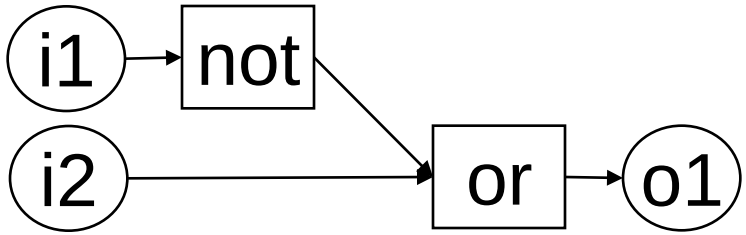
Propositional Logic

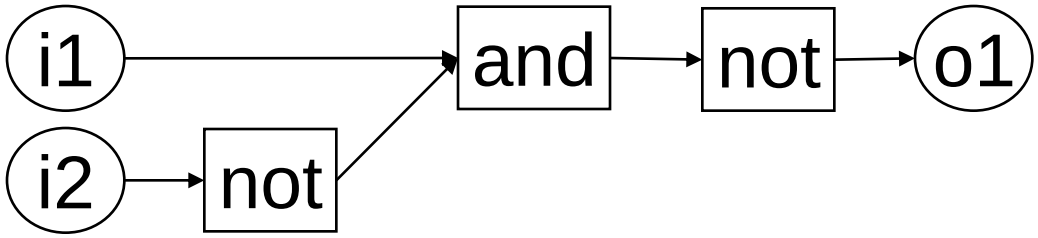
In this lecture we solve a standard propositional problem of circuit equivalence and a puzzle about common knowledge.

Problems and puzzles crystallize larger issues, eg., consistency and co-ordination

Problem: Logic Circuits



Are these two circuits equivalent ?



Problem: Common Knowledge

A husband unfaithful to his wife will be shot by her on the day she finds out.

Day 1: Three wives are told that at least one of their husbands is unfaithful. None knows whether her husband is unfaithful but each knows about the others.

Day 2: The three wives meet again and learn that no husband has been shot.

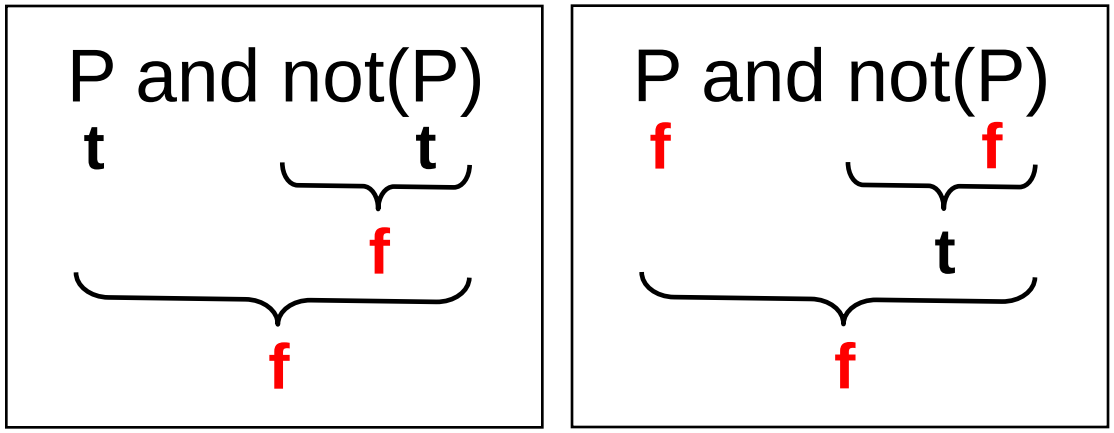
Day 3: The three wives meet again and learn that no husband has been shot. That night there is shooting.

How many husbands were unfaithful ?

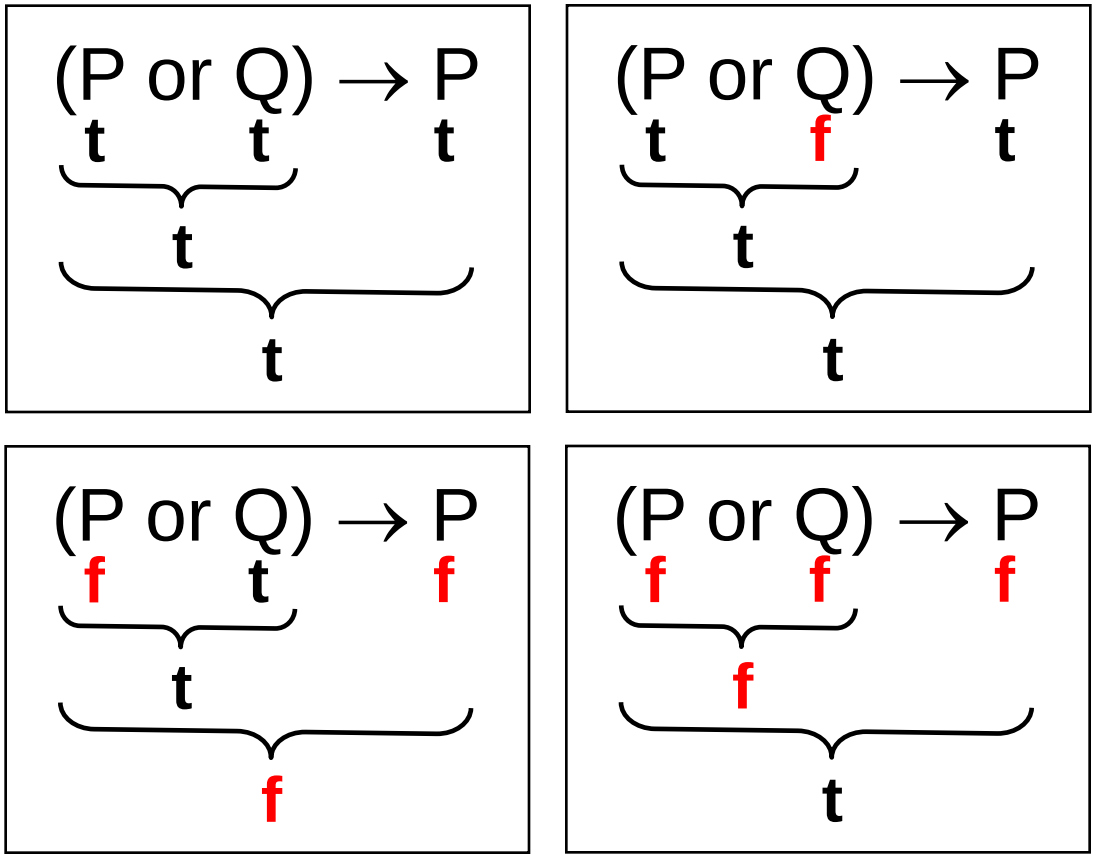
Truth Tables

P	Q	not(P)	P and Q	P or Q	$P \rightarrow Q$	$P \leftrightarrow Q$
t	t	f	t	t	t	t
t	f	f	f	t	f	f
f	t	t	f	t	t	f
f	f	t	f	f	t	t

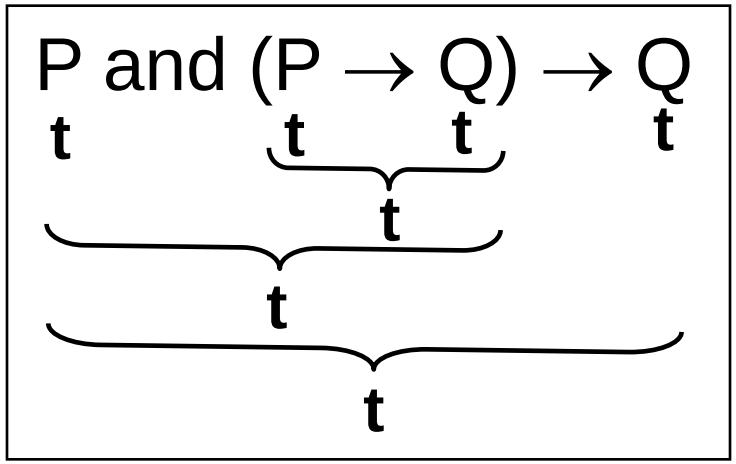
Inconsistencies

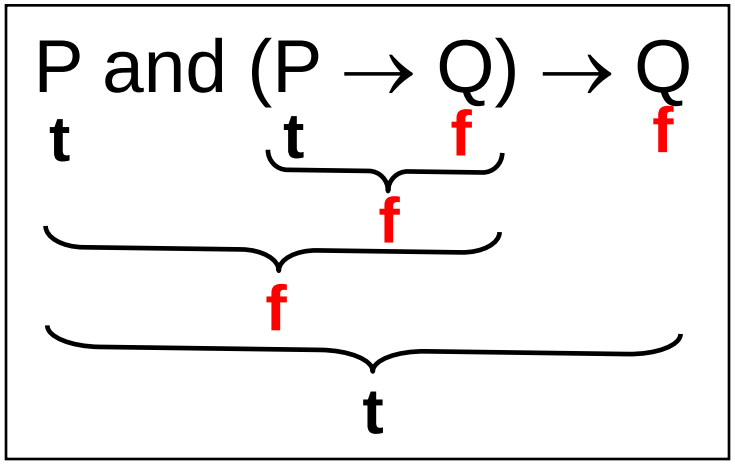


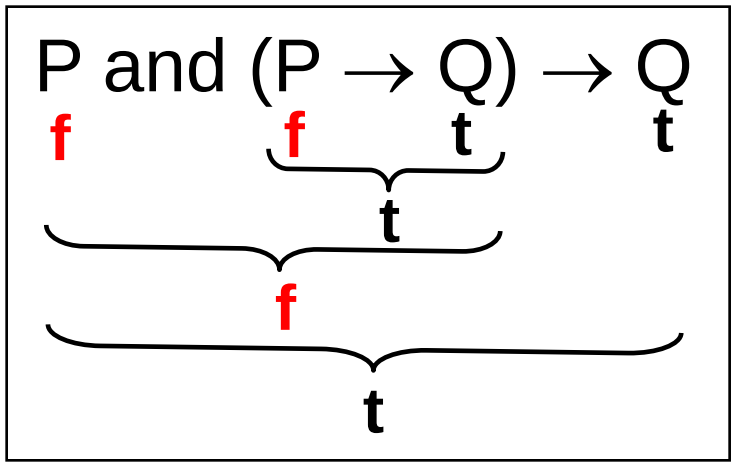
Contingencies

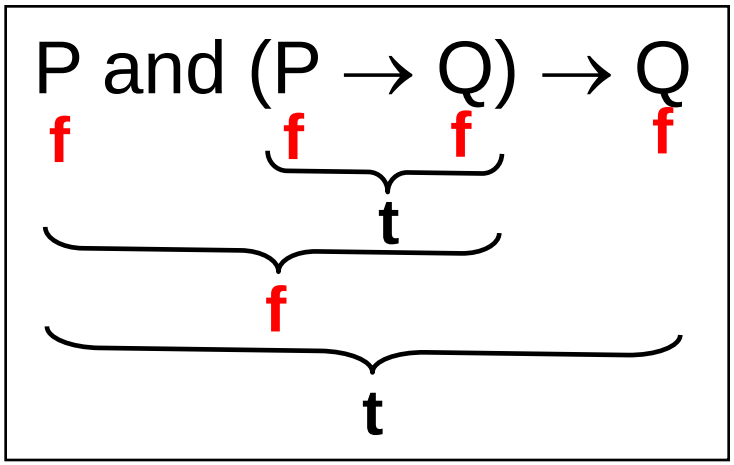


Tautologies









Clicker Question 1

Which of the following is a contingency?

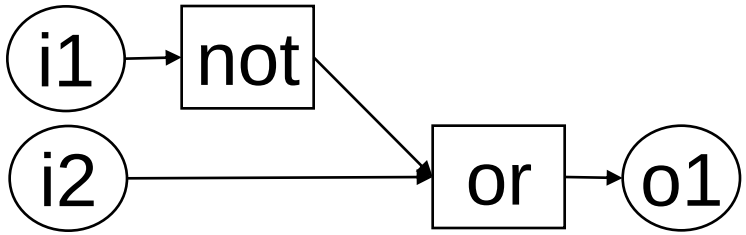
(a) $P \rightarrow P$

(b) Q or (not(Q)

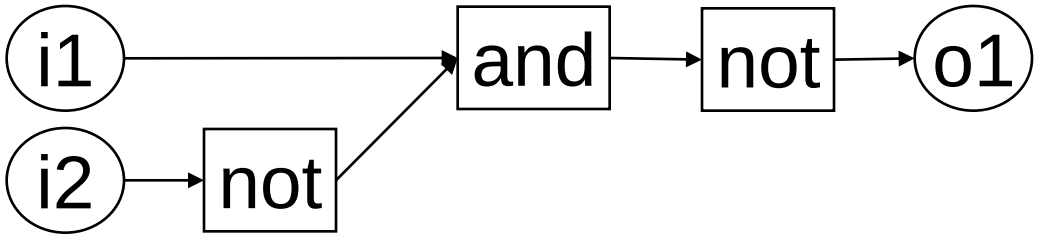
(c) $P \rightarrow Q$

(d) (P and Q) and (not(Q))

Solution: Logic Circuits (1)



equivalent ?



not(i1) or i2

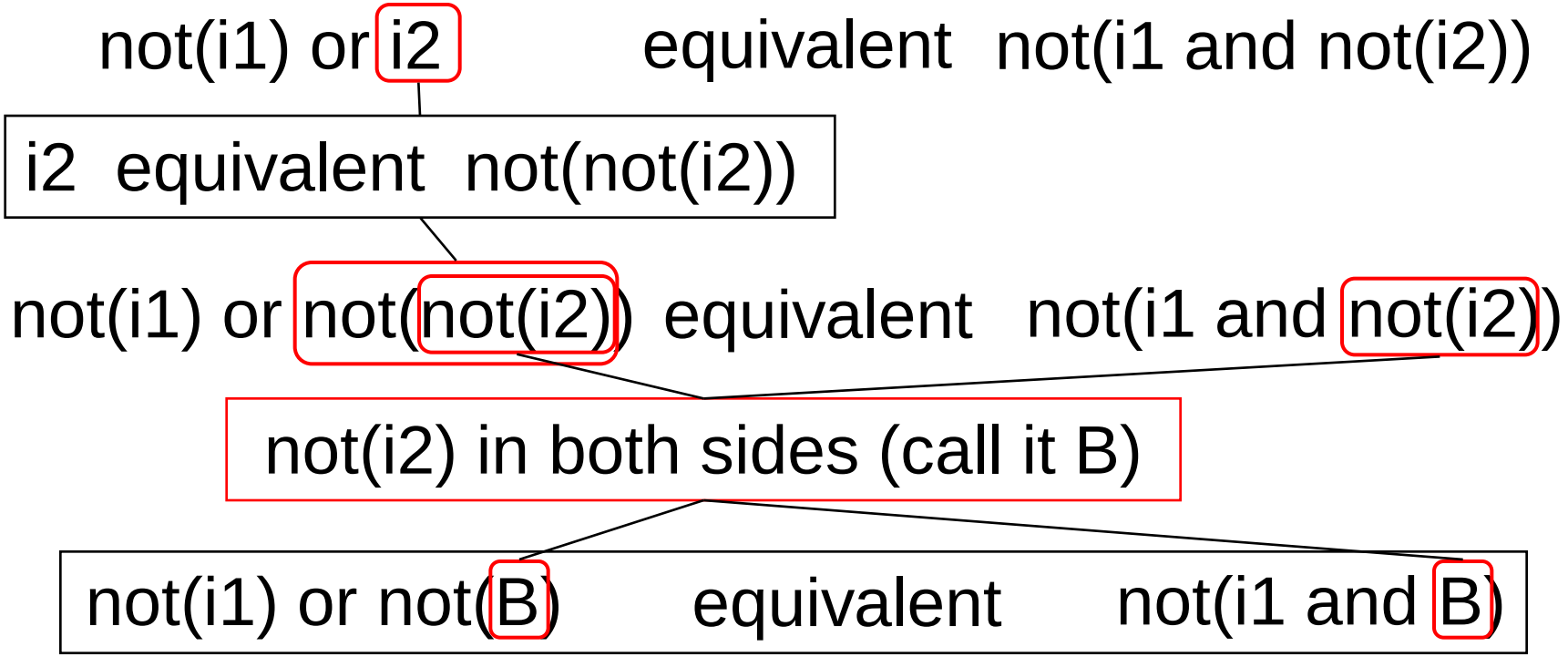
equivalent ?

not(i1 and not(i2))

Solution: Logic Circuits (2)

i1	i2	not(i1)	not(i2)	not(i1) or i2	i1 and not(i2)	not(i1 and not(i2))
t	t	f	f	t	f	t
t	f	f	t	f	t	f
f	t	t	f	t	f	t
f	f	t	t	t	f	t

Solution: Logic Circuits (2)



Here we are performing a proof using equivalences

Problem: Common Knowledge

A husband unfaithful to his wife will be shot by her on the day she finds out.

Day 1: Three wives are told that at least one of their husbands is unfaithful. None knows whether her husband is unfaithful but each knows about the others.

Day 2: The three wives meet again and learn that no husband has been shot.

Day 3: The three wives meet again and learn that no husband has been shot. That night there is shooting.

How many husbands were unfaithful ?

Solution: Common Knowledge

cheat1	cheat2	cheat3	
t	t	t	**BANG!**
t	t	f	**X – day 3**
t	f	t	**X – day 3**
t	f	f	**X – day 2**
f	t	t	**X – day 3**
f	t	f	**X – day 2**
f	f	t	**X – day 2**
f	f	f	**X – day 1**

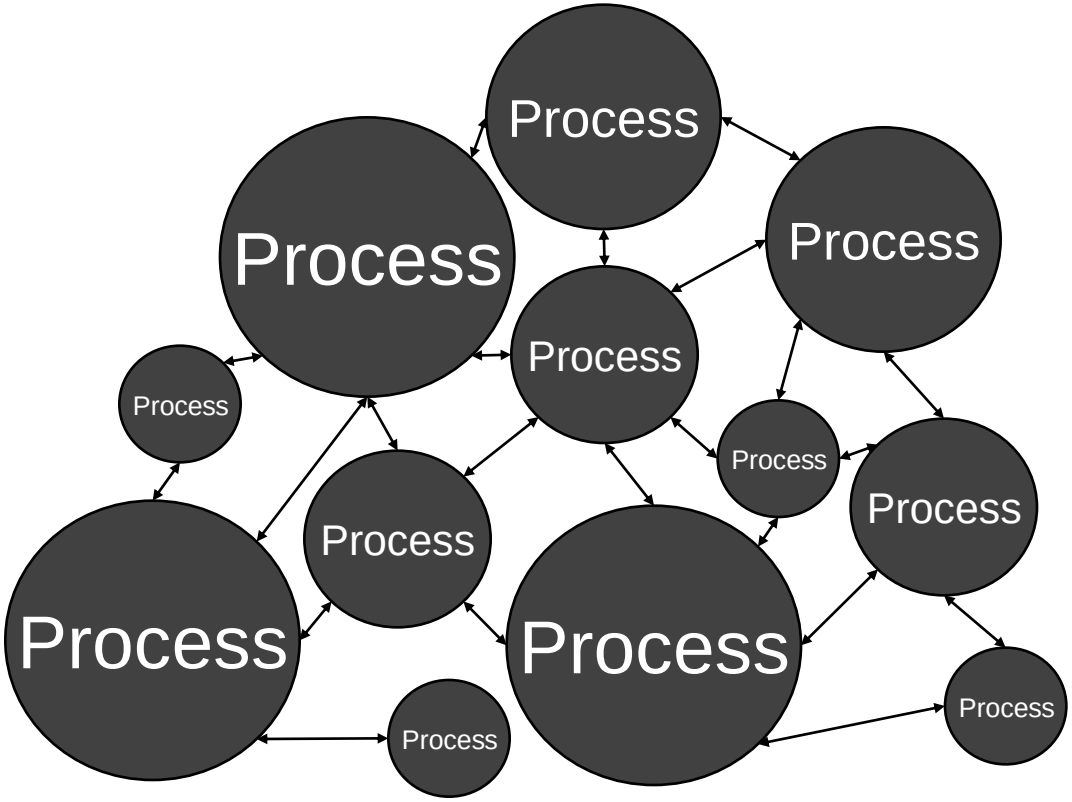
(See notes for full truth table)

Clicker Question 2

Same setting as before, but now 5 wives are told about infidelity on Day 1. And there is shooting at the end of Day 1! How many straying husbands?

(a) 1
(b) 2
(c) 4
(d) 5

Real Common Knowledge



Things to Practice

- Writing English statements in propositional logic (exercises in tutorials).
- Being able to work out the truth of any propositional expression using truth tables (exercises in tutorials).
- Understanding why some expressions are equivalent (see equivalences in notes).

Recreational Reading

Raymond Smullyan,

"What is the Name of This Book? The Riddle of Dracula and Other Logical Puzzles"

&

"Alice in Puzzle-Land"