Perceptrons

Informatics 1 CG: Lecture 5

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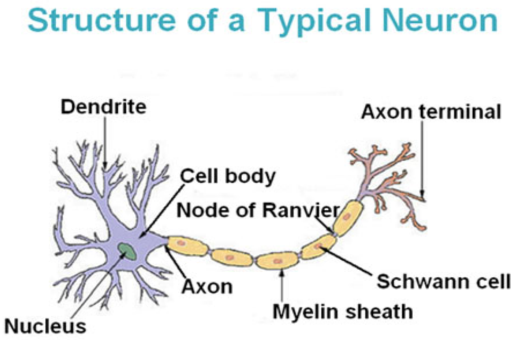
Reading:

Steven Pinker's Words and Rules, Chapter 2
Kevin Gurney's Introduction to Neural Networks, Chapters 2 and 4

Recap: Words and Rules

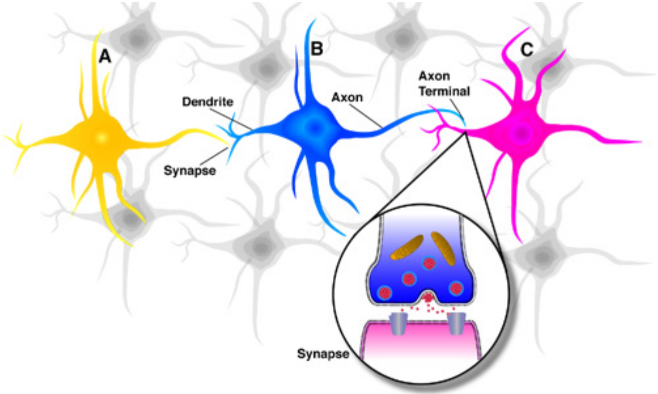
- Does the theory of words and rules explain the dichotomy between regular and irregular verbs?
- Is SPE a plausible theory of how the past tense is formed?
- What does evidence from language development tell us about regular and irregular verbs?
- Maybe a rule is not necessary to explain the past tense.
- Maybe children simply analogise from verbs they already know (e.g., from correct forms like *folded, molded, scolded* to over-regularisations like *holded*).
- All-rules versus all-memory approach.

A Single Neuron



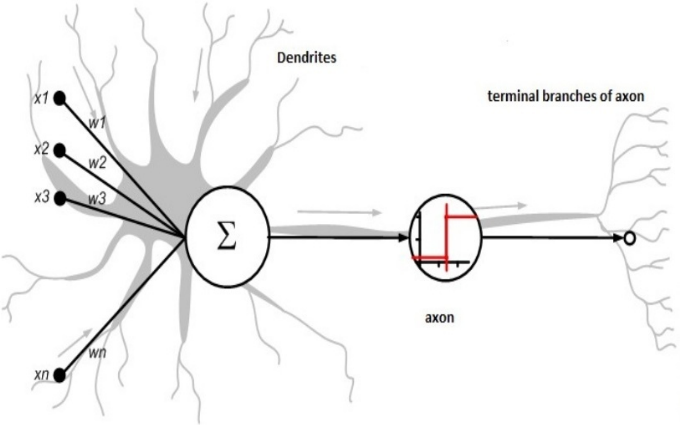
- Neuron receives inputs and combines these in the cell body.
- If the input reaches a threshold, then the neuron may fire (produce an output).
- Some inputs are excitatory, while others are inhibitory.

Biological Neural Networks



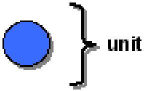
- In biological neural networks, connections are synapses.
- Input connection is conduit through which a member of a network receives information (INPUT)
- Output connection is a conduit through which a member of a network sends information (OUTPUT).

Connectionism is the name for a computer modeling approach based on how information processing occurs in neural networks (connectionist networks are called artificial neural networks).



Anatomy of a Connectionist Model

Units are to a connectionist model what neurons are to a biological neural network — the basic information processing structures.



Anatomy of a Connectionist Model

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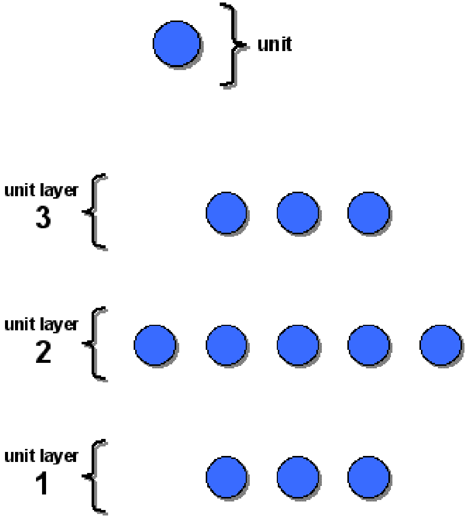
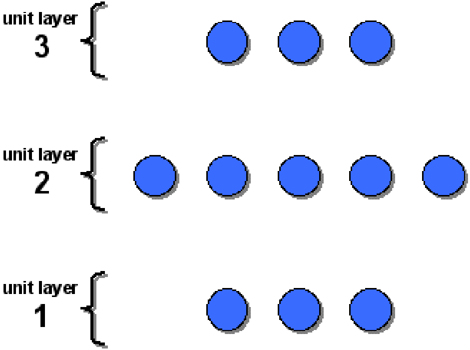
Biological neural networks are organized in layers of neurons. Connectionist models are organized in layers of units, not random clusters.

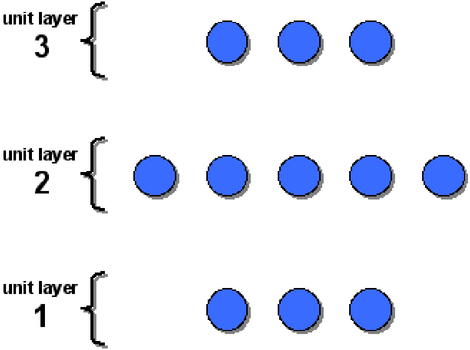


But what you see here still isn't a network. Something is missing.



Anatomy of a Connectionist Model

But what you see here still isn't a network. Something is missing. Network connections are conduits through which information flows between members of a network.



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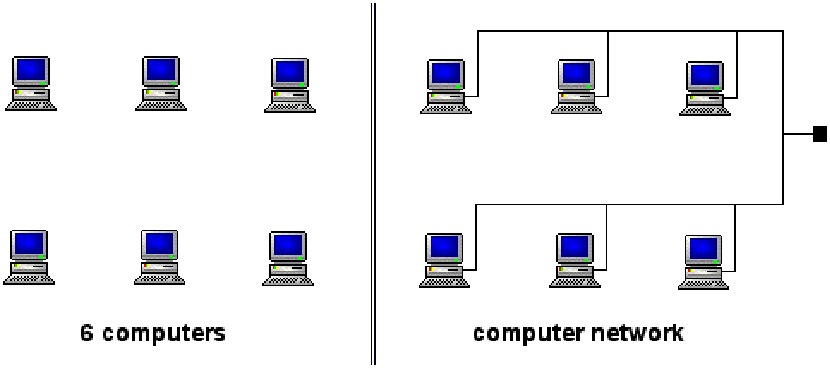

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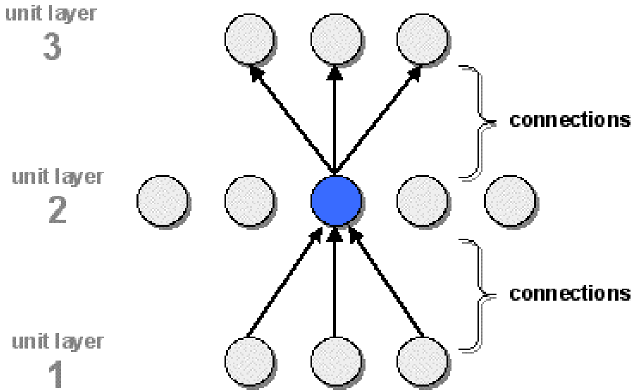
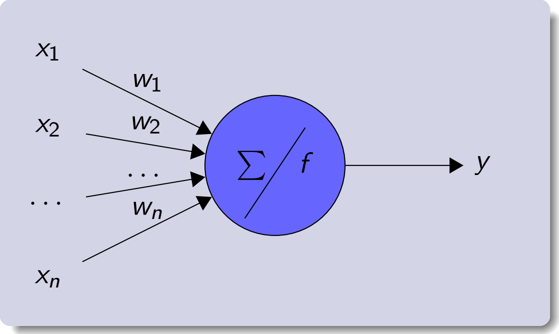


- Connections are represented with lines
- Arrows in a connectionist model indicate the flow of information from one unit to the next.

Perceptron: An Artificial Neuron

Perceptron was developed by Frank Rosenblatt in 1957 and can be considered as the simplest artificial neural network.



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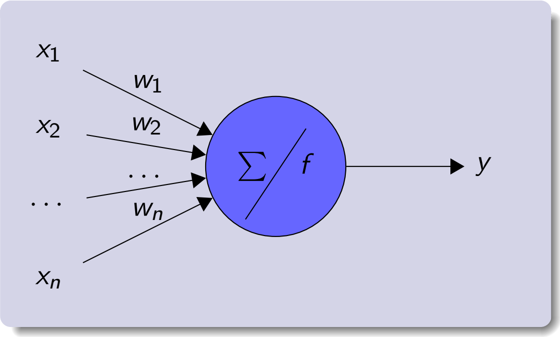

Input function:

$$u(\mathbf{x}) = \sum_{i=1}^{n} w_i x_i$$

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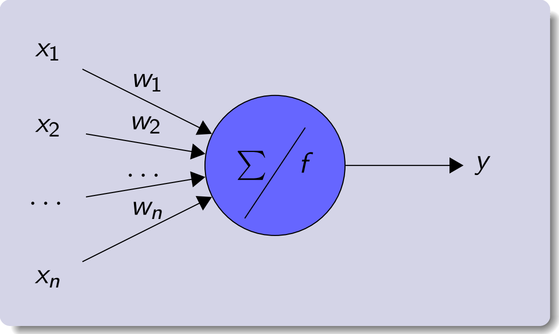

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Activation function: threshold

$$y = f(u(\mathbf{x})) = \begin{cases} 1, & \text{if } u(\mathbf{x}) > \theta \\ 0, & \text{otherwise} \end{cases}$$

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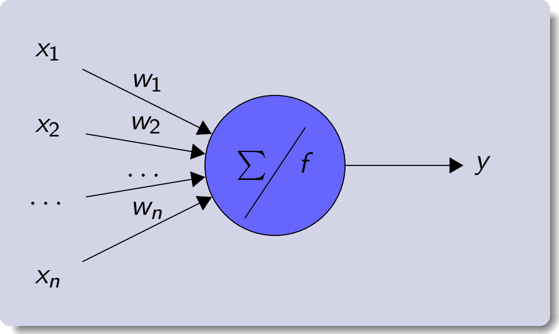

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Activation state:
0 or 1 (-1 or 1)

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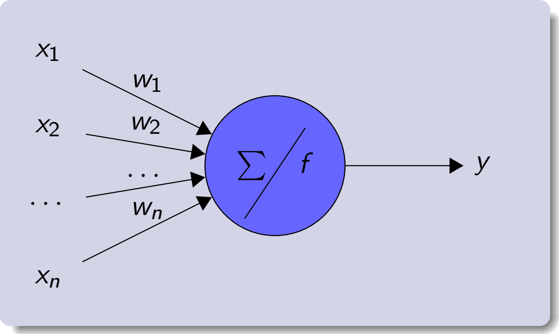
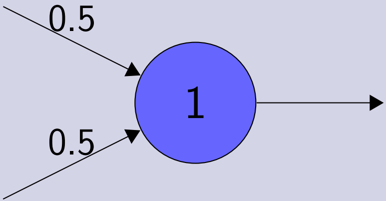


- Inputs are in the range [0, 1], where 0 is "off" and 1 is "on".
- Weights can be any real number (positive or negative).

Perceptrons for Logic

Perceptron for AND



f if $\sum \geq \theta$ then 1 else 0

x_1	x_2	x_1 AND x_2
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0	1	0
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Perceptrons for Logic

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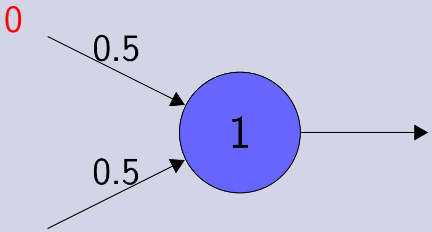

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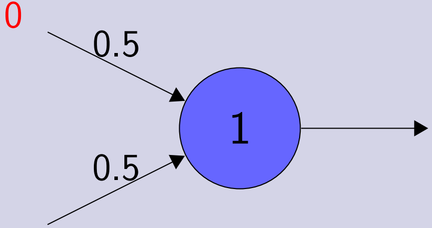

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$0 \cdot 0.5 + 1 \cdot 0.5 = 0.5$

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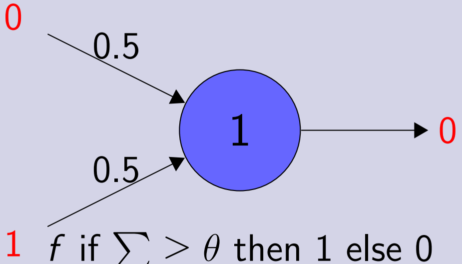

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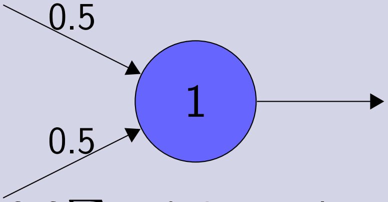

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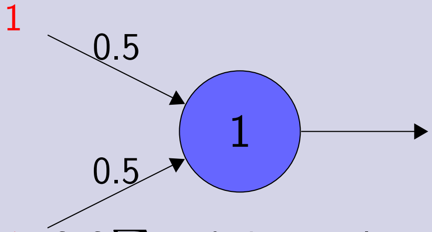

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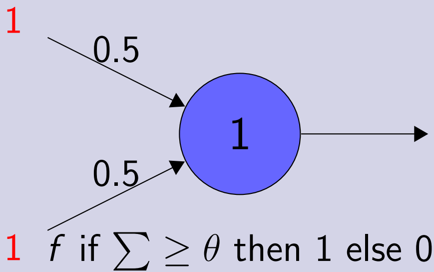

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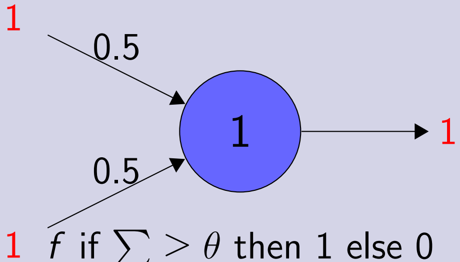

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Perceptrons for Logic

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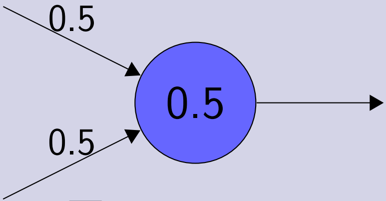

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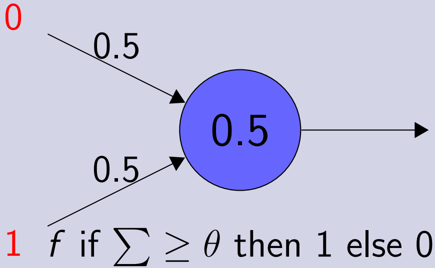
Perceptron for OR



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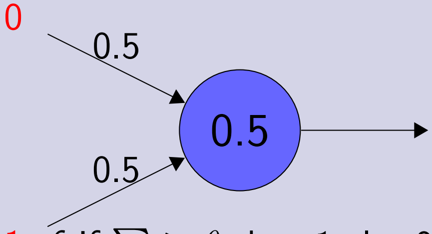

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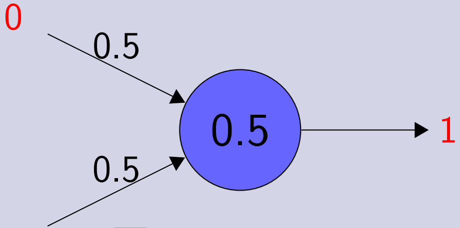

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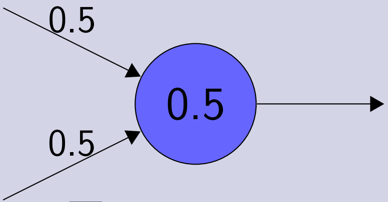
Perceptrons for Logic

Perceptron for XOR



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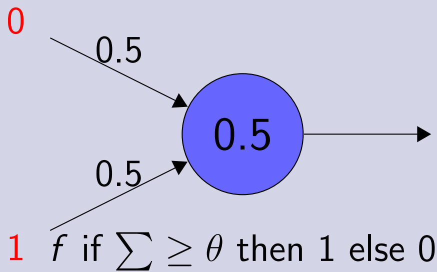
XOR is an exclusive OR because it only returns a true value of 1 if the two values are exclusive, i.e., they are both different.

Perceptron for XOR



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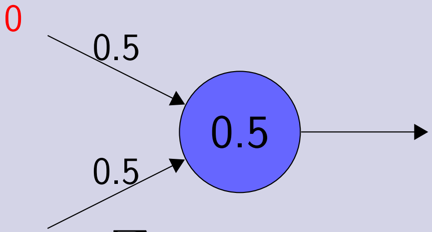

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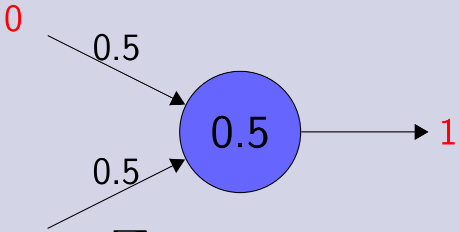

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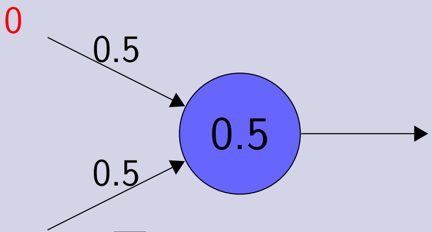

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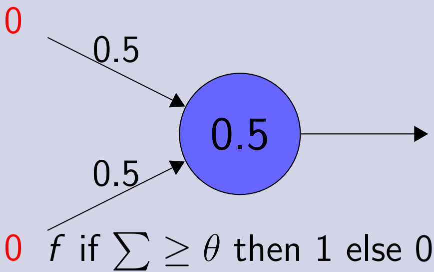

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Perceptrons for Logic

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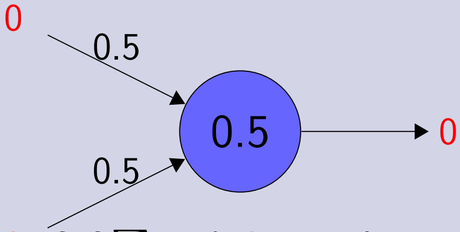

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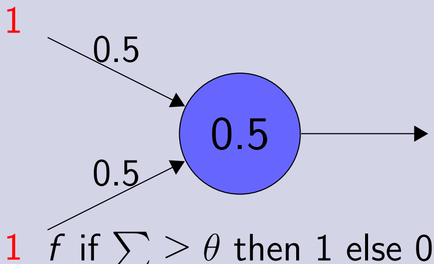

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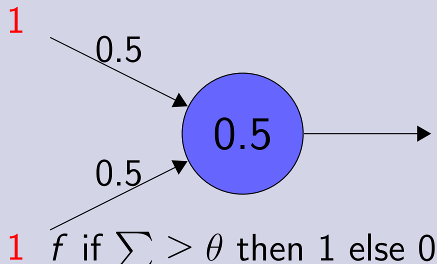

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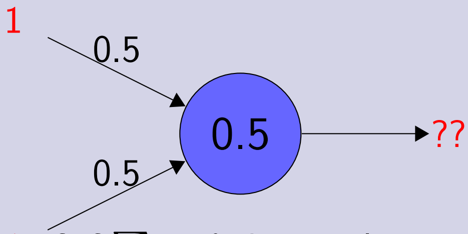

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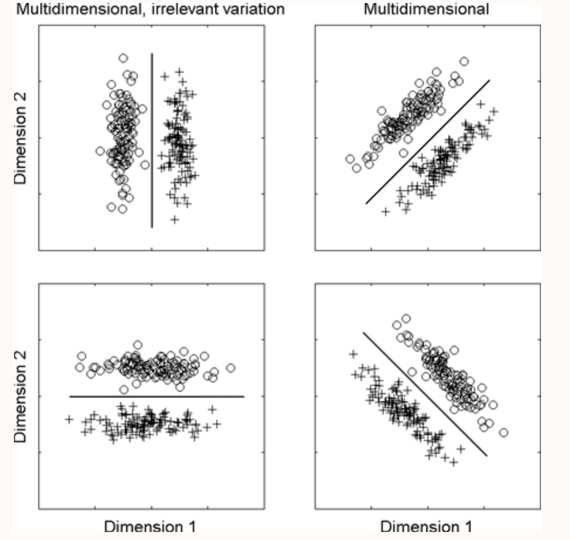
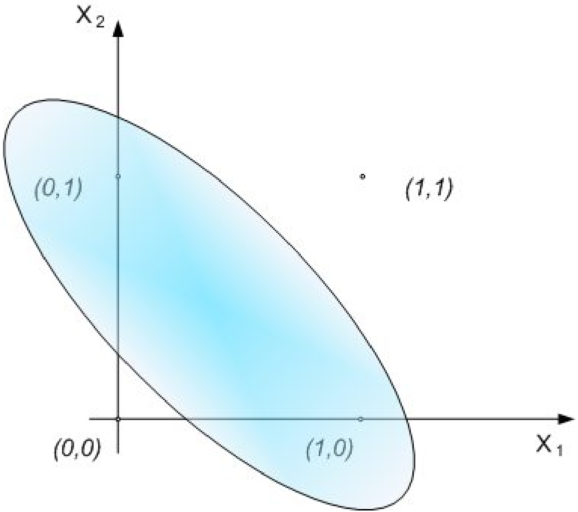
Perceptrons as Classifiers

Perceptrons are linear classifiers, i.e., they can only separate points with a hyperplane (a straight line).



The XOR probem again



What is the Perceptron Really Seeing?

Sequence of exemplars presented to the Perceptron:

N	input x	target t
1	(0,1,0,0)	1
2	(1,0,0,0)	0
3	(0,1,1,1)	0
4	(1,0,1,0)	0
5	(1,1,1,1)	1
6	(0,1,0,0)	1
...	...	...

- The above Perceptron has 4 inputs (binary) $\approx$ feature vector representing each exemplar.
- The Perceptron sees 6 exemplars or training items
- We know what the right answer is $\approx$ target
- What would happen if we used random weights/threshold?

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Learning

Q_1: But... choosing weights and threshold θ for the perceptron is not easy! How to learn the weights and threshold from examples?

A_1: We can use a learning algorithm that adjusts the weights and threshold based on examples.

`http://www.youtube.com/watch?v=vGwemZhPlsA&feature=youtu.be`

Learning: A trick to learn θ

$$\sum_{i=1}^{n} w_i x_i > \theta$$

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$$w_1 x_1 + w_2 x_2 + \ldots w_n x_n + \theta(-1) > 0$$

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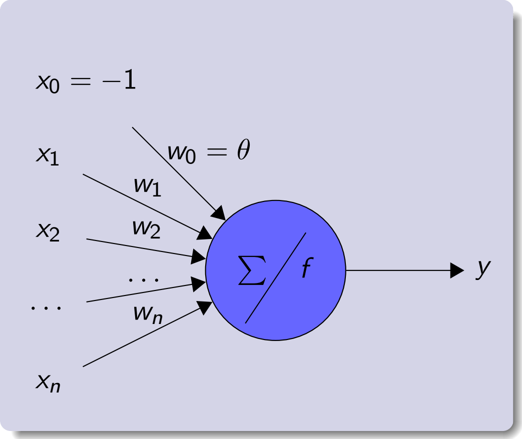

$$w_1 x_1 + w_2 x_2 + \dots w_n x_n - \theta > 0$$

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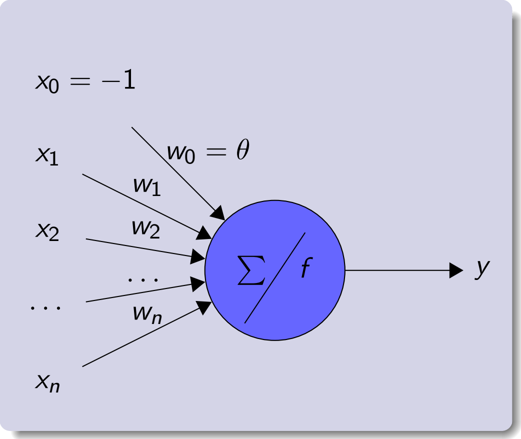

- We can consider θ as a weight to be learnt!
- The input is fixed as -1. The activation function is then:

$$y = f(u(\mathbf{x})) = \begin{cases} 1, & \text{if } u(\mathbf{x}) > 0 \\ 0, & \text{otherwise} \end{cases}$$

Learning Rule

Learning happens by adjusting weights. The threshold can be considered as a weight.

Perceptron's Learning Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i = \eta(t - o)x_i$$

- η, $0 < \eta \leq 1$ is a constant called learning rate.
- t is the target output of the current example.
- o is the output obtained by the Perceptron.

Learning Rule

Perceptron's Learning Rule

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$o = 1$ and $t = 1$
$o = 0$ and $t = 1$

- Learning rate η is positive; controls how big changes Δw_i are.
- If $x_i > 0$, $\Delta w_i > 0$. Then w_i increases in an attempt to make $w_i x_i$ become larger than θ.
- If $x_i < 0$, $\Delta w_i < 0$. Then w_i reduces.

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$$\begin{aligned} o = 1 \text{ and } t = 1 \quad \Delta w_i &= \eta(t-o)x_i = \eta(1-1)x_i = 0 \\ o = 0 \text{ and } t = 1 \quad \Delta w_i &= \eta(t-o)x_i = \eta(1-0)x_i = \eta x_i \end{aligned}$$

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Learning Rule: Exercise

Perceptron's Learning Rule

$$w_i \leftarrow w_i + \Delta w_i$$

$$\Delta w_i \;= \eta(t - o)x_i$$

Consider a Perceptron with only one input x_1, weight $w_1 = 0.5$, threshold $\theta = 0$ and learning rate $\eta = 0.6$. Consider also the training example $\{x_1 = -1, t = 1\}$. For now, let's temporarily ignore the learning of the threshold and consider it fixed.

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  $w_1 x_1 = 0.5(-1) = -0.5 \leq \theta \rightarrow o = 0$
- The new weight w_1 after applying the learning rule:

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Learning Rule: Exercise

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- The new weight w_1 after applying the learning rule:
  $\Delta w_1 = 0.6(1 - 0)(-1) = -0.6 \rightarrow w_1 = 0.5 - 0.6 = -0.1$
- The new output of the Perceptron for the input -1:
  $w_1 x_1 = -0.1(-1) = 0.1 \geq \theta \rightarrow o = 1$

Learning Algorithm

```
Initialize all weights randomly.
repeat
    for each training example do
        Apply the learning rule.
    end for
until the error is acceptable or a certain number
of iterations is reached
```

This algorithm is guaranteed to find a solution with zero error in a limited number of iterations as long as the examples are linearly separable.

Summary

What does this have to do with the words versus rules debate?

- Connectionism is a computer modeling approach inspired by neural networks.
- Anatomy of a connectionist model: units, connections
- The Perceptron as a linear classifier.
- A learning algorithm for Perceptrons

Next lecture: multilayer perceptrons (neural networks).