$$\Gamma \models \Delta$$

We can use the rules to show this is universally valid,
or, if it is not, to generate
a counterexample, a model in which

$$\Gamma \not\models \Delta$$

$$\text{some } \bigwedge \Gamma \text{ is not } \bigvee \Delta$$

Can we use the rules to show this is somewhere valid?
We say the sequent is **satisfiable** if we can
find a model in which

$$\text{some } \bigwedge \Gamma \text{ is } \bigvee \Delta$$

Can we use the rules to show this is somewhere valid?
We say the sequent is satisfiable if we can
find a model where

$$\text{some } \bigwedge \Gamma \text{ is } \bigvee \Delta$$

$$\Gamma \not\models \neg \bigvee \Delta$$

$$\Gamma \models \neg \bigvee \Delta$$

We can use the rules to show this is universally valid,
or, if it is not, to generate
a counterexample, which shows

$$\Gamma \not\models \neg \bigvee \Delta$$

$$\Gamma \models \neg \bigvee \Delta \qquad \Gamma, \bigvee \Delta \models$$

We can use the rules to show this is universally valid,

$\Gamma, \bigvee \Delta$ is inconsistent

or, if it is not, to generate
a counterexample, a model in which

$$\Gamma \not\models \neg \bigvee \Delta$$

$$\text{some } \bigwedge \Gamma \text{ is } \bigvee \Delta$$

$$\models$$

$$\bigwedge \varnothing \models \bigvee \varnothing$$

$$\top \models \bot$$

which is only valid in the empty universe

$$\models$$

$$\Gamma \models \Delta \quad (\Gamma = \Delta = \varnothing)$$

$$\bigwedge \varnothing \models \bigvee \varnothing$$

$$\top \models \bot$$

which is only valid in the empty universe

$$\varnothing \models \varnothing$$

$$a \models b \quad (a = b = \varnothing = \bot)$$

$$\bot \models \bot$$

which is universally true

This is a type error
— but for a mathematician
a set is just a set
there is only one emptyset