

We can apply Bayes filter to the two-node map problem in the lecture slides as follows. We want to know $Bel(x_t = home)$, given the following information:

1. At time $t - 1$, $Bel(x_{t-1} = home) = 0.2$ and so $Bel(x_{t-1} = away) = 0.8$
2. We have taken action u_t 'go home' and we know (from the diagram):

$$P(x_t = home | u_t = gohome, x_{t-1} = home) = 1$$

$$P(x_t = away | u_t = gohome, x_{t-1} = home) = 0$$

$$P(x_t = home | u_t = gohome, x_{t-1} = away) = 0.9$$

$$P(x_t = away | u_t = gohome, x_{t-1} = away) = 0.1$$
3. We have the measurement z_t 'seen home' where:

$$P(z_t = home | x_t = home) = 0.7$$

$$P(z_t = home | x_t = away) = 0.1$$

$$\begin{aligned}
 Bel(x_t = home) &= \eta P(z_t = home | x_t = home) \sum_{x_{t-1}} P(x_t | u_t, x_{t-1}) Bel(x_{t-1}) \\
 &= \eta 0.7 [P(x_t = home | u_t = gohome, x_{t-1} = home) Bel(x_{t-1} = home) \\
 &\quad + P(x_t = home | u_t = gohome, x_{t-1} = away) Bel(x_{t-1} = away)] \\
 &= \eta 0.7 (1 \times 0.2 + 0.9 \times 0.8) \\
 &= \eta 0.644
 \end{aligned}$$

$$\begin{aligned}
 Bel(x_t = away) &= \eta P(z_t = home | x_t = away) \sum_{x_{t-1}} P(x_t | u_t, x_{t-1}) Bel(x_{t-1}) \\
 &= \eta 0.1 [P(x_t = away | u_t = gohome, x_{t-1} = home) Bel(x_{t-1} = home) \\
 &\quad + P(x_t = away | u_t = gohome, x_{t-1} = away) Bel(x_{t-1} = away)] \\
 &= \eta 0.1 (0 \times 0.2 + 0.1 \times 0.8) \\
 &= \eta 0.008
 \end{aligned}$$

We then can calculate the normalisation factor η , as we know $Bel(x_t = home) + Bel(x_t = away) = 1$ (the robot must be in one or other state) so $\eta = \frac{1}{0.644+0.008} = 1.534$

So our final $Bel(home) = 1.534 \times 0.644 = 0.988$