Resources

- Lecturer: Rahul Santhanam (rsanthan@inf.ed.ac.uk)
- TA: Areti Manataki (A.Manataki@ed.ac.uk)
- All notes and tutorials are on the course Web site at: www.inf.ed.ac.uk/teaching/courses/inf1/cl/
- Tutorial each week
- Lecture slides will be on course web site
- Schedule of lectures in weeks 2-5 rather non-standard: refer to First Year Handbook for details

Describing Information Systems in Logic

Logic is the ``calculus of computer science''
[Manna & Waldinger 85].

In the next seven lectures you will encounter a way of describing information and the processing of information that is precise while remaining independent of any particular computing machine.

This lecture starts you describing information in logic.

In the remaining six lectures we solve problems via inference.

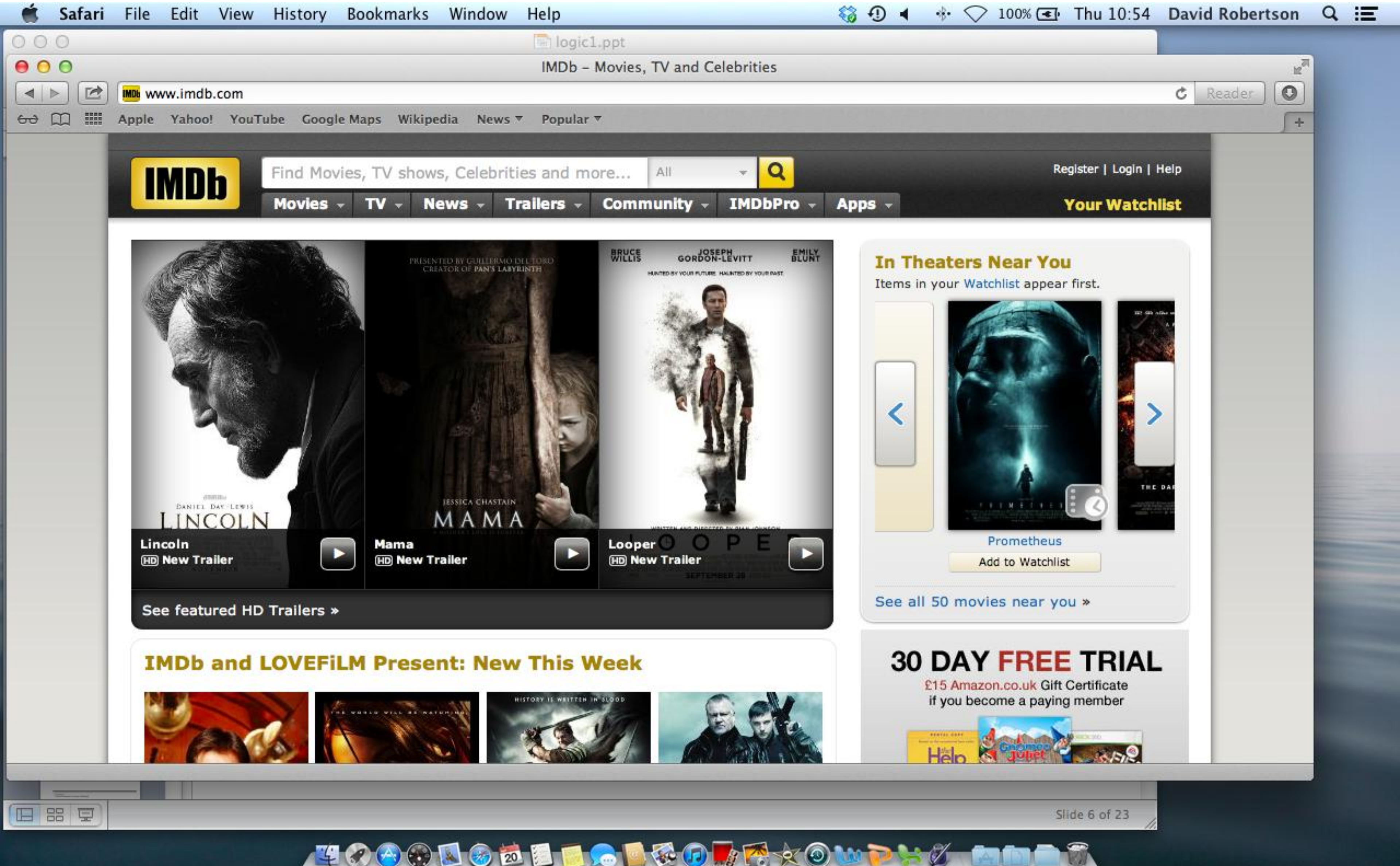
A Brief History of Logic

- **500 BC - 19th Century**: Logic as discipline in human argument.
- **mid-late 19th Century**: Logic begins to be understood as a mathematical language.
- **late 19th - mid 20th Century**: Study of mathematical proof and its limitations.
- **mid 20th Century – date**: Logic in informatics.

What Use is Logic?

- The gates in computer circuits implement Boolean logic.
- Some problems resist solution no matter how fast we build our computers, but we can explore these problems in logic.
- Some programming languages (e.g. database query languages) are close to logic.
- Logic is used to define semantics of languages so these can be compared.
- Logic is used in verifying systems and security protocols.
- The lingua franca for many areas of informatics research is logic.

Motivating Problem



Motivating Problem (Data)

film(F) means F is a film
actor(F, P) means that person P acted in film F
director(F, P) means that person P directed film F
oscar(X) means that X got an Oscar

```
film('Mars Attacks')
director('Mars Attacks','Tim Burton')
actor('Mars Attacks','Jack Nicholson')
actor('Mars Attacks','Glenn Close')
oscar('Jack Nicholson')
oscar('Glenn Close')
```

Motivating Problem (Query)

How could we describe precisely the following questions about our data?

- Which Oscar-winning films were directed by an actor?
- Which Oscar winning actors have directed themselves?
- Which directors have directed more than one film?
- Which films have more than one director?
- Have all directors been actors?

Notation

$$\forall X.p(a, X) \text{ and } q(X) \rightarrow \exists Y.r(X, Y)$$

p(a,X)	p is a predicate name a is a constant X is a variable
not P	Negation of P
P and Q	P and Q are true (conjunction)
P or Q	P or Q is true (disjunction)
$P \rightarrow Q$	P implies Q
$P \leftrightarrow Q$	P and Q are equivalent
$\forall X.\ s(X)$	For all X, s is true
$\exists X.\ s(X)$	For some X, s is true

Precedence of Operators

We take the precedence ordering to be the following:

- '$\rightarrow$' and '$\leftrightarrow$' dominate most.
- 'and' and 'or' are next most dominant.
- 'not' is least dominant.
- If equal dominance, the operator on the right dominates.

So we can write:

((a and b) or c) $\rightarrow$ d

as

a and b or c $\rightarrow$ d

Why There are Many Ways to Describe the Same Concept

- Ambiguity in Understanding the World
- Boundary Choices
- Equivalences Between Formal Expressions
- Differences in Logical Systems

Ambiguity in Understanding the World

You observe that this classroom is crowded. Is it:

- Full?
- Almost full?
- At over 80% of capacity?
- Being held to ransom by a geek who believes in ruminating over logic at lunchtime?

Boundary Choices

oscar(P) means that P got an Oscar.

Doesn't tell us which film, F
oscar_winner(P, F) $\rightarrow$ oscar(P)

or which time, T.
oscar_win(P, F, T) $\rightarrow$ oscar_winner(P, F)

At some stage, however, we must choose a level of description adequate for the task we wish to undertake and consider other information to be outside the boundary of our model.

Equivalences Between Formal Expressions

$a \rightarrow b$ is equivalent to not(a) or b

So which is it better to say?

Depends on how you want to use it to solve a problem and who has to read it.

Differences between Logical Systems

- Propositional logic or predicate logic?
  - In propositional logic, easier to decide truth or falsity of statements
  - However, predicate logic provides richer framework for representing data
- Various other logics to choose from as well

Similarity Between Expressions

Exploring dualities between common forms of expression to deepen our understanding of what the expressions mean.

In a later lecture you find out how to prove the dualities we introduce now.

Implication Versus Conjunction With Negation

$a \rightarrow b$ is equivalent to not(a) or b

(Suggested read: "What the Tortoise Said to Achilles" by Lewis Carroll)

Conjunction Versus Disjunction

a and b	is equivalent to	not(not(a) or not(b))
a or b	is equivalent to	not(not(a) and not(b))

Universal Quantification Versus Conjunction

If all instances of X are $a_1, a_2, \ldots a_n$ then

$\forall X.\ p(X)$ is equivalent to $p(a_1)$ and $p(a_2)$ and … $p(a_n)$

Existential Quantification Versus Disjunction

If all instances of X are $a_1, a_2, \ldots a_n$ then

$\exists X.\ p(X)$ is equivalent to $p(a_1)$ or $p(a_2)$ or … $p(a_n)$

Solving Our Problem (Query 1)

Which Oscar-winning films were directed by an actor?

Prove that there exists an Oscar winning film and its director and there exists a film in which that director was a film actor.

$\exists$F,P. oscar(F) and director(F,P) and $\exists$F1.actor(F1, P)

Solving Our Problem (Query 2)

Which Oscar winning actors have directed themselves?

Prove that there exists a film and an oscar winning actor in it and the same person is a director of that film.

$\exists$F,P. oscar(P) and actor(F,P) and director(F, P)

Solving Our Problem (Query 3)

Which directors have directed more than one film?

Prove that there exists two films with the same director and these are not the same film.

$$\exists F1,F2,P.\ \text{director}(F1, P) \text{ and } \text{director}(F2, P) \text{ and } \text{not}(F1 = F2)$$

Solving Our Problem (Query 4)

Which films have more than one director?

Prove that there exists a film for which there exists two directors and these are not the same person.

$$\exists F,P1,P2.\ \text{director}(F, P1) \text{ and } \text{director}(F, P2) \text{ and } \text{not}(P1 = P2)$$

Solving Our Problem (Query 5)

Have all directors been actors?

Prove that all directors of films are actors in some film

$$\forall P,F1.director(F1,P) \rightarrow \exists F2.actor(F2,P)$$