i,j,n}$$

every number occurs in each column

```
(bigand n (1 .. 9) (bigor i (1 .. 3) (bigor j (1 .. 3) (p i j n))))
```

$$\bigwedge_{n\in\{1,..,9\}} \bigvee_{i\in\{1,..,3\}} \bigvee_{j\in\{1,..,3\}} p_{i,j,n}$$

every number occurs in top-left square

```
(bigand n (1 .. 9) (bigor i (4 .. 6) (bigor j (1 .. 3) (p i j n))))
```

$$\bigwedge_{n\in\{1,..,9\}} \bigvee_{i\in\{4,..,6\}} \bigvee_{j\in\{1,..,3\}} p_{i,j,n}$$

every number occurs in top-centre square

```
(bigand n (1 .. 9) (bigor i (7 .. 9) (bigor j (1 .. 3) (p i j n))))
```

$$\bigwedge_{n\in\{1,..,9\}} \bigvee_{i\in\{7,..,9\}} \bigvee_{j\in\{1,..,3\}} p_{i,j,n}$$

every number occurs in top-right square

```
(bigand n (1 .. 9) (bigor i (1 .. 3) (bigor j (4 .. 6) (p i j n))))
```

$$\bigwedge_{n\in\{1,..,9\}} \bigvee_{i\in\{1,..,3\}} \bigvee_{j\in\{4,..,6\}} p_{i,j,n}$$

every number occurs in middle-left square

```
(bigand n (1 .. 9) (bigor i (4 .. 6) (bigor j (4 .. 6) (p i j n))))
```

$$\bigwedge_{n\in\{1,..,9\}} \bigvee_{i\in\{4,..,6\}} \bigvee_{j\in\{4,..,6\}} p_{i,j,n}$$

```
(bigand n (1 .. 9) (bigor i (7 .. 9) (bigor j (4 .. 6) (p i j n))))
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$$\bigwedge_{n\in\{1,..,9\}} \bigvee_{i\in\{7,..,9\}} \bigvee_{j\in\{4,..,6\}} p_{i,j,n}$$

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(bigand n (1 .. 9) (bigor i (7 .. 9) (bigor j (7 .. 9) (p i j n))))
```
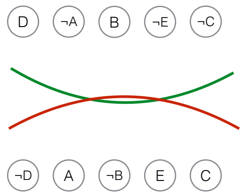

$$\bigwedge_{n\in\{1,..,9\}} \bigvee_{i\in\{7,..,9\}} \bigvee_{j\in\{7,..,9\}} p_{i,j,n}$$

729 atoms
structural constraints include
many, many occurrences of literals
How Many?

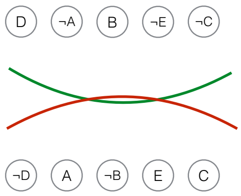
A **valuation** makes some atoms true and the rest false.
Once we have a valuation, for each atom, we can compute the truth value of every expression.
If an atom is true its negation is false, and vice versa.



We draw a line to visualise a valuation, placing the true literals above the line, and the false literals below it.

Every binary constraint

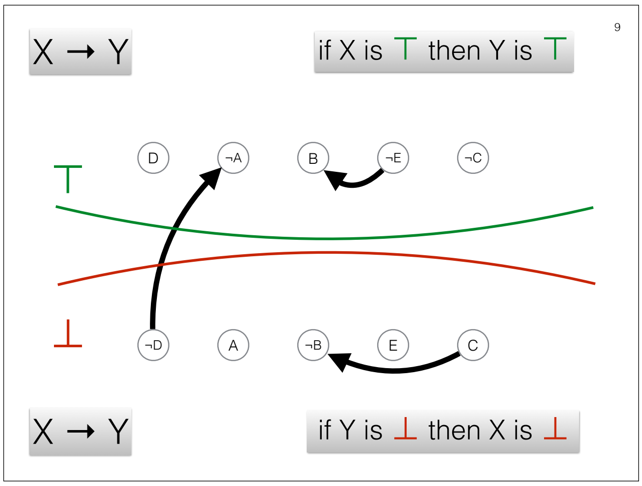
We draw a line to visualise a valuation, placing the true literals above the line, and the false literals below it.



An implication between literals is represented by an arrow.

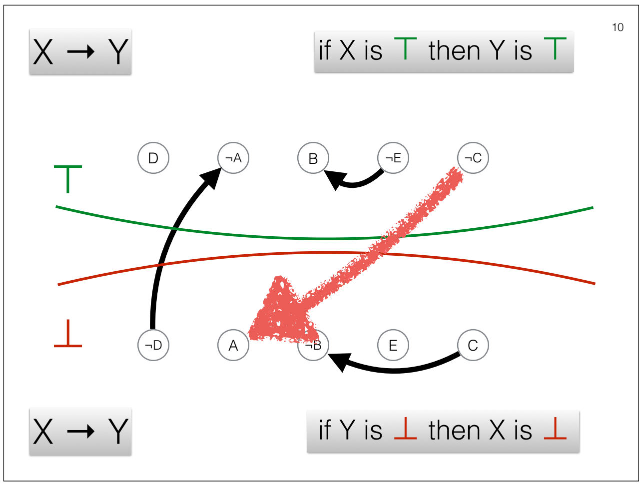
The valuation makes the implication true, unless the arrow goes from true to false.

Every binary constraint



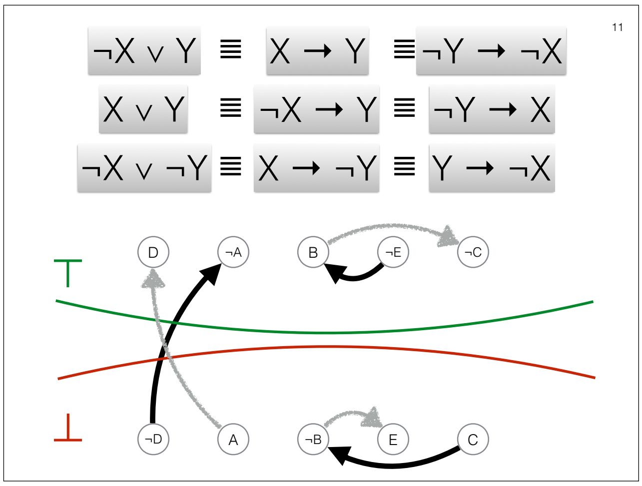
This valuation makes B and D true, and A, C,and E false.

It makes ¬D → ¬A, C→ ¬B, and ¬E → B true.



This valuation makes B and D true, and A, C, and E false.

It makes ¬D → ¬A, C→ ¬B, and ¬E → B true, and ¬C → A is false



The arrows actually come in pairs, since each arrow is just one way of expressing a binary constraint:

A

12

¬A ∨ C

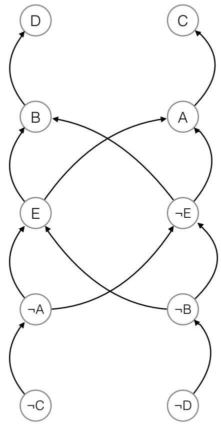
¬B ∨ D

¬E ∨ B

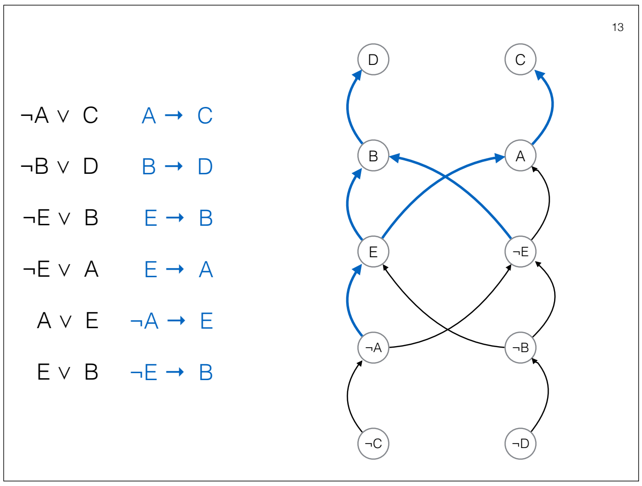
¬E ∨ A

A ∨ E

E ∨ B



If we start with the constraints, we can draw the diagram



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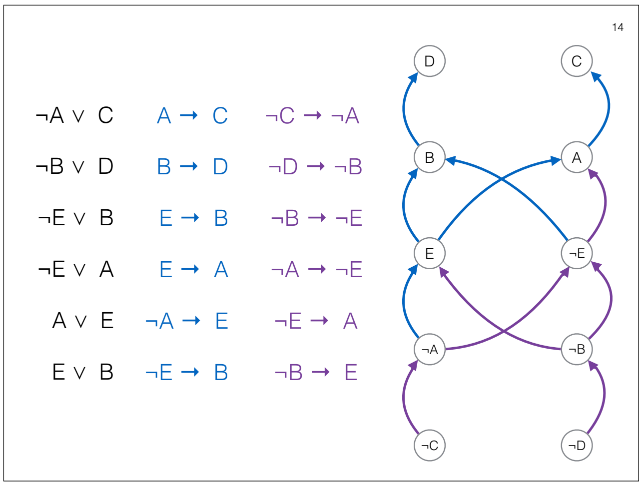
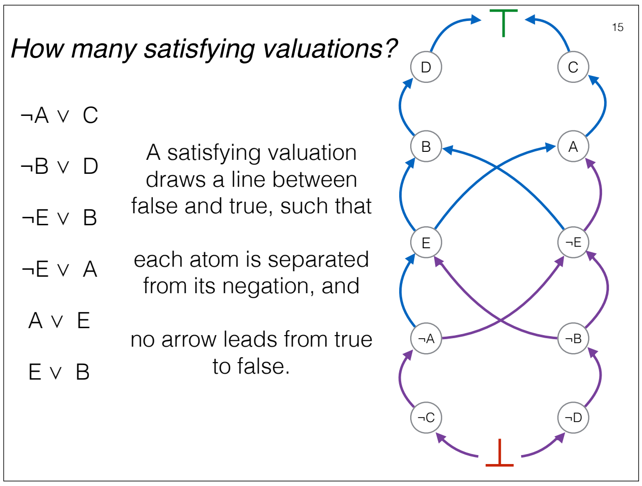


If we start with the constraints, we can draw the diagram. The diagram shows us how the constraints fit together. What if we just want to calculate?



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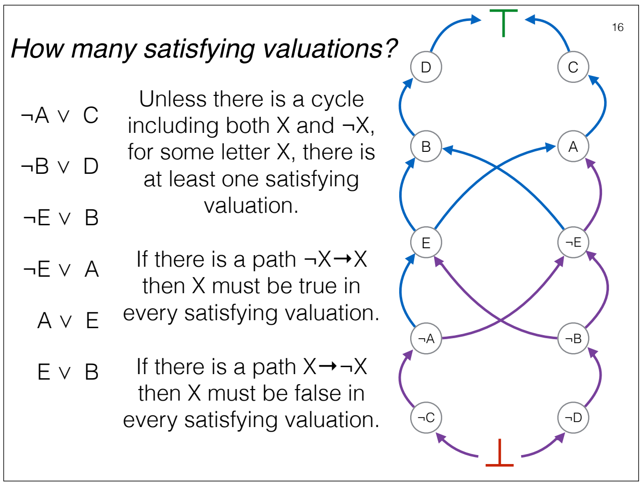

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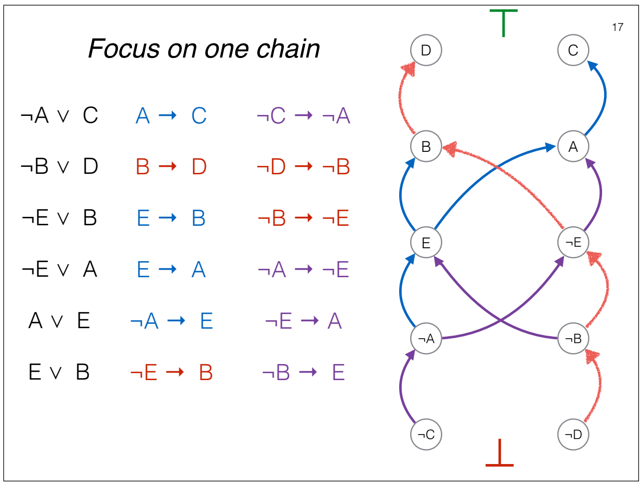
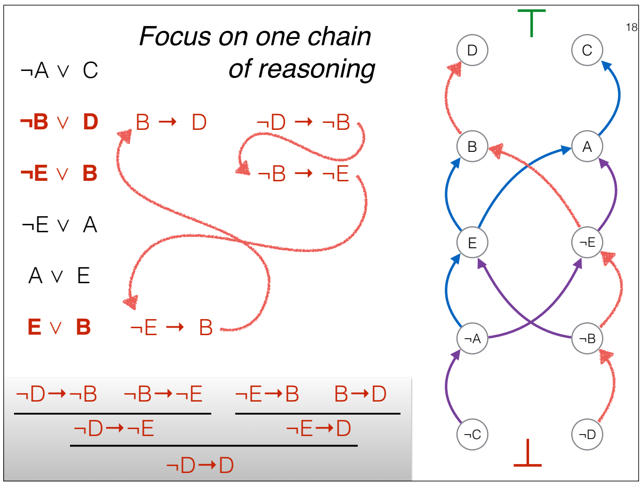


The diagram makes us see chains of reasoning



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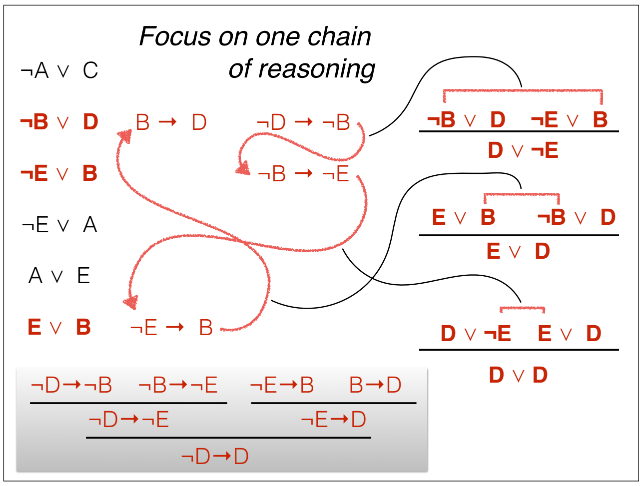
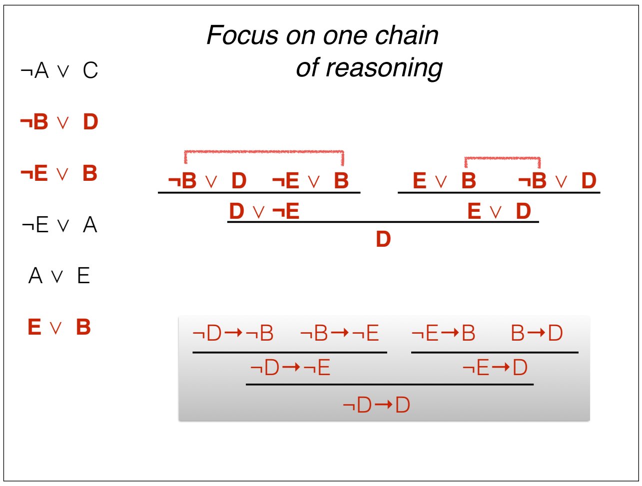


The diagram makes us see chains of reasoning.

We add more constraints, corresponding to the transitive closure of our set of arrows.

Notice that we can use the same constraint.



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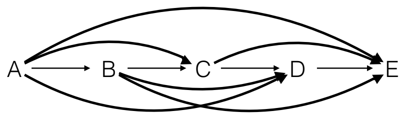
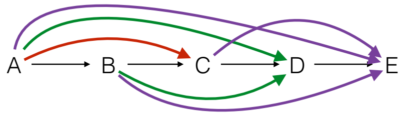


$$\dfrac{\dfrac{\mathbf{\neg A} \lor \mathbf{B} \qquad \mathbf{\neg B} \lor \mathbf{C}}{\mathbf{\neg A} \lor \mathbf{C}} \qquad \dfrac{\mathbf{\neg C} \lor \mathbf{D} \qquad \mathbf{\neg D} \lor \mathbf{E}}{\mathbf{\neg C} \lor \mathbf{E}}}{\mathbf{\neg A} \lor \mathbf{E}}$$

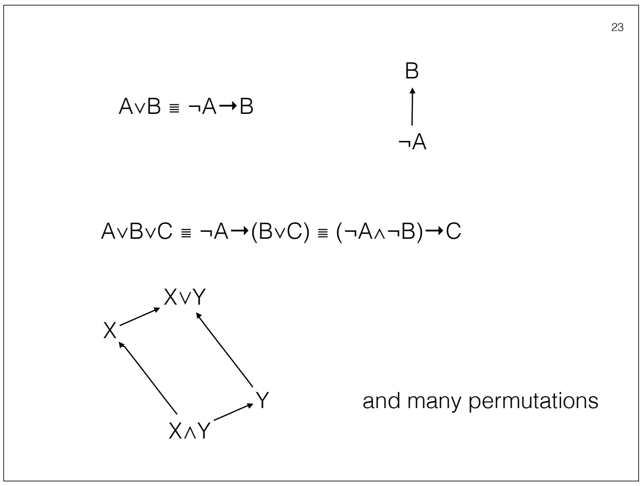
$$\dfrac{\neg A \lor C \qquad \neg C \lor D}{\mathbf{\neg A} \lor \mathbf{D}} \qquad\qquad \dfrac{\neg B \lor C \qquad \neg C \lor E}{\mathbf{\neg B} \lor \mathbf{E}}$$

$$\dfrac{\neg B \lor C \qquad \neg C \lor D}{\mathbf{\neg B} \lor \mathbf{D}}$$



	B	C	D
¬A∨B	**¬A∨C**	**¬A∨D**	**¬A∨E**
¬B∨C		**¬B∨D**	**¬B∨E**
¬C∨D			**¬C∨E**
¬D∨E			

We keep adding clauses obtained by resolution.
Davis Putnam - choose a variable then add all instances.
Different orders for resolution will give the same results.



Once we have more than 2 literals in a clause things get more complicated.

Premises

$X \vee \neg D$ $Y \vee D$

$X \vee Y$

Conclusion

A valid inference	Any assignment of truth values that makes all the premises true will make the conclusion true.

The conclusion follows from the premises

Premises

$X \lor \neg D$ $\quad$ $Y \lor D$

$X \lor Y$

Conclusion

For any valid inference	Any assignment of truth values that makes the conclusion false will make at least one of the premises false.

Premises

$$\frac{X \lor \neg D \qquad Y \lor D}{X \lor Y}$$

Conclusion

where D does not occur in X or Y

*A **special property** of this inference*

If some assignment abc of values for ABC makes the conclusion false then the assignments abc⊤ and abc⊥ for ABCD each make one or other of the two premises false.

Resolution

$$\frac{U \lor \mathbf{V} \lor \mathbf{W} \lor \mathbf{X} \lor \neg\mathbf{C} \qquad\qquad X \lor \mathbf{Y} \lor \mathbf{Z} \lor \mathbf{C}}{U \lor \mathbf{V} \lor \mathbf{W} \lor X \lor \mathbf{Y} \lor \mathbf{Z}}$$

(A?B:C)