Maps and Planning

IAR Lecture 7

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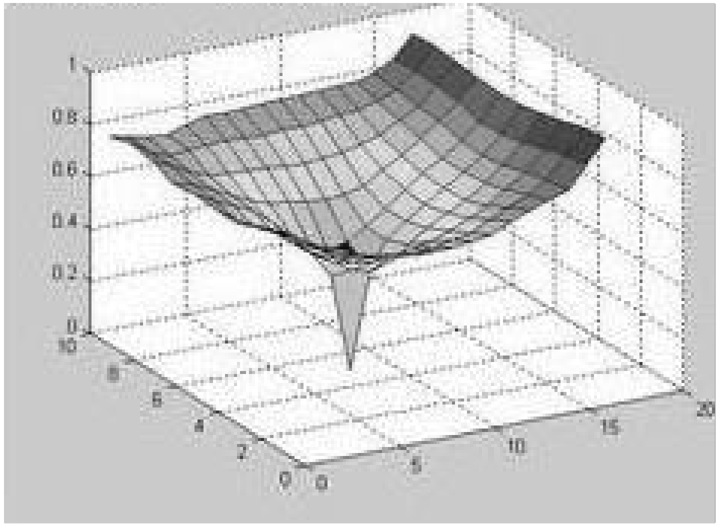
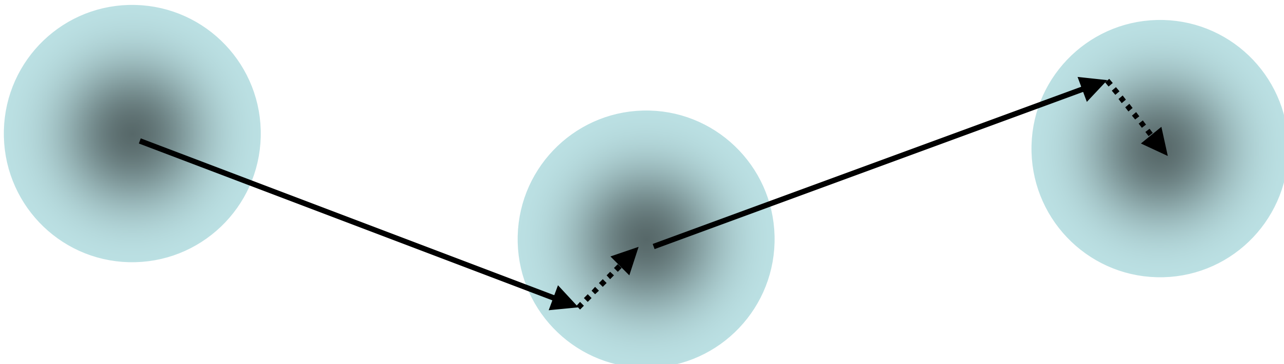
From local strategies to routes to maps

- (From lecture 4) Beacon or memory of home view allows navigation from a surrounding catchment area
- Multiple beacons or memories can be linked together to form routes

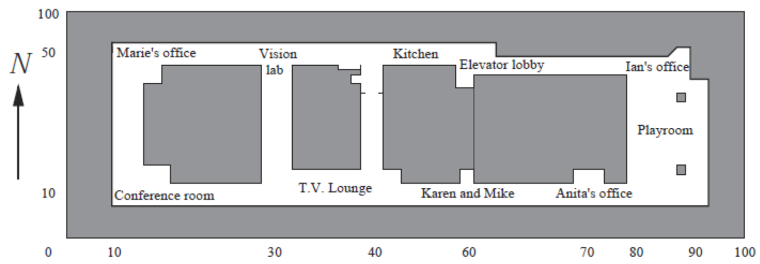


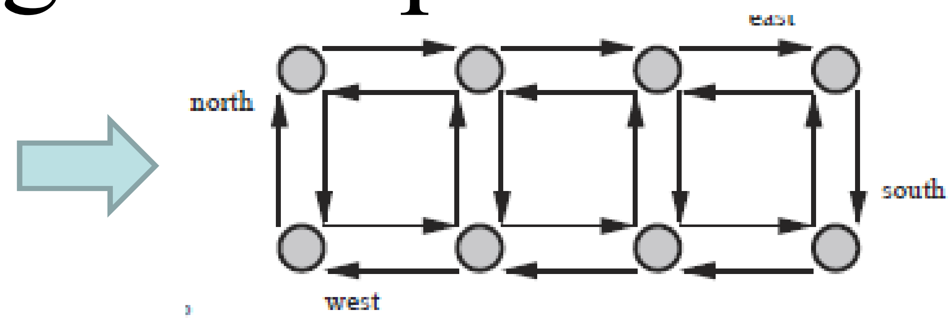


From local strategies to routes to maps

- If we can combine routes (or close loops in routes) by recognising overlapping locations we create a graph representation of the world
- Problem is to reliably recognise when encounter the same location:
  - From different approach directions
  - With possible alterations to appearance (e.g. lighting)
  - In ‘wrong place’ according to odometry
- But not to confuse locations that are different but look similar
- And ideally, to do this with an efficient algorithm.

Topological maps

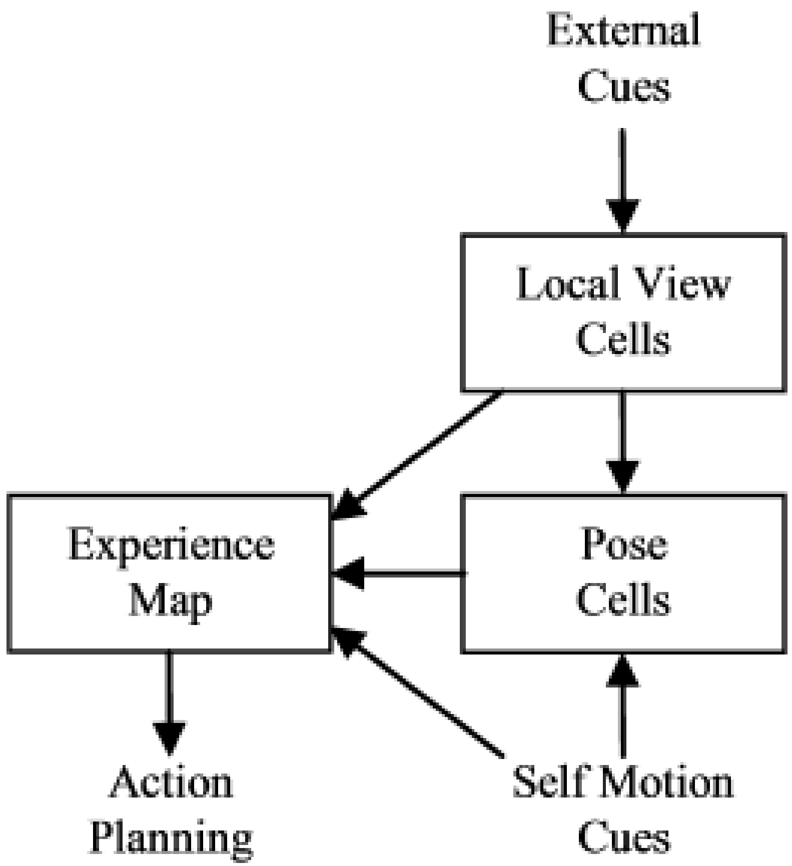




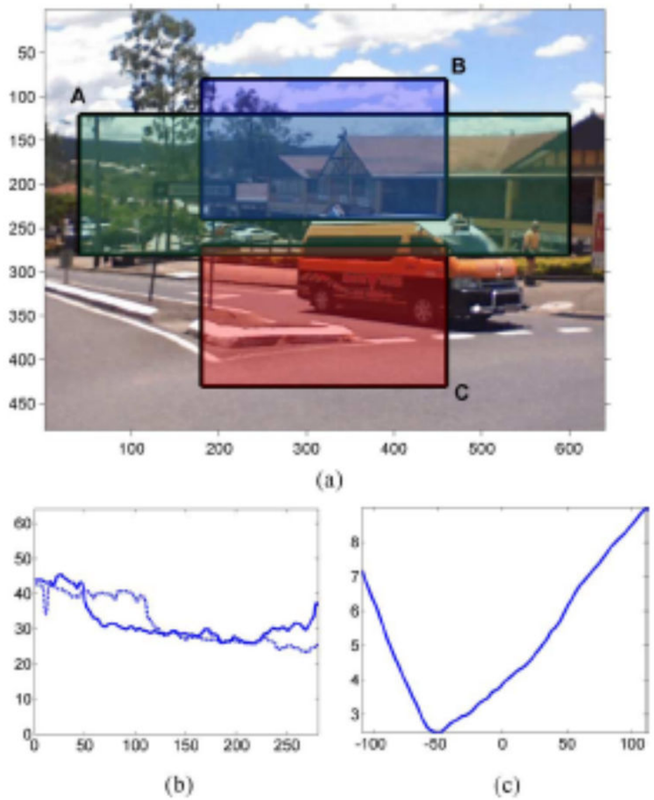
- Represent known locations and the connections between them as nodes and edges in a graph
- The edges could represent simple adjacency, the raw actions needed to get from one node to the next; or direction, distance, path convenience etc.
- Can determine a possible (or even the optimal) route by standard graph search methods

Example: RAT-SLAM

- For each new local view and/or pose, store an ‘experience’ node, linked to the previous node by a transition derived from the self motion (experience nodes are like rat place cells)
- Recognise when same view and pose occur to close loops in the experience map
- When closing loops, align the transitions and poses for geometric consistency to correct for drift



RAT-SLAM



• Both local views and self-motion are derived from vision

• Use different parts of visual field for:

A: Local view: compare to previously stored templates; either recognise or store as new template

B: Rotation estimate: find sideways pixel shift that produces best match.

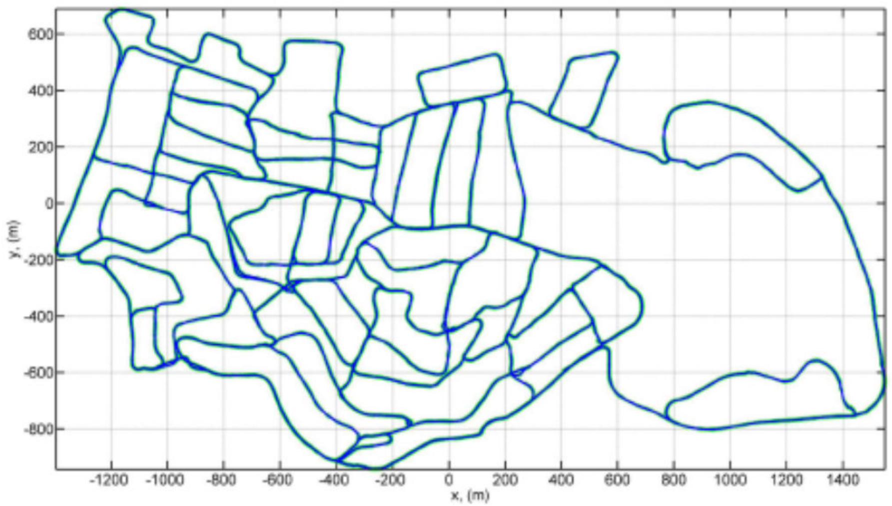
C: Speed estimate: for best rotation, take image difference.

• In each case reduce image to one-dimensional scanline of normalised intensity across columns.

RatSLAM Results 2008

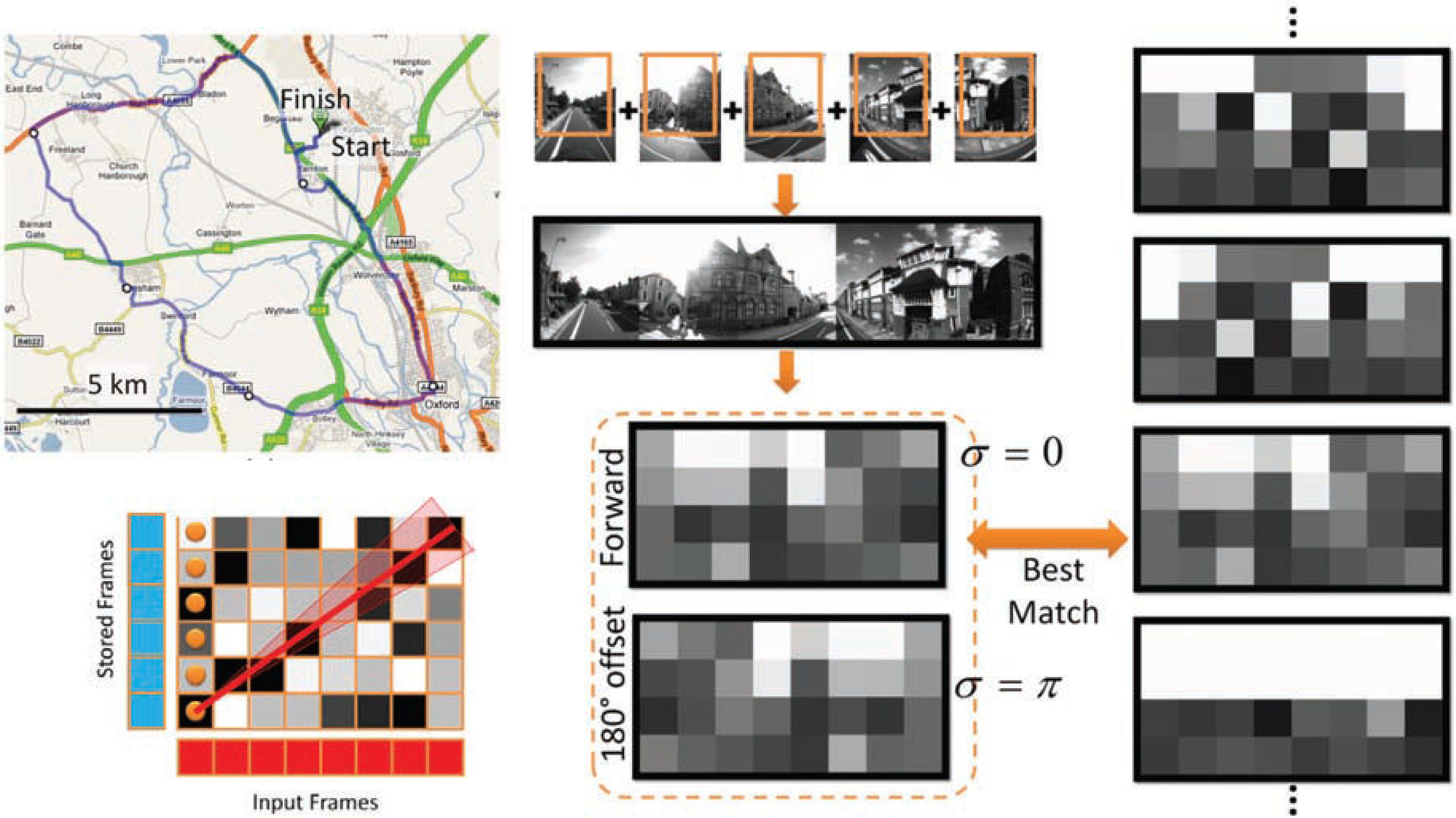
- Using built-in laptop webcam, drove for 100 minutes through 3km by 1.6km area of Brisbane
- Visualising the 'experience map' shows method produces a fairly accurate map that could be used for navigation

(a)



(b)

Sequence SLAM (Milford 2013)
Improve matching by looking for sequences

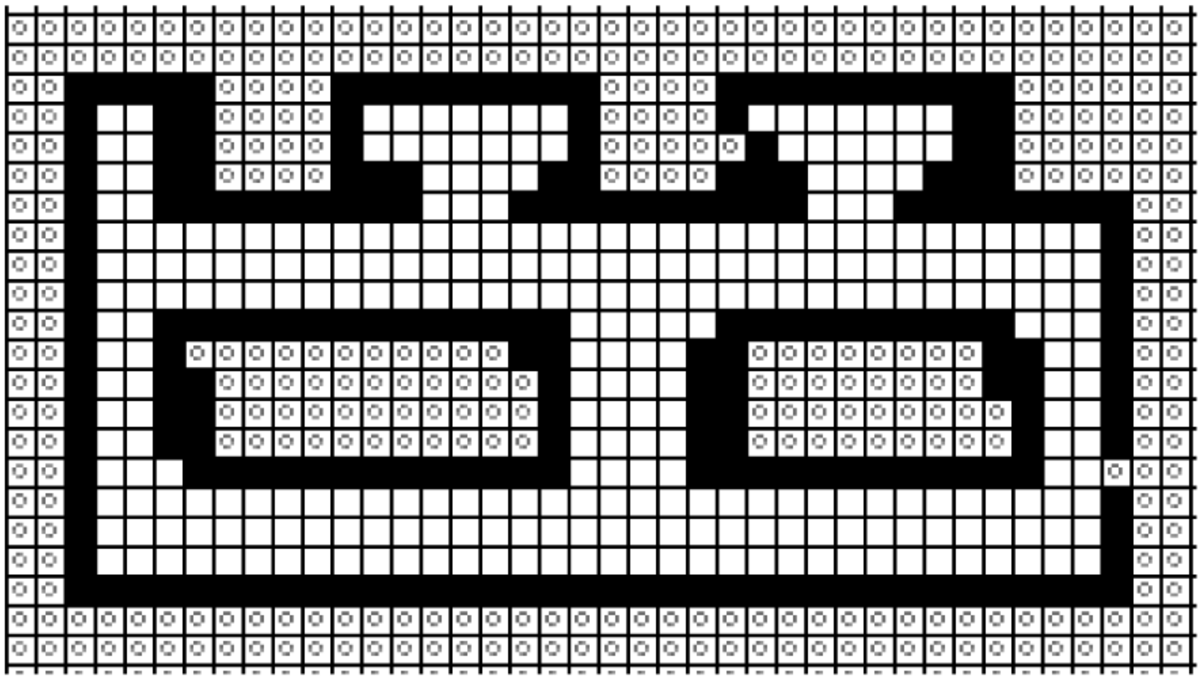


From topological to metric maps

- A graph in which edges represent the distances and directions between locations in consistent global co-ordinates is effectively a metric map.
- We describe the location of the robot and objects in its world in some kind of absolute coordinates (e.g. cartesian or polar)
- For simple robot, moving on ground plane and able to rotate on the spot, could consider this as 2 degree of freedom *configuration space* (i.e., where the robot can move to)*:*
  - Only obstacle location matters (not identity)
  - Assume robot is holonomic or can rotate on spot
  - Treat robot as a single point and expand obstacle boundaries by robot's maximum dimension

Example: Occupancy grid representation

- Divide space into a regular rectangular grid at some specific resolution
- Mark each grid square as either ‘occupied’, ‘empty’ or ‘unknown’



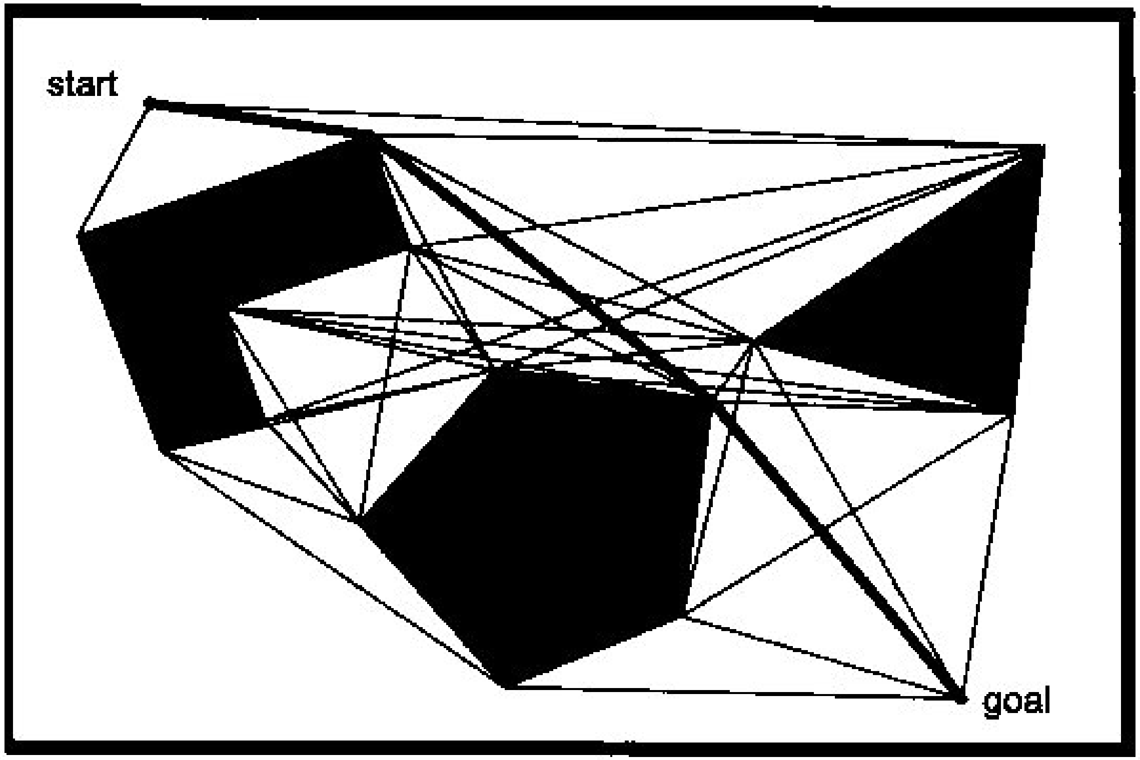
Topological	Raw sensor data	Store few locations	Connect by raw motor action
↑	Extract features	Store continuous links	Connect with some metric information
↓	Use sensor model	Make inferences between nodes	Nodes have global position
Metric	Convert to spatial data	Continuous representation	Absolute metric

From metric to topological maps

- For planning a route, one popular approach is to convert a metric map to a topological map
- Several different methods:
  - Visibility graph
  - Voronoi diagram
  - Cell decomposition
  - Or treat each empty grid square as a node
- Can then apply standard graph search e.g. A*

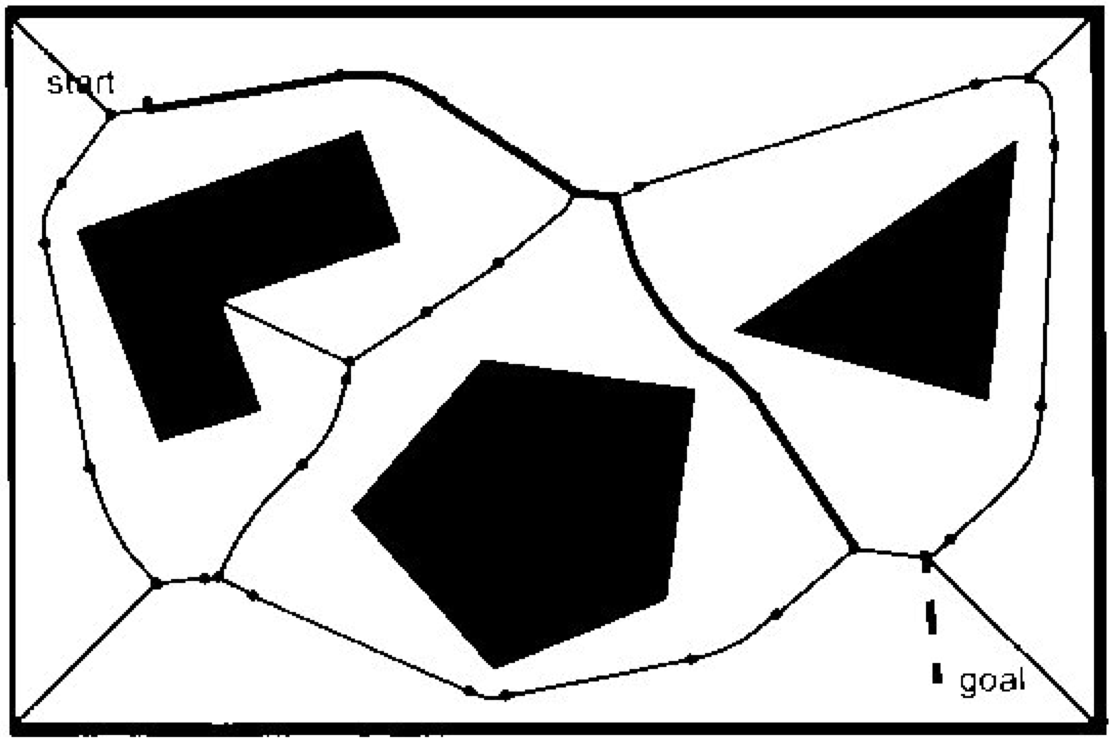
From metric to topological maps

Visibility graph: edges join all vertices that can 'see' each other. Defines shortest possible paths.



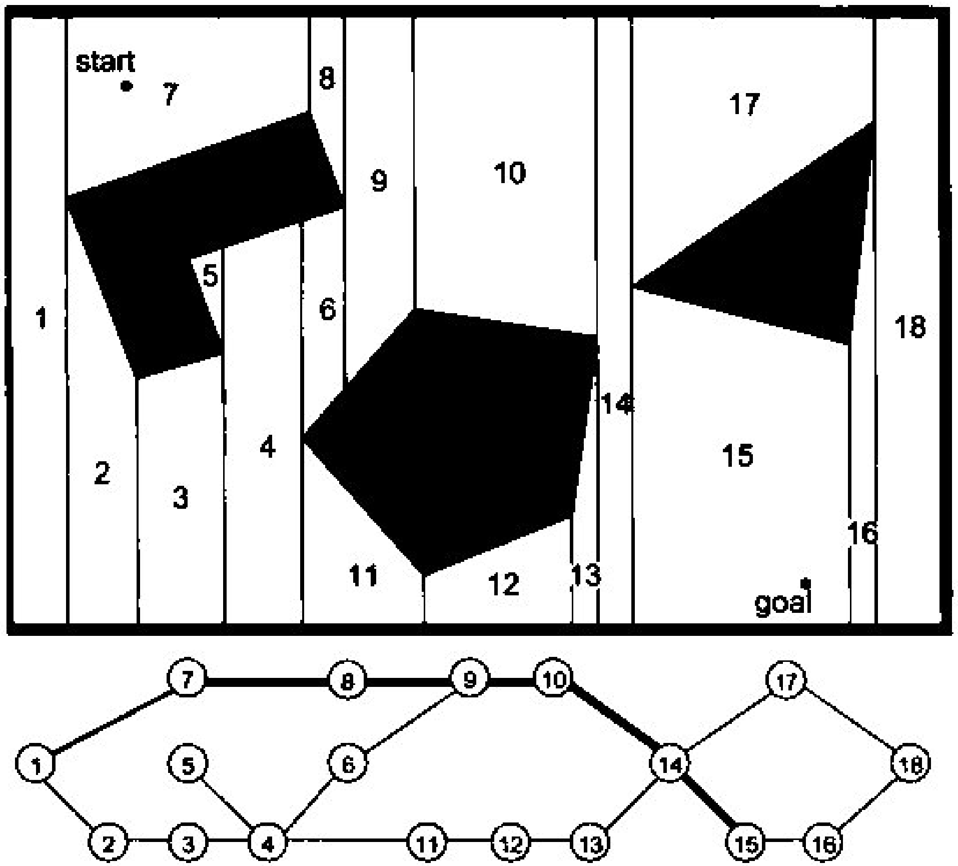
From metric to topological maps

- Voronoi graph – generate edges that are equidistant from obstacles, meeting at vertices.



From metric to topological maps

- Cell decomposition: define free and occupied geometric areas and determine which are adjacent



Path planning

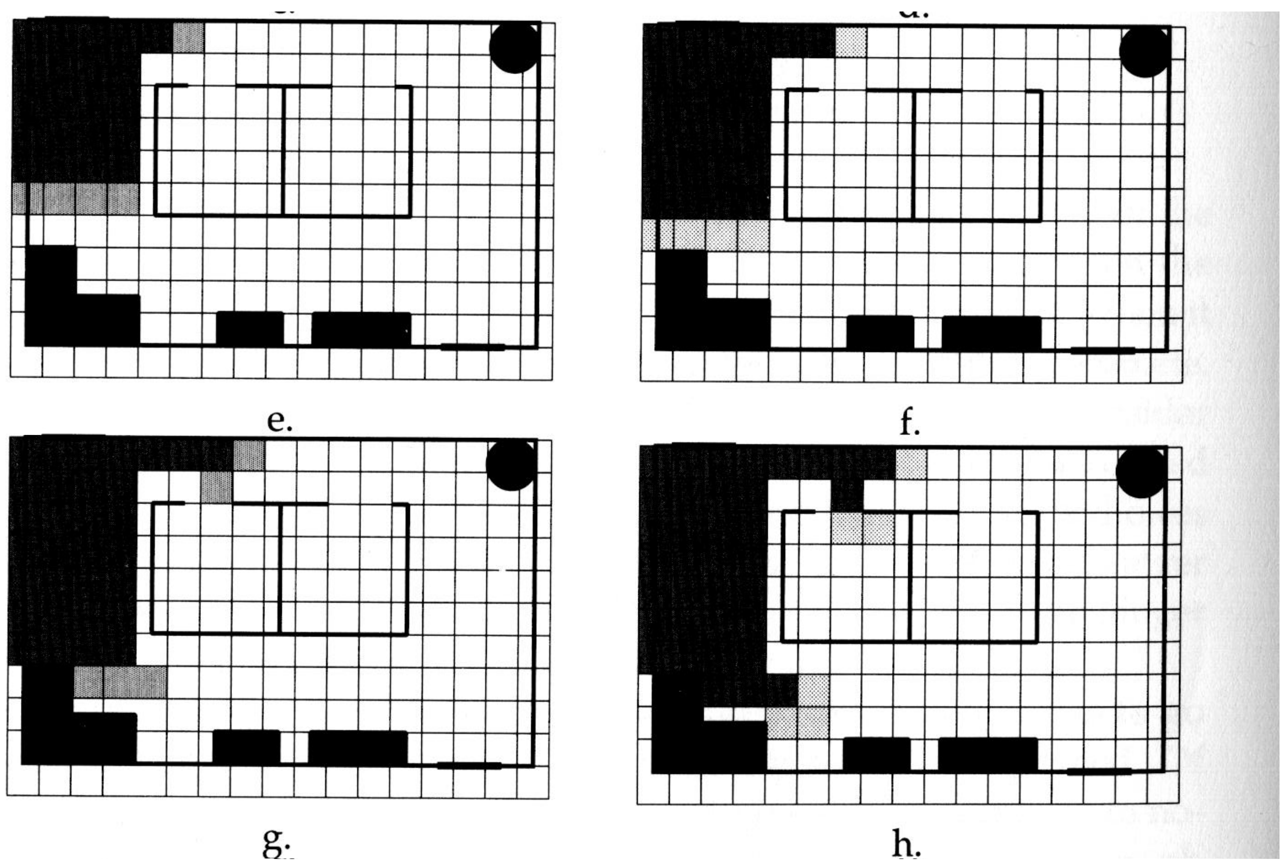
- For a graph with nodes connected by edges

$$\mathbf{f(n)=g(n)+\varepsilon h(n)}$$

- f(n) is the "goodness" of the path via node n
- g(n) is the "cost" of going from the Start to node n
- h(n) is the cost of going from n to the Goal
- **ε** is relative weighting of these costs
- Use c(n,n'): cost from node n to adjacent node n'

Breadth-first search

If **ε=0**, and c(n,n') is constant for all n (e.g. in grid) then breadth first search will find optimal route.



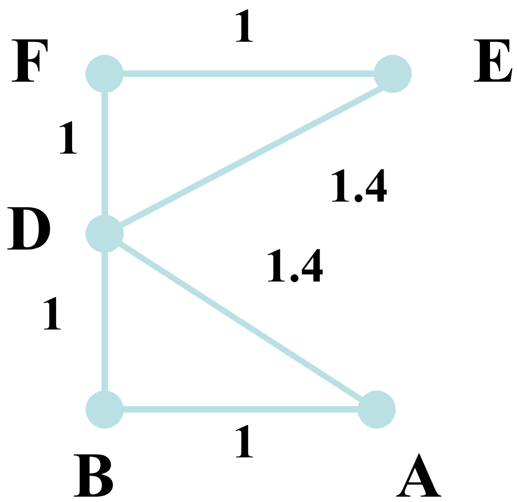
A* Heuristic Function

If **ε=1**, and c(n,n') is not constant, A* is a more efficient search. Like breadth-first except always expand the 'best' (least cost) node first (note same method with **ε=0** is Dijkstra's algorithm)

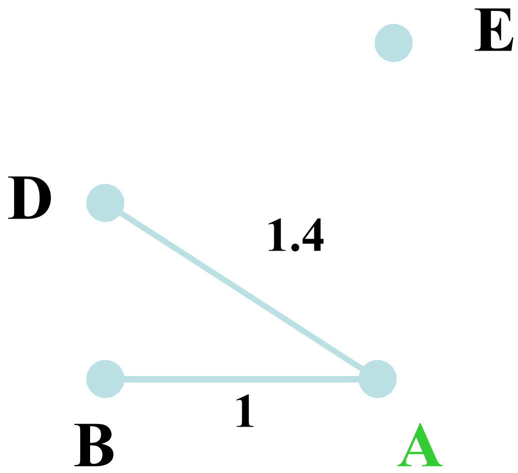
$$\mathbf{f^*(n)=g^*(n)+h^*(n)}$$

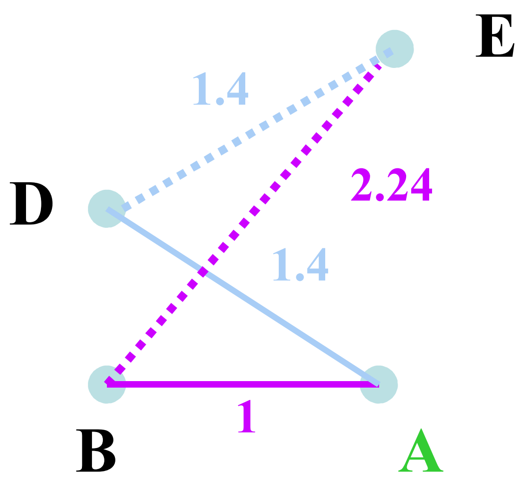
- g*(n) is easy: just sum up the path costs to n
- h*(n) is tricky
  - But if began with metric map, may know the *direct* distance between any two nodes, even if not what path is needed to get between them.
  - Thus a minimal estimate of the remaining cost we can use for h*(n) is the direct distance between n and Goal

Example: A to E



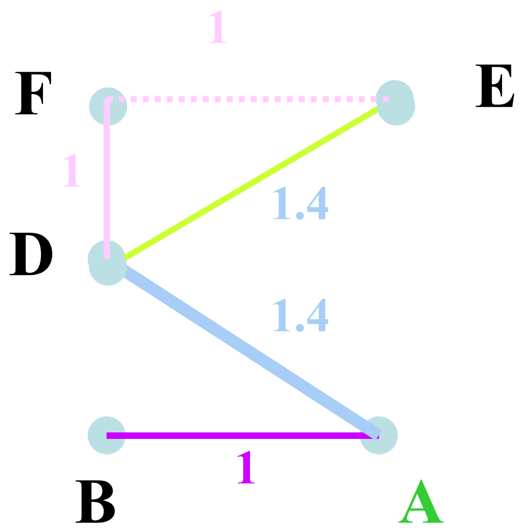
- But since you're starting at A and can only look 1 node ahead, this is what you see:





- Two choices for n: B, D
- Do both
  - f*(B)=1+2.24=3.24
  - f*(D)=1.4+1.4=2.8
- Expand the most plausible path first => A-D-?-E

- A-D-?-E
  - "stand on D"
  - Can see 2 new nodes: F, E
  - f*(F)=(1.4+1)+1=3.4
  - f*(E)=(1.4+1.4)+0=2.8



- Three paths
  - A-B-?-E >= 3.24
  - A-D-E = 2.8
  - A-D-F-?-E >=3.4

- A-D-E is the winner!
  - Don't have to look farther because expanded the shortest first, others couldn't possibly do better without having negative distances, violations of laws of geometry…

Planning as optimisation

- The problem of planning can be formulated as an optimisation problem:
  - Robot has goals, but actions have costs
  - Express both as a single 'pay-off' function, e.g.,

$$r(x,u) = \begin{cases} +100 & \textit{If reach the desired state} \\ -1 & \textit{Otherwise, i.e., cost for each time step} \end{cases}$$

Aim is to maximise the cumulative expected pay-off:

$$R_T = E\left[\sum_{\tau=1}^{T} \gamma^{\tau} r_{t+\tau}\right]$$

Where $0<\gamma<1$ is a discount factor, making distant reward less attractive.

Optimal Policy

- 1-step optimal policy:

$$\pi_1(x) = \underset{u}{\operatorname{argmax}}\ r(x,u)$$

- Value function of 1-step optimal policy:

$$V_1(x) = \gamma \max_u r(x,u)$$

- T-step optimal policy is defined recursively:

$$\pi_T(x) = \underset{u}{\operatorname{argmax}} \left[r(x,u) + \sum_{i=1}^{N} V_{T-1}(x_i)\, p(x_i \mid u, x) \right]$$

N is number of possible states

*Value of the **next** state*

State transition probabilities

- In theory this can sometimes be solved; in practice usually obtain the value function by iteration till the approximation converges

Summary

- Local navigation strategies suffice for some robot tasks
- Local strategies can be linked to form routes
- Routes can be linked to form maps:
  - A map is needed to plan novel routes (exception?)
- Choice of map representation has many consequences:
  - How can it be acquired?
  - How much information must be stored?
  - How can it be used to find novel routes?

References:

M. J. Milford, G. Wyeth (2008) ”Mapping a Suburb With a Single Camera Using a Biologically Inspired SLAM System,“ *IEEE Transactions on Robotics*, 24: 1038-1052

M. J. Milford (2013) Vision-based place recognition: how low can you go? The International Journal of Robotics Research 32 (7), 766-789

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