Probabilistic approaches

IAR Lecture 8

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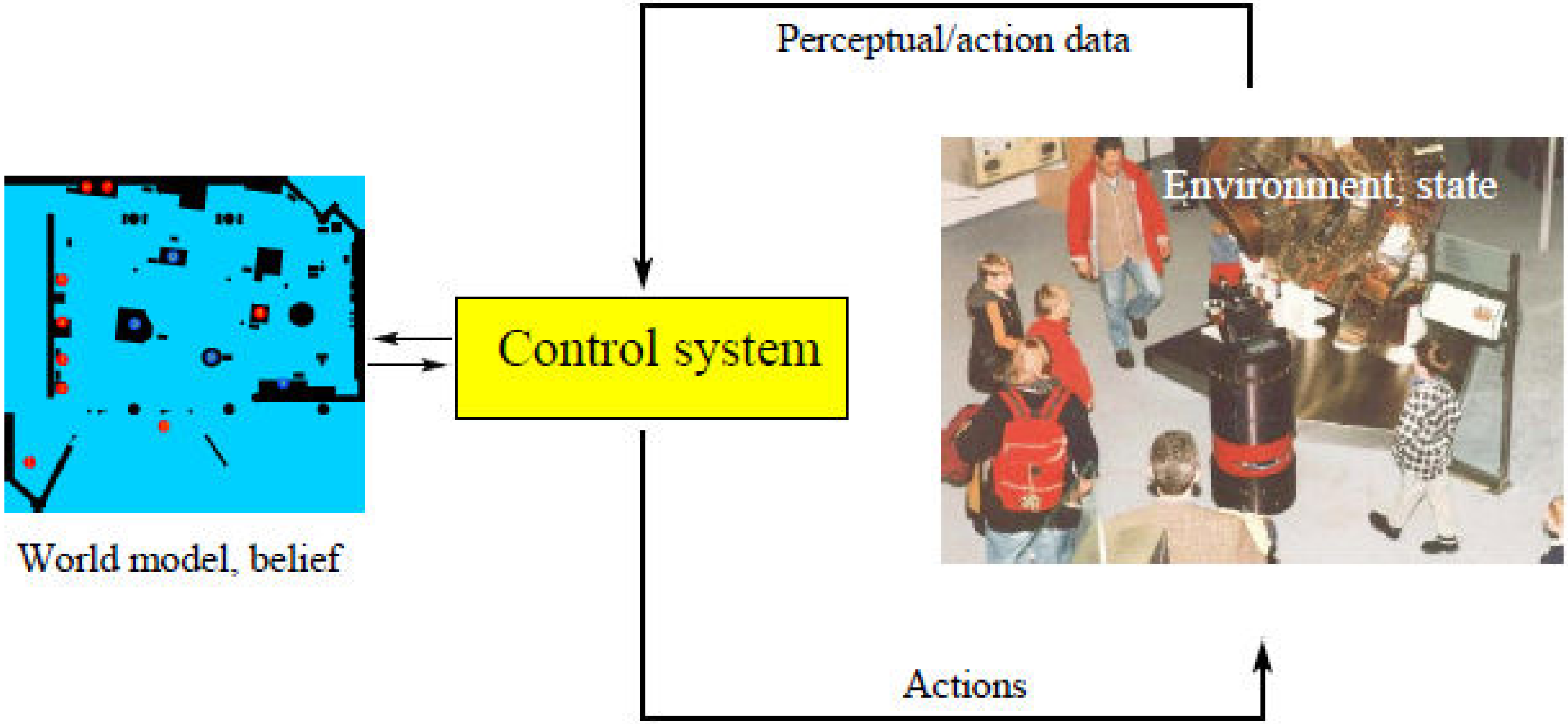
Historically there have been different approaches to dealing with the inherent uncertainty in robotics

Model-based Principled but brittle	Assume everything is known, or engineer robot or situation so this is approximately true	sense→plan→act
Reactive Robust and cheap but unprincipled	Assume nothing is known, use immediate input for control in multiple tight feedback loops	sense→act sense→act
Hybrid Best and worst of both ?	Plan for ideal world, react to deal with run-time error	plan ⇩ sense→act
Probabilistic Principled, robust but computationally expensive	Explicitly model what is not known	sense→ plan → act with uncertainty

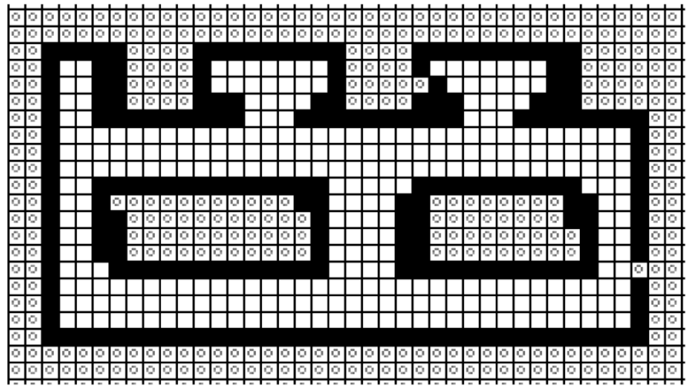
- A control system that includes a world model can interact more competently and flexibly with the world, e.g. by planning and anticipating.

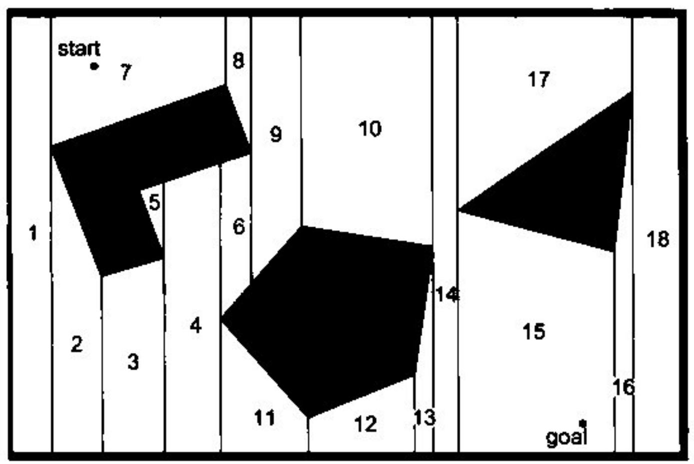


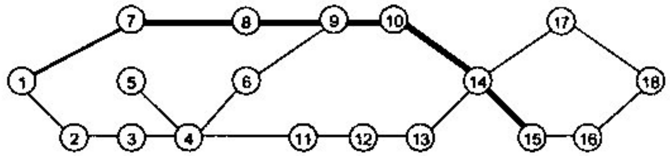
- But an incorrect or unreliable world model can be worse than no model.
- Crucial idea is that you need to know the limits of what you know.

E.g.Assuming I have a map, where am I?

- How can a robot know where it is?
- E.g. determine its *pose, [x,y,θ]*
- In general this is a problem of position estimation (i.e. same methods could apply to external tracking or object localisation)
- Usually, the immediately available sensory evidence will not be enough to determine position precisely and unambiguously.
- Basic method is to infer location from sensor measurements over time, while moving in space.

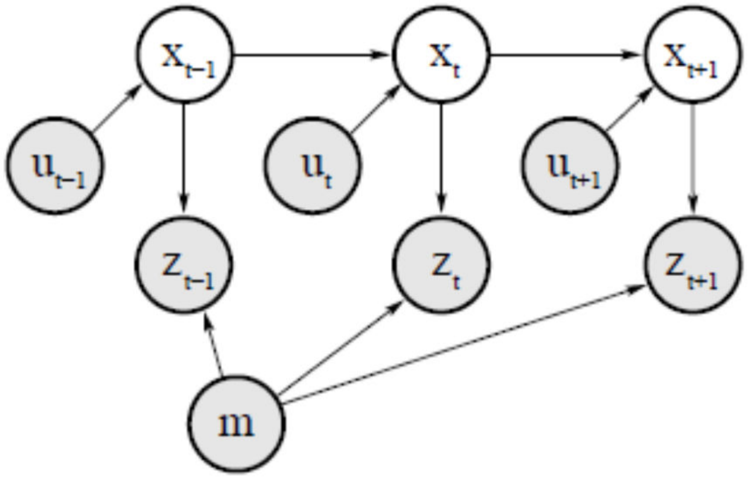






‘Markov’ localisation

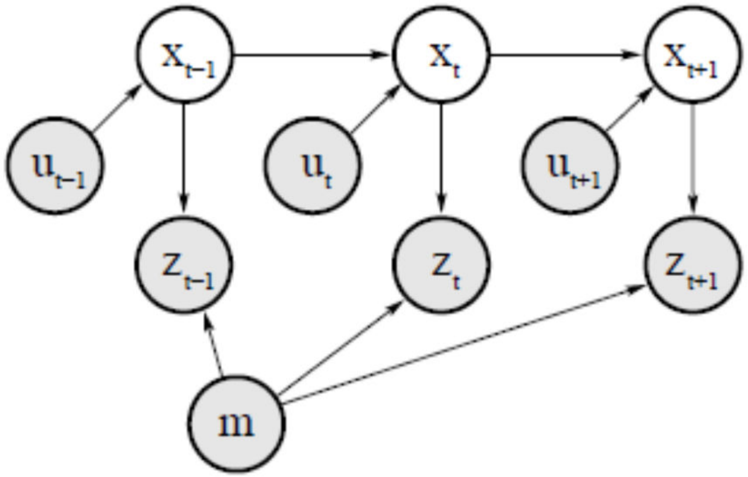
- Control action *u* will put the robot in state *x,* which for a given map *m* should result in sensor measurements, *z*.



- Markov assumption is that current state sufficiently represents the history of all previous states, so that only current control action determines the state transition, and new state determines the measurement (i.e., at x_t, $u_{1:t-1}$ and $z_{1:t-1}$ no longer matter)
- This is almost always false, but is a very useful approximation

‘Markov’ localisation

- Control action *u* will put the robot in state *x,* which for a given map *m* will result in sensor measurements, *z.*



- **LOCALISATION PROBLEM:** Assuming the robot knows the map *m*, the control actions *u,* and the measurements *z*, it wants to infer its current state (its location or pose) *x*

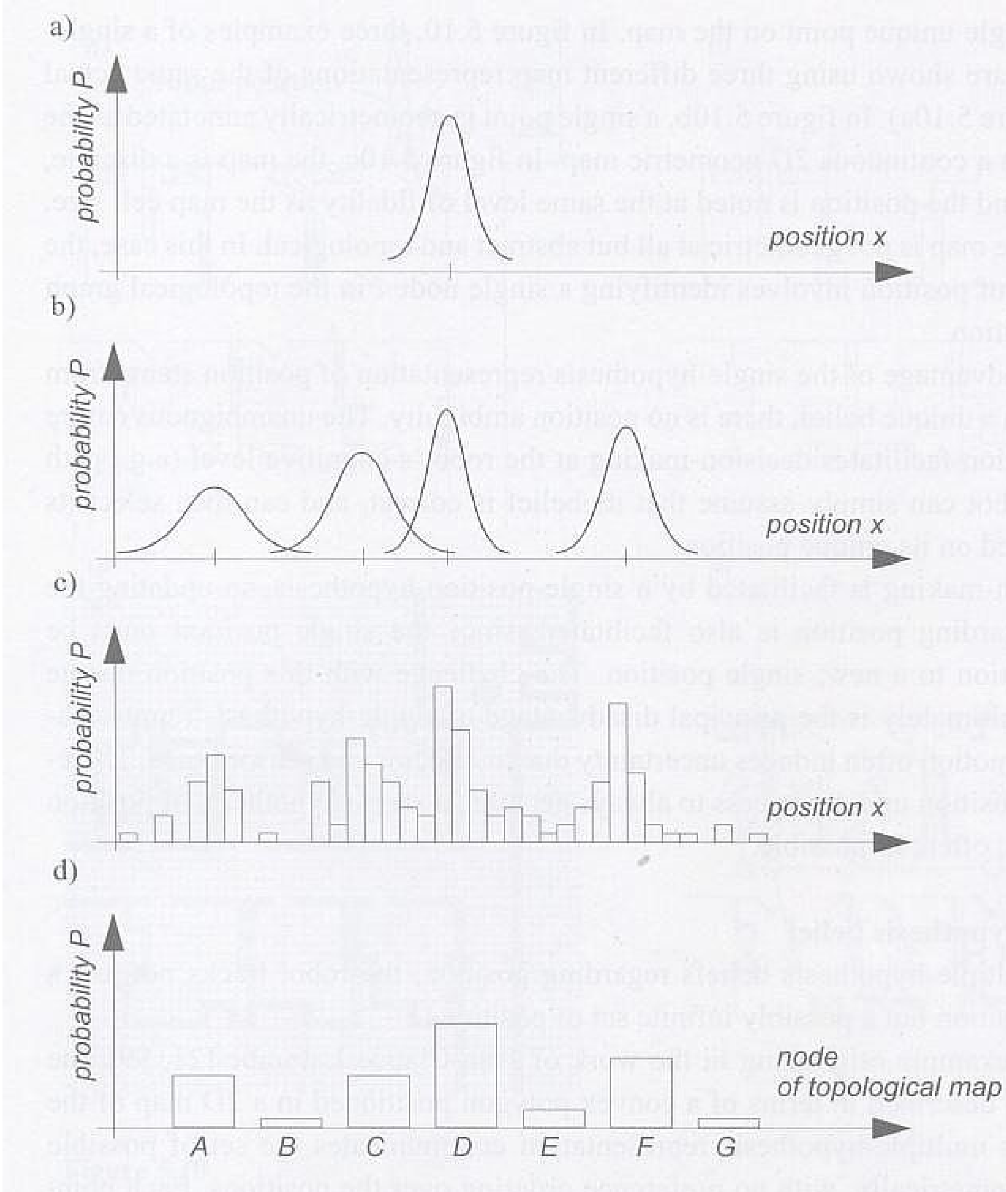
Representing the robot's belief $Bel(x_t)$ about its current state

Single hypothesis

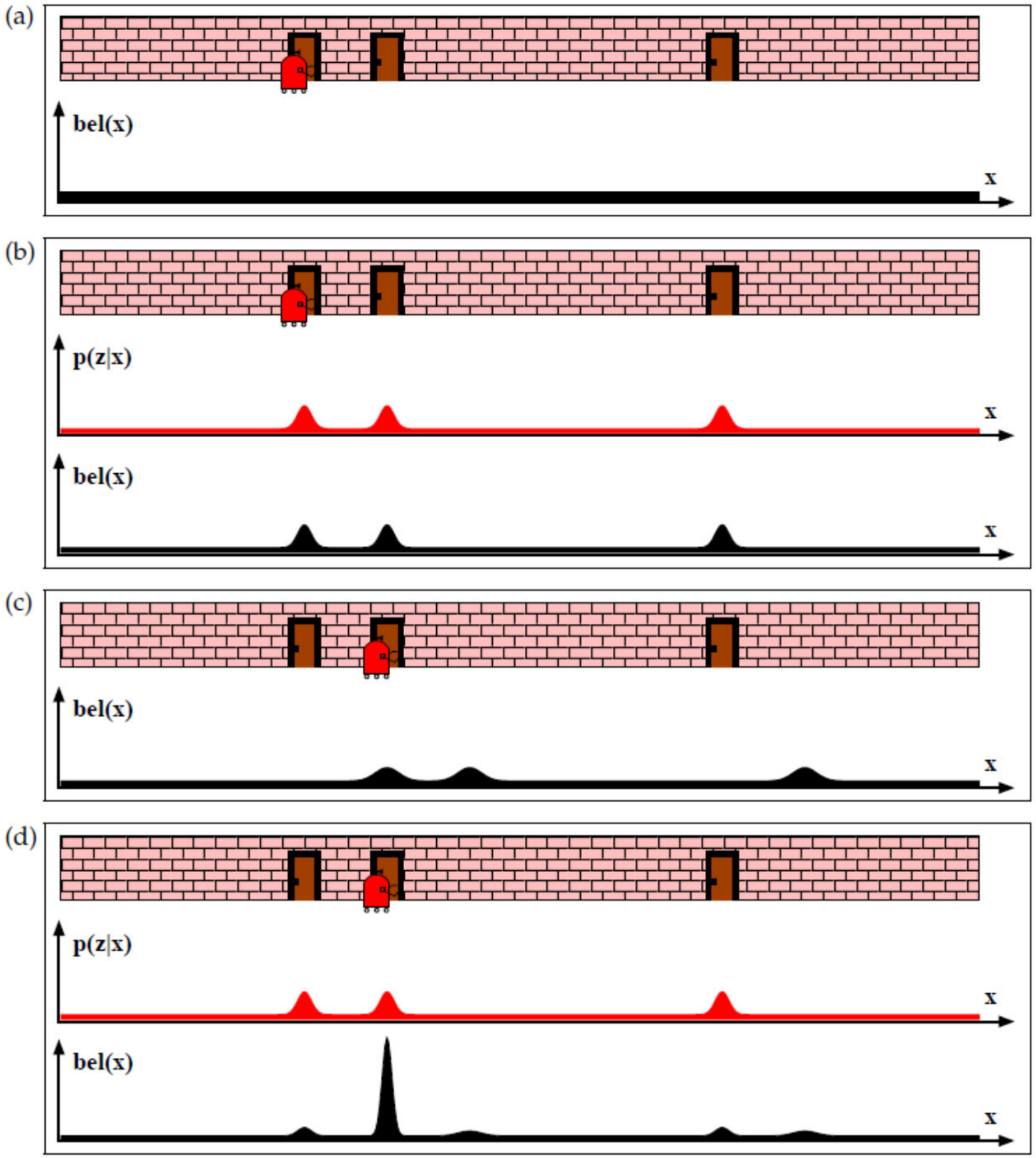
Multiple hypotheses

Probability of each grid location

Probability of each node in a topological map



Siegwart & Nourbakhsh 2004



1. Robot starts with equal probability for every possible location x

2. Measurement z indicates robot is near a door: get three peaks in position estimate

3. Robot moves to the right: updates position estimate but becomes less certain

4. New measurement indicates robot is near door: this makes one possible position more likely than the others

z = observation
u = action
x = state

Bayes Filter (1)

Bayes Theorem

$$P(a \mid b) = \frac{P(b \mid a)\ P(a)}{P(b)}$$

Conditioned on c

$$P(a \mid b, c) = \frac{P(b \mid a, c)\ P(a \mid c)}{P(b \mid c)}$$

$$Bel(x_t) = P(x_t \mid z_{1:t}, u_{1:t})$$

Bayes

$$= \frac{P(z_t \mid x_t, z_{1:t-1}, u_{1:t})\ P(x_t \mid z_{1:t-1}, u_{1:t})}{\boxed{P(z_t \mid z_{1:t-1}, u_{1:t})}}$$

Normalising term not dependent on state x

$$= \boxed{\eta}\ \boxed{P(z_t \mid x_t, z_{1:t-1}, u_{1:t})}\ P(x_t \mid z_{1:t-1}, u_{1:t})$$

Measurement z does not depend on history

Markov

$$= \eta\ \boxed{P(z_t \mid x_t)}\ P(x_t \mid z_{1:t-1}, u_{1:t})$$

z = observation
u = action
x = state

Bayes Filter (2)

Theorem of total probability

$$P(a) = \int P(a \mid b) P(b)\, db$$

Conditioned on c

$$P(a \mid c) = \int P(a \mid b, c) P(b \mid c)\, db$$

$$Bel(x_t) = P(x_t \mid z_{1:t}, u_{1:t})$$

$$= \eta\, P(z_t \mid x_t)\, P(x_t \mid z_{1:t-1}, u_{1:t})$$

Total prob.

$$= \eta\, P(z_t \mid x_t) \int \boxed{P(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t})} P(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1}$$

Markov

Effect of control u does not depend on history

$$= \eta\, P(z_t \mid x_t) \int \boxed{P(x_t \mid x_{t-1}, u_t)} P(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, dx_{t-1}$$

z = observation
u = action
x = state

Bayes Filter (3)

$$Bel(x_t) = P(x_t \mid z_{1:t}, u_{1:t})$$

$$= \eta\, P(z_t \mid x_t) \int P(x_t \mid x_{t-1}, u_t) \boxed{P(x_{t-1} \mid z_{1:t-1}, u_{1:t})}\, dx_{t-1}$$

Markov

Prior state x_{t-1} does not depend on current control u_t

$$= \eta\, P(z_t \mid x_t) \int P(x_t \mid x_{t-1}, u_t) \boxed{P(x_{t-1} \mid z_{1:t-1}, u_{1:t-1})}\, dx_{t-1}$$

This describes the belief at time t-1

$$= \eta\, P(z_t \mid x_t) \int P(x_t \mid x_{t-1}, u_t) \boxed{Bel(x_{t-1})}\, dx_{t-1}$$

Bayes Filter Summary

z = observation
u = action
x = state

$$\boxed{Bel(x_t)} = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Bayes $$= \eta \; P(z_t \mid x_t, u_1, z_1, \ldots, u_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Markov $$= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Total prob. $$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov $$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov $$= \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \, P(x_{t-1} \mid u_1, z_1, \ldots, z_{t-1}) \; dx_{t-1}$$

$$\boxed{= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}}$$

Bayes Filter is usually described as a two step process

- Prediction

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx_{t-1}$$

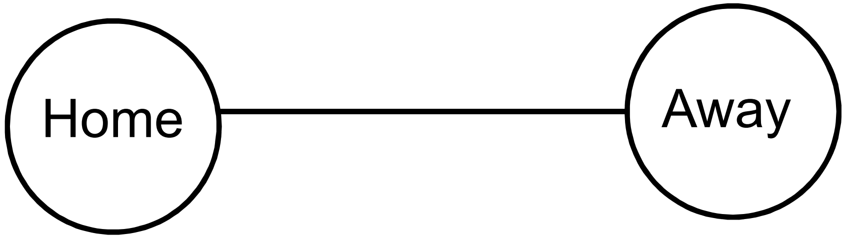
Or for discrete state values:

$$= \sum_x p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})$$

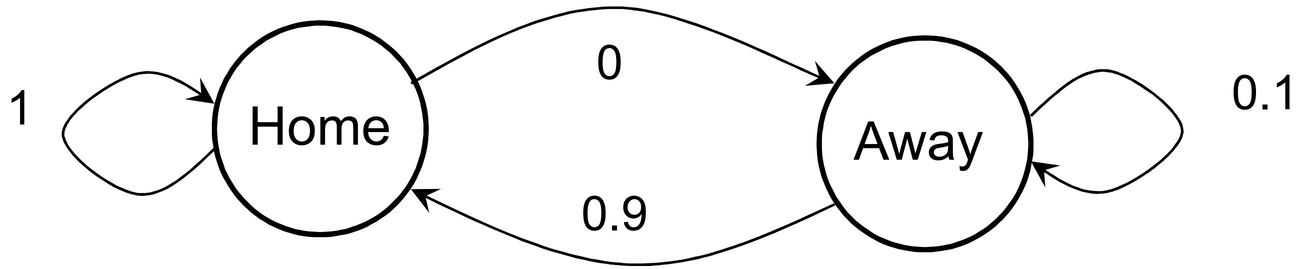
- Correction

$$bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}(x_t)$$

Example: two node topological map



- Effect of action "go home"



- Perception: P("see home"|home)=0.7, P("see home"|away)=0.1
- If robot starts with *Bel(home)=0.2*, takes the action "go home", and sees home, what is the new *Bel(home)*?

References:

Sebastian Thrun, Wolfram Burgard and Dieter Fox, “Probabilistic Robotics”, MIT Press, Cambridge MA, 2005

Roland Siegwart & Illah Nourbakshsh “Introduction to Autonomous Robotics” MIT Press, Cambridge MA, 2011