

Sensing for action

IAR Lecture 3

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Sensing for Action

- Sensors transduce energy from one form to another
- From the robot control point of view we have some information – a measured value – that represents some property of the world
- This relationship is rarely a direct one:

E.G. We say the IR sensor is a ‘range’ or ‘distance’ sensor:

Distance to object →

Light scattering →

Amount of light reflected →

Resistance of sensor element →

Voltage →

Analog to Digital Conversion →

Calculation →

Distance value

But note! We may not need to know the actual distance to perform the appropriate action, such as “avoid”

Describing sensors

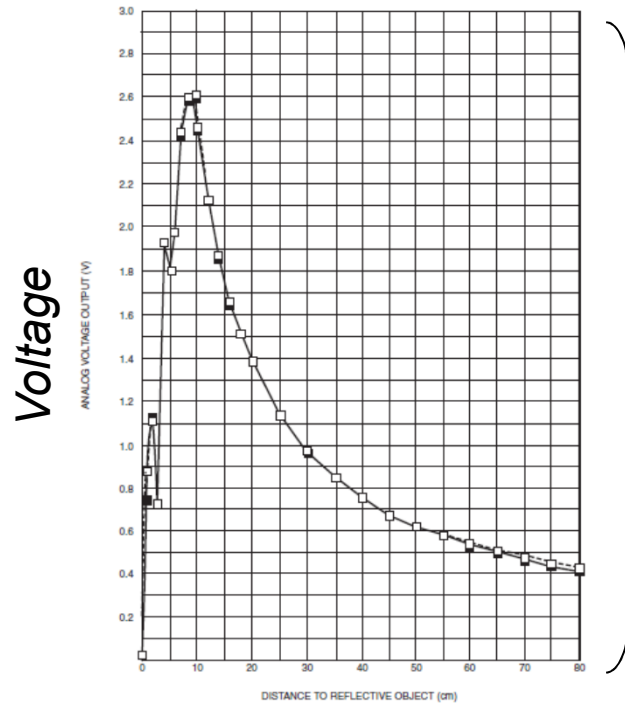
- *Sensitivity*:
 - ratio of output change to input change
 - Usually a trade-off with *range* (min to max)
- *Resolution*:
 - Limit in resolving power of output scale
- *Precision*:
 - Repeatability of measurements (under same conditions)
- *Accuracy*:
 - (lack of) error in measurements

Accuracy

- Sometimes described in terms of the mean of the error (*precision* relates to the variance of error)
- Calibration can remove some but not all inaccuracy. E.g. for a linear sensor there may remain:
 - Uncertainty about the offset
 - Uncertainty about the slope (% error)
 - Uncertainty about deviations from linearity
- Combined with imprecision, inaccuracy may limit the *effective* resolution much more than the output scale

Example: IR sensors on the Khepera

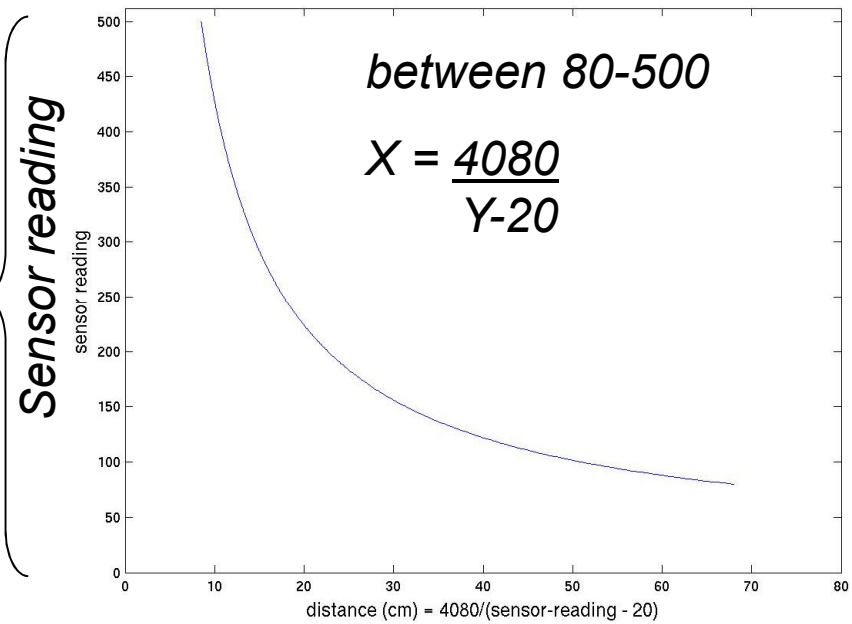
Sensor data sheet



Real distance to object

*10 bit Analog
to Digital
Conversion*

Calculation of distance from reading



Estimated distance to object

Describing sensors

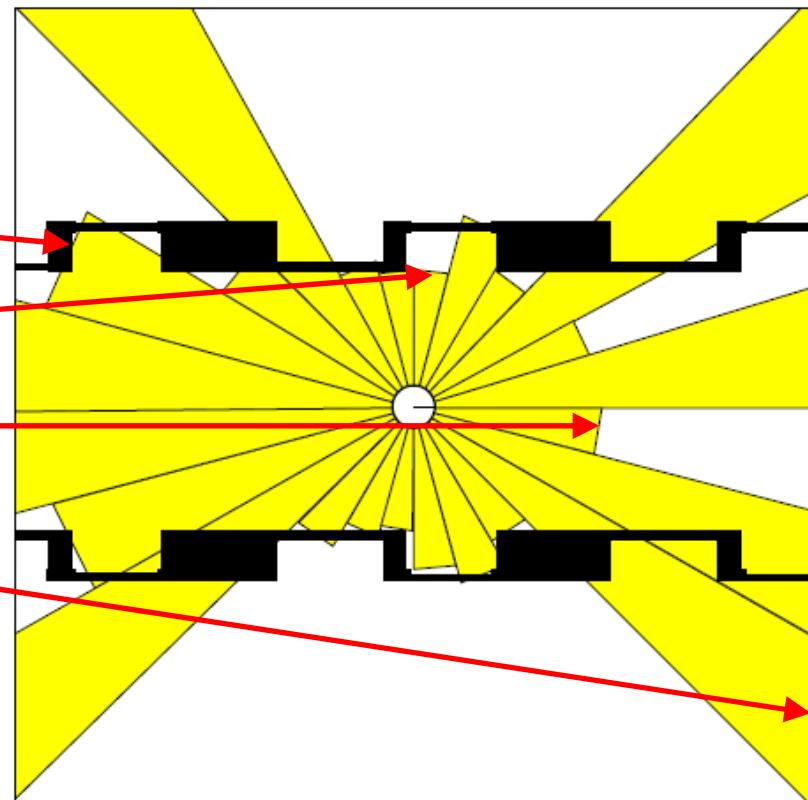
- *Selectivity*
 - Inaccuracy is often the result of *cross-sensitivity* to environmental properties *other* than the target property
 - E.g. many transducers are affected by temperature
 - For IR, reflectance of the object and ambient light will alter the ‘distance’ reading

Describing in form of a *sensor model*

- E.g. what is the probability distribution of the sensor reading from a range sensor, given the wall distance?

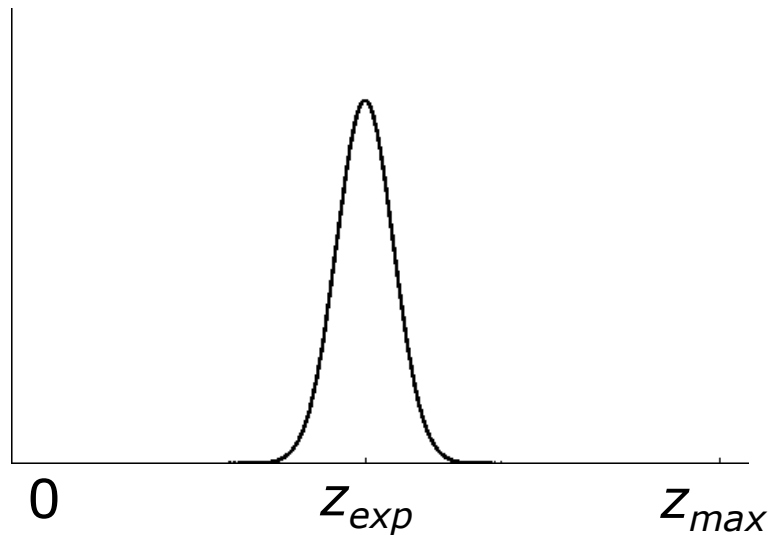
Possible sources of error:

1. Noise in the actual distance measure
2. Person passing
3. Random measurements
4. Maximum range measurements



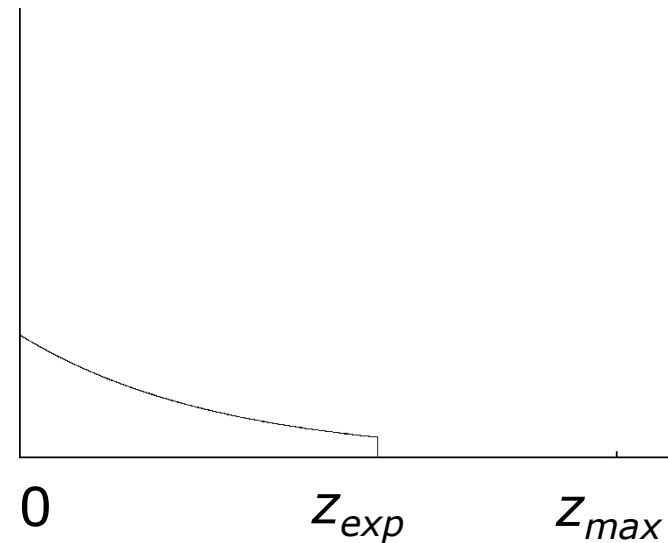
Describe each error source as a distribution

Measurement noise



$$P_{hit}(z | x, m) = \eta \frac{1}{\sqrt{2\pi b}} e^{-\frac{1}{2} \frac{(z - z_{exp})^2}{b}}$$

Unexpected obstacles

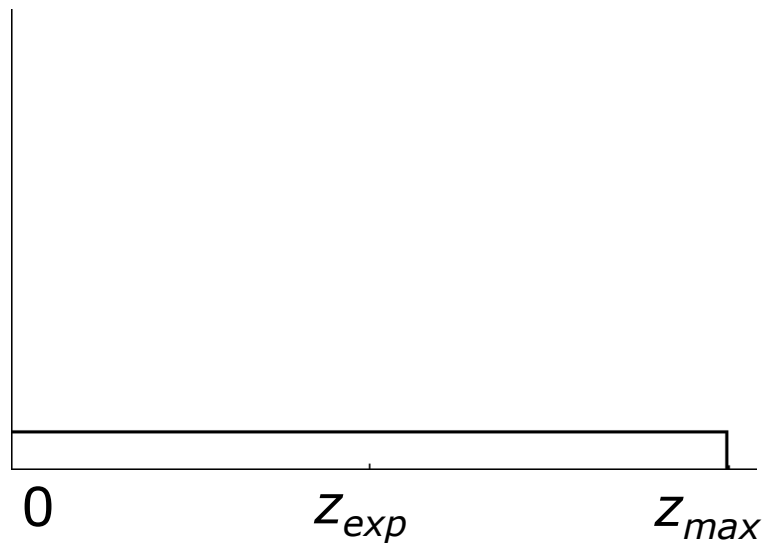


$$P_{unexp}(z | x, m) = \begin{cases} \eta \lambda e^{-\lambda z} & z < z_{exp} \\ 0 & otherwise \end{cases}$$

z is sensor signal, x is robot pose, m is world model, $\{x, m\}$ define expected z_{exp}

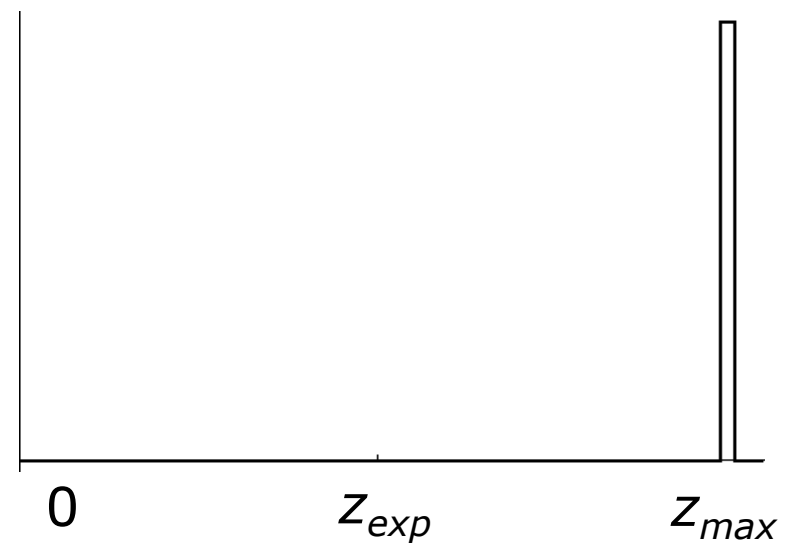
Describe each error source as a distribution

Random measurement



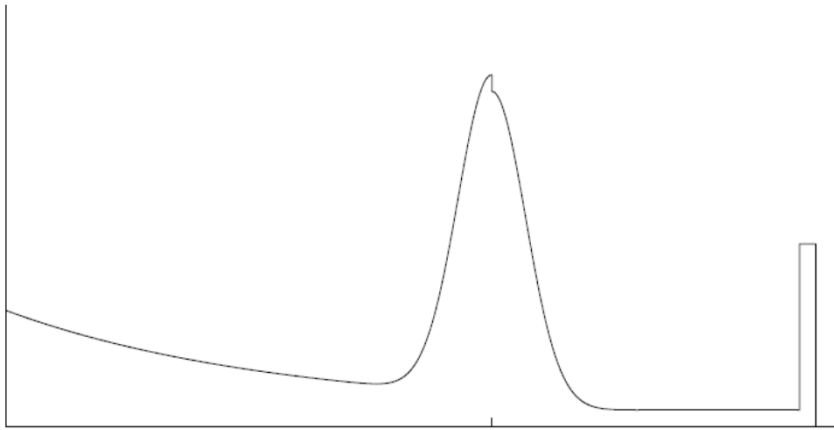
$$P_{rand}(z | x, m) = \eta \frac{1}{z_{max}}$$

Max range



$$P_{max}(z | x, m) = \begin{cases} 1 & \text{if } z = z_{max} \\ 0 & \text{otherwise} \end{cases}$$

Combine as a mixture density



$$P(z | x, m) = \begin{pmatrix} \alpha_{\text{hit}} \\ \alpha_{\text{unexp}} \\ \alpha_{\text{max}} \\ \alpha_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} P_{\text{hit}}(z | x, m) \\ P_{\text{unexp}}(z | x, m) \\ P_{\text{max}}(z | x, m) \\ P_{\text{rand}}(z | x, m) \end{pmatrix}$$

Note, this seems to assume we know the real distance, z_{exp}

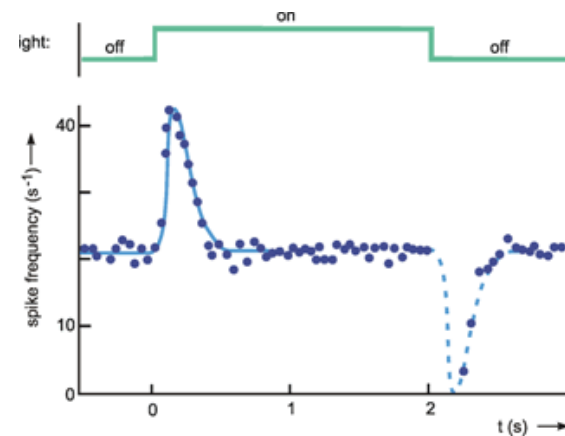
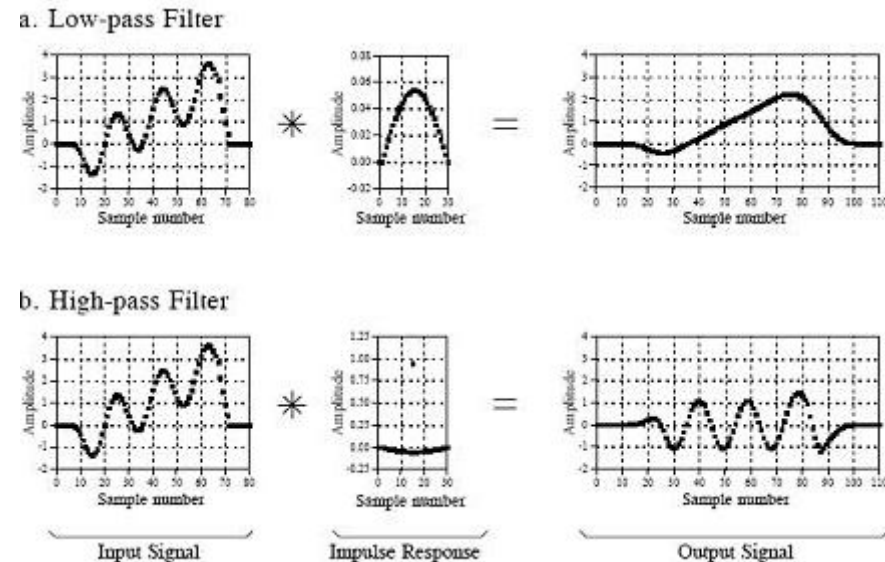
- May be in context of calibration; use to learn parameters α
- Knowing $P(z|z_{\text{exp}})$ can be applied (through Bayes theorem) to determine $P(z_{\text{exp}}|z) = P(z|z_{\text{exp}})P(z_{\text{exp}})/P(z)$ (see later lectures)

Sensor/signal conditioning

- E.g. linear transformation: $output = offset + gain \times input$
- Many signals may need non-linear transformation
- Might need to tune linear or non-linear parameters through learning methods (again, this can be action-relative)
- ‘Intelligence’ might be introduced at this level to make sensing adaptive, i.e., sensor/system itself detects:
 - Is the output a reasonable value (e.g. relative to previous measurements or other sensor reports)?
 - Is the full range being used?
 - Is the sensor stuck at one extreme?

Sensor/signal conditioning

- Low-pass filtering:
 - Low-pass: usually against noise or other rapid fluctuations
- Highpass filtering:
 - interested in fluctuations not background



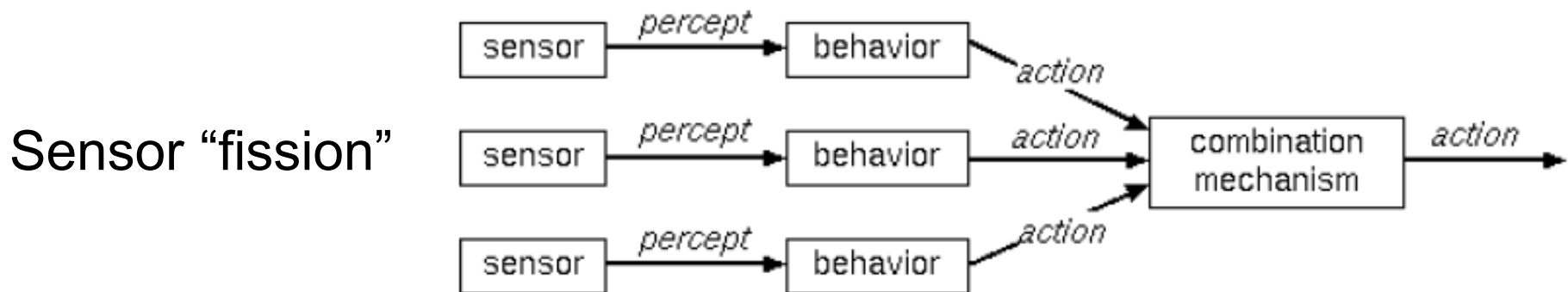
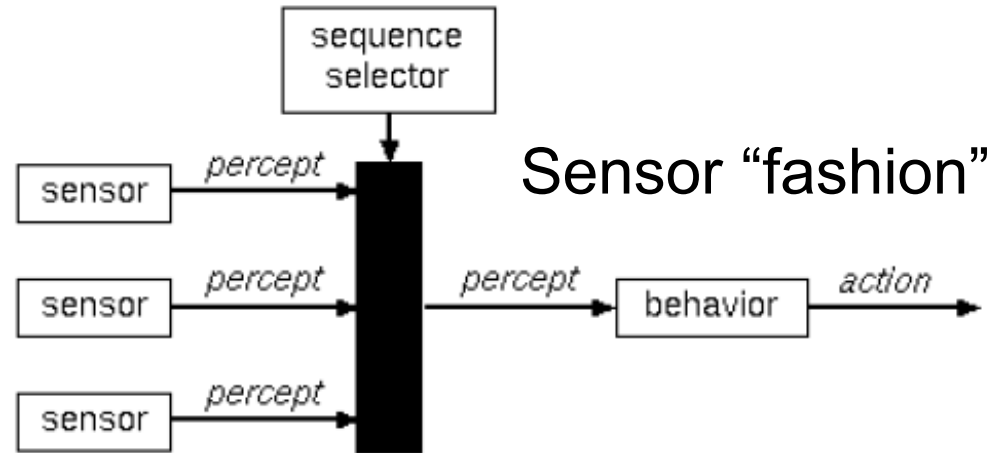
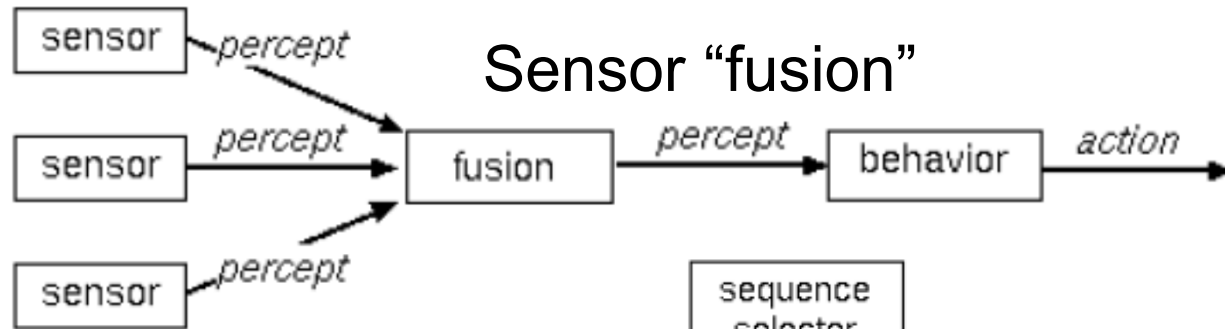
Response of *Limulus* sensory neuron to light

Sensor processing

Implies a more complex transformation than conditioning:

- Logic functions (e.g. triggers for action)
- Data reduction (e.g. extracting features)
- Decision making (e.g. classification)

Combining sensors



Sensor fusion

The information provided by different sensors might be:

- **Complementary:** sensors that measure different attributes of same target → Fusion could provide richer description
- **Co-operative:** can derive new feature by combining several attributes (e.g. triangulation) → Fusion could disambiguate
- **Competitive/redundant:** different sensors that measure the same attribute → Fusion could provide better estimate of actual value

Sensor fusion

A standard approach is to use a weighted average.

Assume N sensors provide measurements z of property x with some Gaussian distributed noise

$$z_i = x + \varepsilon_i, \varepsilon_i \approx N(0, \sigma_i)$$

Combined estimate is weighted average:

$$\hat{x} = \sum_{i=1}^N w_i z_i, \quad \sum_{i=1}^N w_i = 1$$

Maximum likelihood estimation says optimal weighting is:

$$w_i = \frac{1/\sigma_i^2}{\sum_{j=1}^N 1/\sigma_j^2}$$

Note there are also adaptive methods that modify the weights over time, e.g. *democratic cue integration*: sensors with values near the combined estimate increase their weights, those further away decrease.

Simple example with two measurements

Robot uses two different sensors to measure distance to a wall:

z_1 with variance σ_1^2

z_2 with variance σ_2^2

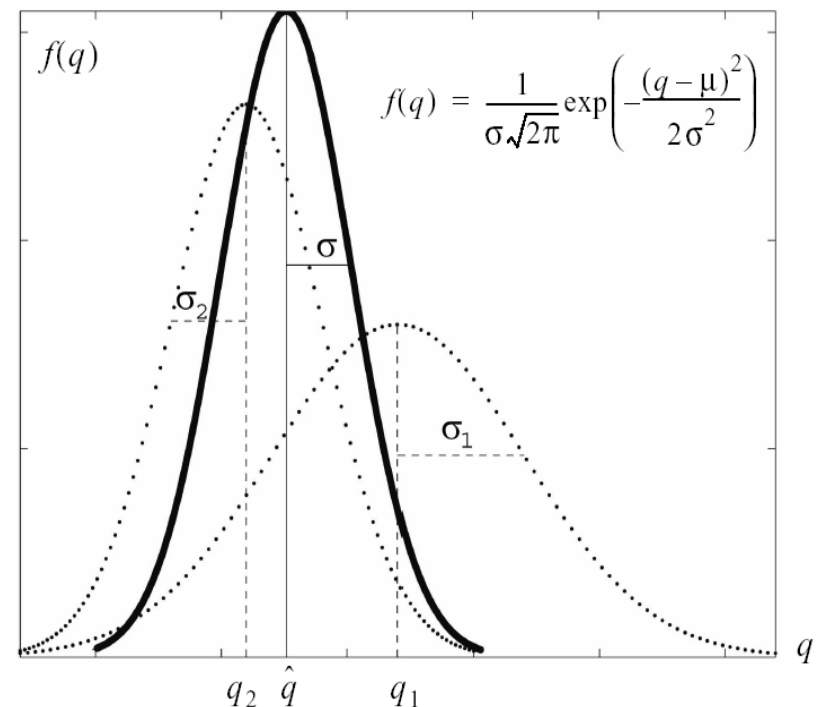
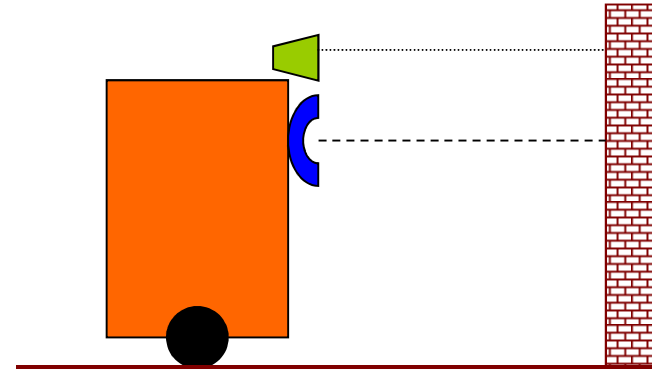
Combined estimate:

$$\hat{x} = \frac{1/\sigma_1^2}{1/\sigma_1^2 + 1/\sigma_2^2} z_1 + \frac{1/\sigma_2^2}{1/\sigma_1^2 + 1/\sigma_2^2} z_2$$

Variance of combined estimate –

$$\sigma^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

– will be less than that of either single measure



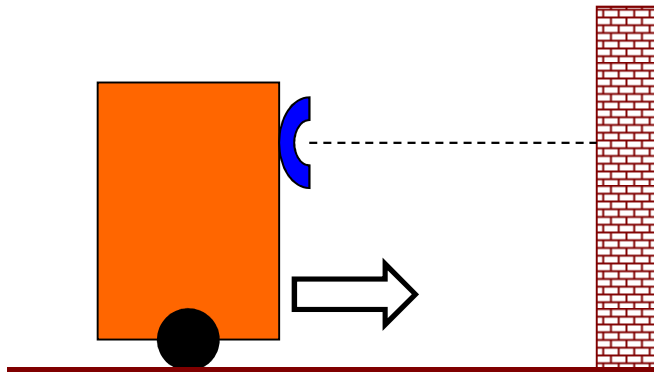
Can rearrange in terms of successive estimates:

$$\begin{aligned}\hat{x} &= \frac{1/\sigma_1^2}{1/\sigma_1^2 + 1/\sigma_2^2} z_1 + \frac{1/\sigma_2^2}{1/\sigma_1^2 + 1/\sigma_2^2} z_2 \\ &= z_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} (z_2 - z_1)\end{aligned}$$

Or in general, updating estimate with the $k+1^{\text{th}}$ measurement:

$$\begin{aligned}\hat{x}_{k+1} &= \hat{x}_k + K_{k+1} (z_{k+1} - \hat{x}_k) \\ \sigma_{k+1}^2 &= \sigma_k^2 - K_{k+1} \sigma_k^2\end{aligned}\quad K_{k+1} = \frac{\sigma_k^2}{\sigma_k^2 + \sigma_z^2} ; \sigma_k^2 = \sigma_1^2 ; \sigma_z^2 = \sigma_2^2$$

What about updating successive estimates for a moving robot?



For a moving robot, can first predict the change, then combine this with the new measurement

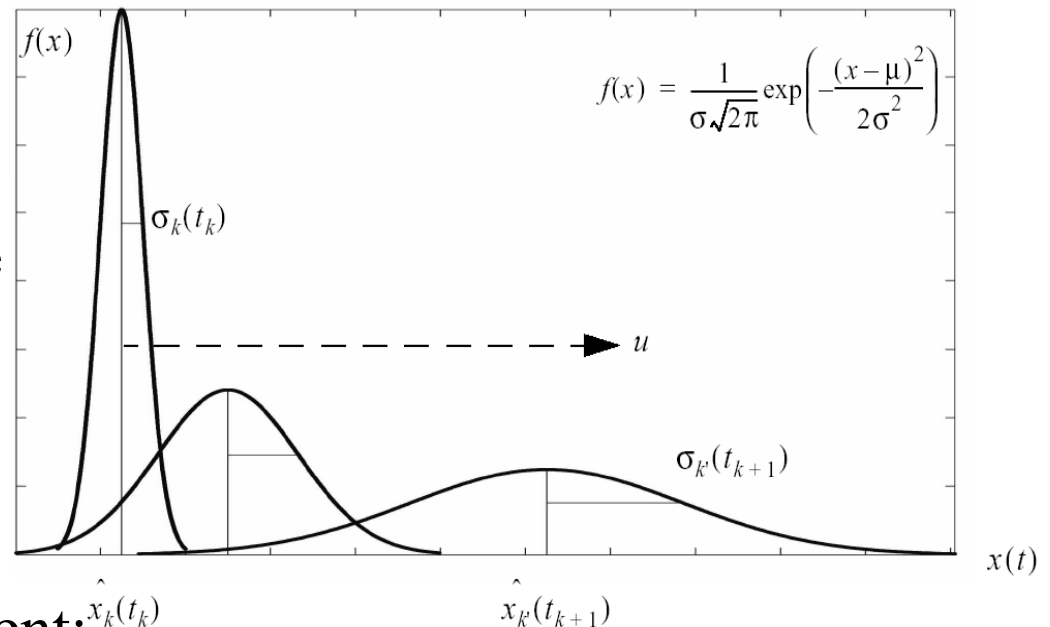
Robot moves at velocity u ,
with noise w

$$\frac{dx}{dt} = u + w$$

Our estimate should incorporate the predicted change:

$$\hat{x}_{k'} = \hat{x}_k + u(t_{k+1} - t_k)$$

$$\sigma_{k'}^2 = \sigma_k^2 + \sigma_w^2[t_{k+1} - t_k]$$



Then update with the measurement:

$$\hat{x}_{k+1} = \hat{x}_{k'} + K_{k+1}(z_{k+1} - \hat{x}_{k'})$$

$$= [\hat{x}_k + u(t_{k+1} - t_k)] + K_{k+1}[z_{k+1} - \hat{x}_k - u(t_{k+1} - t_k)]$$

Seigwart & Nourbakhsh, 2004

$$K_{k+1} = \frac{\sigma_{k'}^2}{\sigma_{k'}^2 + \sigma_z^2} = \frac{\sigma_k^2 + \sigma_w^2[t_{k+1} - t_k]}{\sigma_k^2 + \sigma_w^2[t_{k+1} - t_k] + \sigma_z^2}$$

Generalised form of this is the Kalman filter...

References

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