Exploiting dynamics

IAR Lecture 14

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Dynamical systems

- In general, refers to any system with a state that evolves over time
- More typically, refers to a system described by differential equations:

$$\dot{x} = f(x, \alpha, t)$$

Applied to robotics

- Describe how some behavioural variable changes in time, e.g. robot heading affected by targets and obstacles

$$\dot{\phi} = f_{obstacle}(\phi) + f_{target}(\phi)$$

- Express how the robot's state x changes with the the control commands u (note x,u might be vectors)

$$\dot{x} = f(x, u)$$

- Express the linked interaction of the robot state x_{agent} with the environment state x_{env}, where u_{agent}, u_{env} are parameters of the agent or environment, and S is a sensing function, M a motor function

$$\dot{x}_{agent} = f_{agent}(x_{agent}, S(x_{env}), u_{agent}),$$
$$\dot{x}_{env} = f_{env}(x_{env}, M(x_{agent}), u_{env})$$

Applied to collective robotics

- How do mixed groups of cockroaches and robots distribute themselves under shelters? (Halloy et al., 2007)
- x_i r_i are number of cockroaches, robots under shelter i; x_e r_e number in empty space. Time evolution of these variables depends on R (rate of entering shelter) and Q (rate of quitting shelter), determined by the carrying capacity of the shelter S.

$$dx_i/d_t = R_i x_e - Q_i x_i \quad i = 1, 2$$

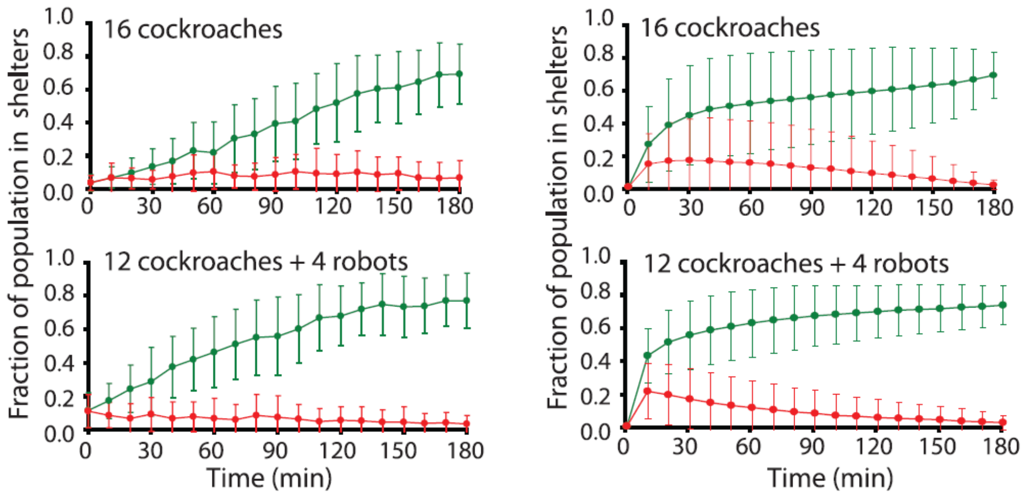
$$dr_i/d_t = R_{ri} r_e - Q_{ri} r_i \quad i = 1, 2$$

$$R_i = \mu_i\{1 - [(x_i + \omega r_i)/S_i]\}$$

$$R_{ri} = \mu_{ri}\{1 - [(x_i + \omega r_i)/S_i]\}$$

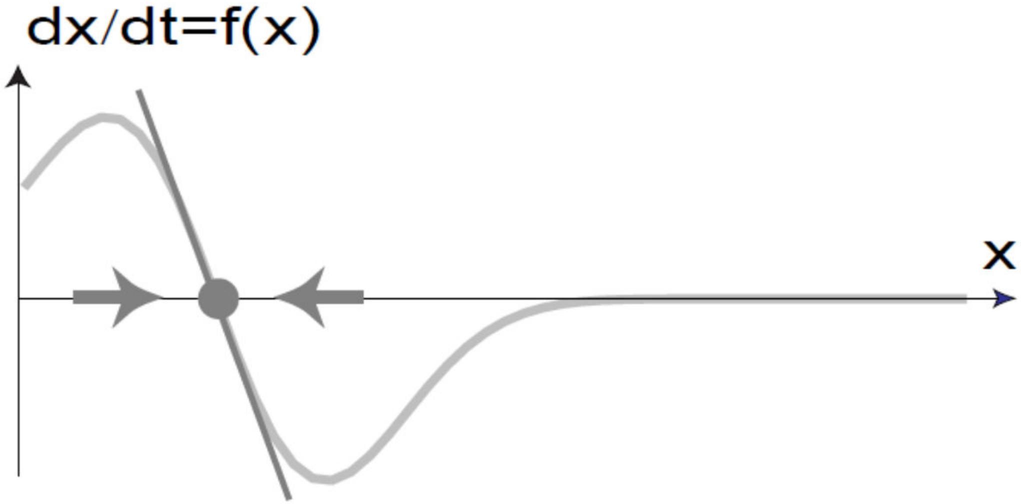
$$Q_i = \theta_i/\{1 + \rho[(x_i + \beta r_i)/S_i]^n\}$$

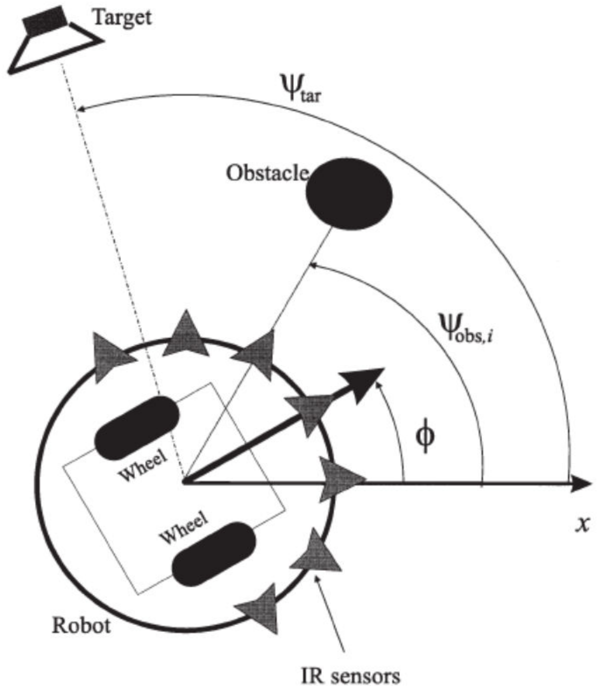
$$Q_{ri} = \theta_{ri}/\{1 + \rho_r[(\gamma x_i + \delta r_i)/S_i]^{n_r}\}$$

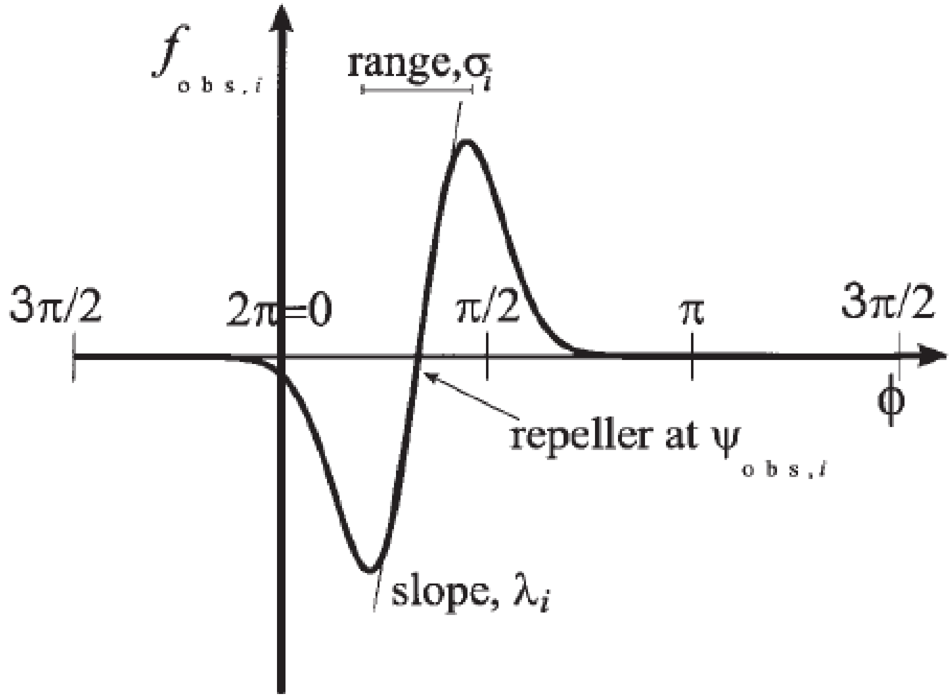


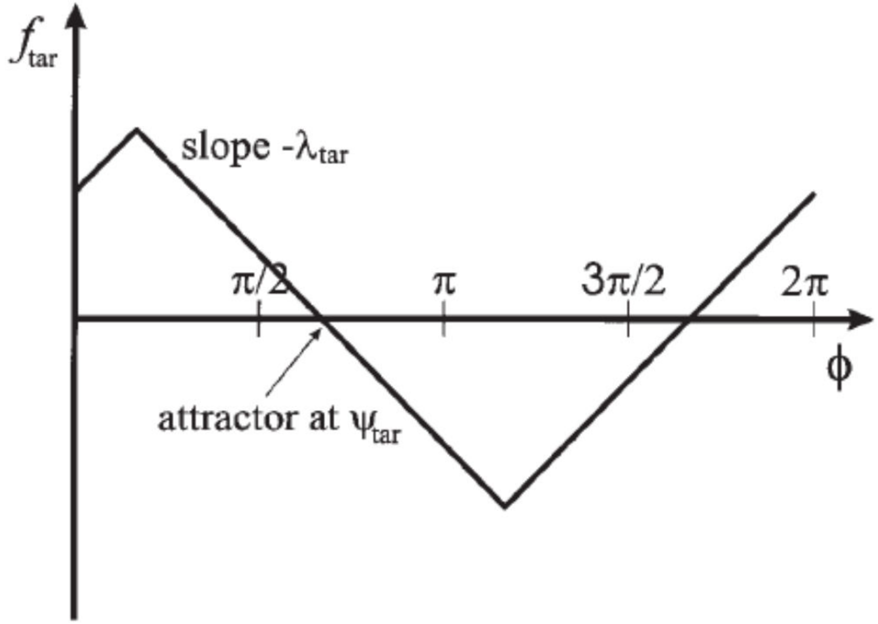
Fixed points and stability

- Where $\dot{x} = f(x) = 0$ the system has a fixed point, i.e., once in that state, no further change will occur
- Such a point may be stable or unstable: if the system is disturbed, does it tend to return to this state or to diverge further?
- E.g. for one dimensional system, stability depends on the slope around the fixed point.







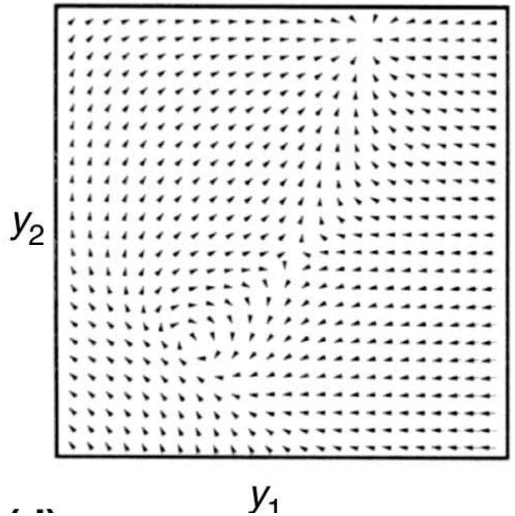


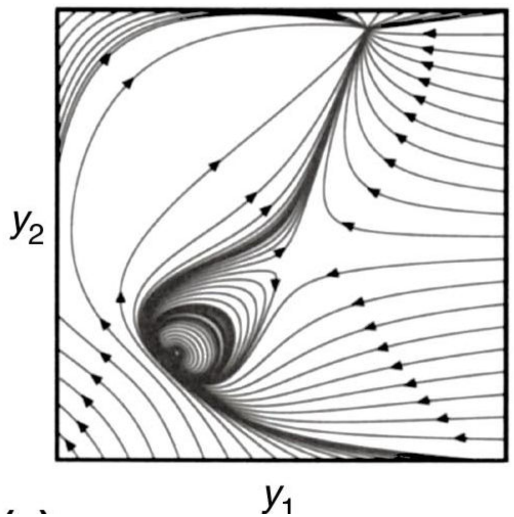
Bichot, Mallot, Schöner (2000) control robot's approach and avoid by defining 'forcelets' that attract or repel

Phase spaces

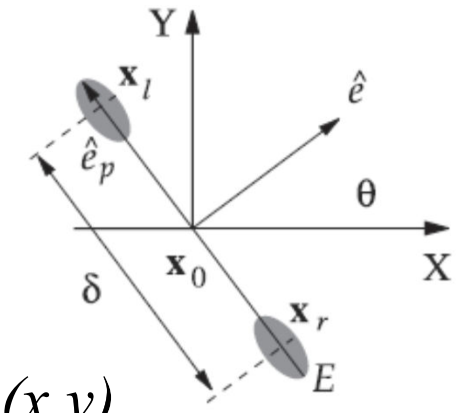
$$\dot{y}_1 = f(y_1, y_2),$$
$$\dot{y}_2 = f(y_1, y_2)$$

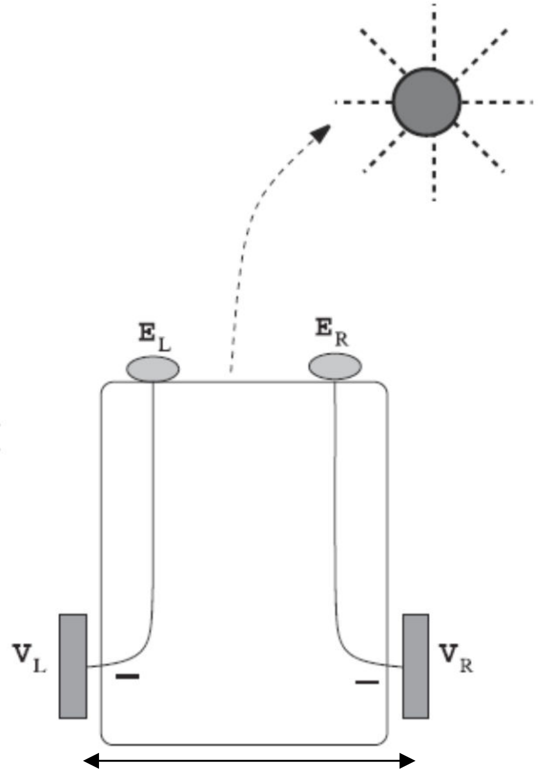
- Dynamical systems can be described in terms of their *phase space*:
  - Each dimension represents one of the variables required to specify the state
  - At each point in the space can define a vector representing the evolution of the state in time
  - The system will follow a trajectory through state space
  - The set of all trajectories (from every possible starting position) is called the *flow*
  - It may be possible to identify interesting properties of the flow without necessarily being able to fully solve the dynamic equations





Example: Braitenberg vehicle (Rañó, 2009)





Wheelbase d

Stimulus $E(\boldsymbol{x})$ for location $\boldsymbol{x} = (x,y)$ describes the environmental effect on the sensors, where E is a smooth function, $E(0)$ is a maximum with gradient $\Delta E(0) = 0$;

Motor output is a smooth decreasing function $F(E(x))$, with minimum 0 at maximum stimulus, $F(E(0))=0$.

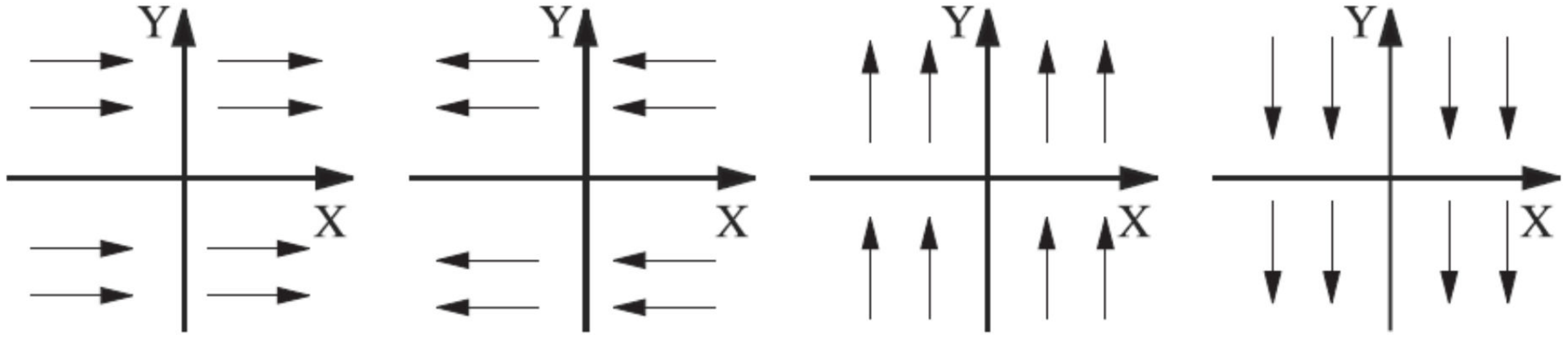
Can derive dynamics:

$$\dot{x} = F(E(x_o))\cos\theta$$

$$\dot{y} = F(E(x_o))\sin\theta$$

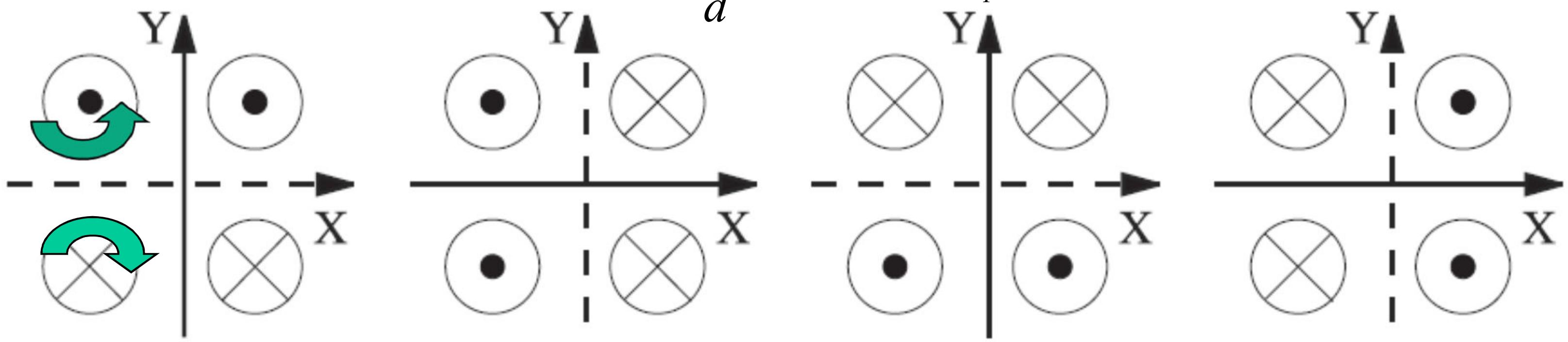
$$\dot{\theta} = -\frac{\delta}{d}\nabla F(E(x_o)).\widehat{e}_p$$

$$\dot{x} = F(E(x_o))\cos\theta \qquad \dot{y} = F(E(x_o))\sin\theta$$



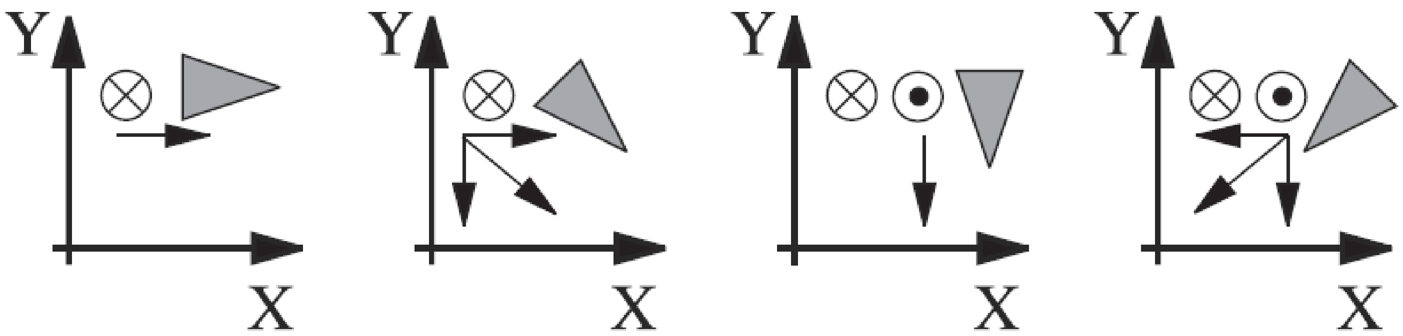
(a) x flow for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (b) x flow for $|\theta| > \frac{\pi}{2}$ (c) y flow for $0 < \theta < \pi$ (d) y flow for $-\pi < \theta < 0$

$$\dot{\theta} = -\frac{\delta}{d}\nabla F(E(x_o)).\widehat{e}_p$$



(a) $\theta = -\pi/2$ (b) $\theta = 0$ (c) $\theta = \pi/2$ (d) $\theta = \pi$

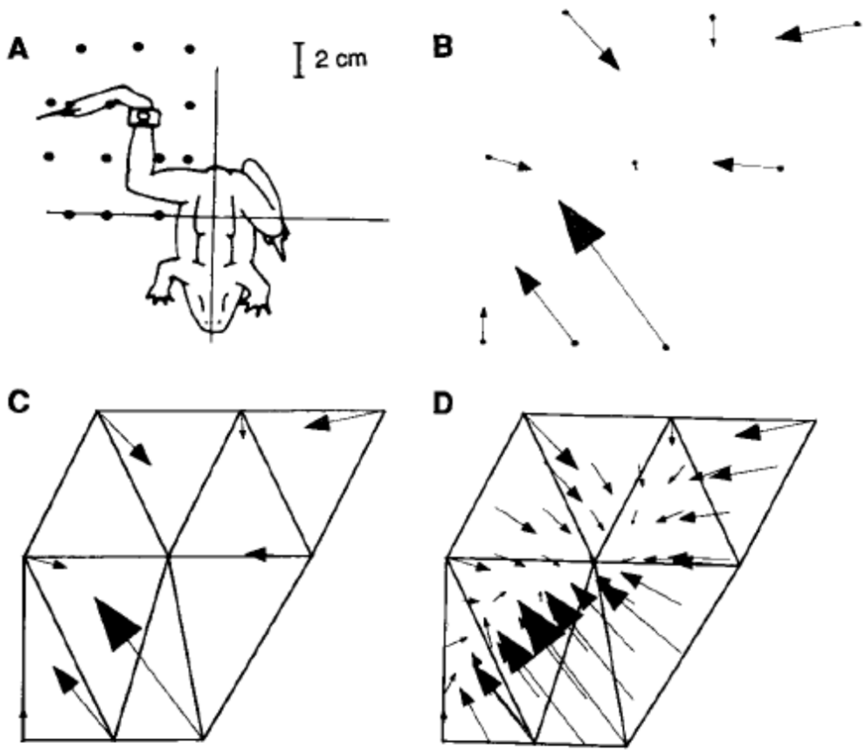
Robot behaviour from *θ=0*, positive *x,y*



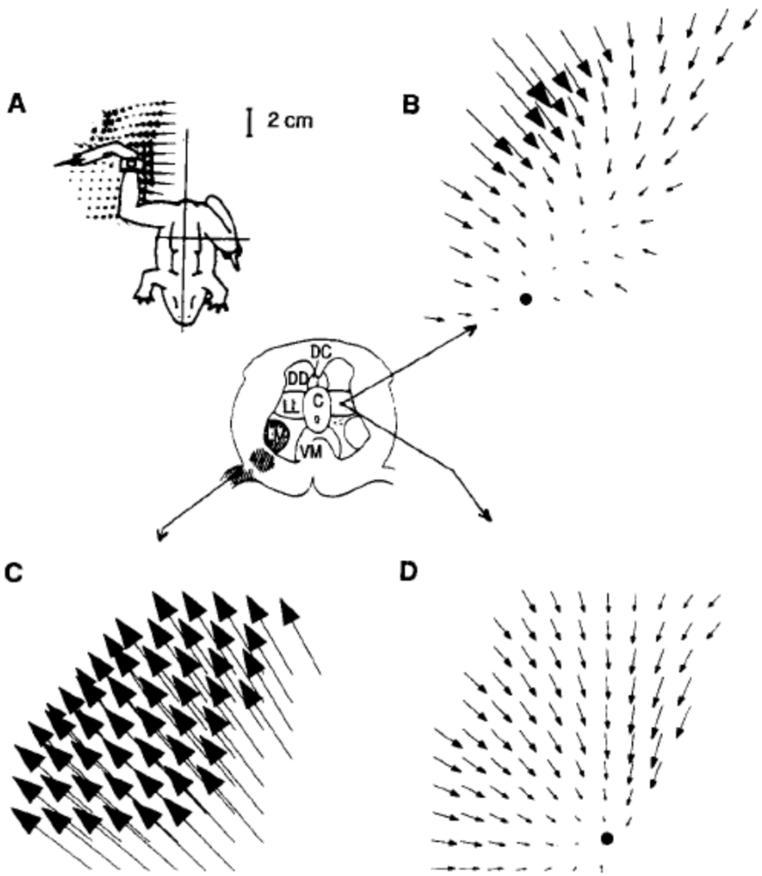
(a) Starting pose (b) Heading orientation (c) Approaching **0** (d) Approaching **0** (ii)

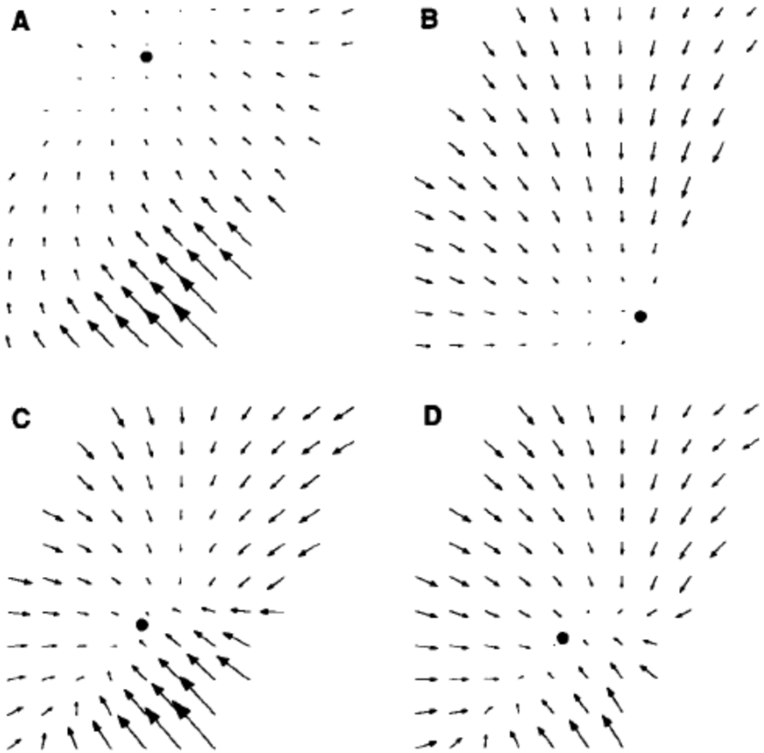
Example: Force fields for limb control (Bizzi, Mussa-Ivaldi, Giszter, 1991)

- Most limbs controlled by muscles in opponent pairs
- These act like dampened springs: depending on muscle stiffness, a perturbed limb will tend to return to particular position (the equilibrium point of the limb-muscle dynamics)
- Could control behaviour by changing stiffness and thus the equilibrium point
- Supporting evidence from measuring organised force fields produced by spinal activation in the frog



Example: Force fields for limb control (Bizzi, Mussa-Ivaldi, Giszter, 1991)





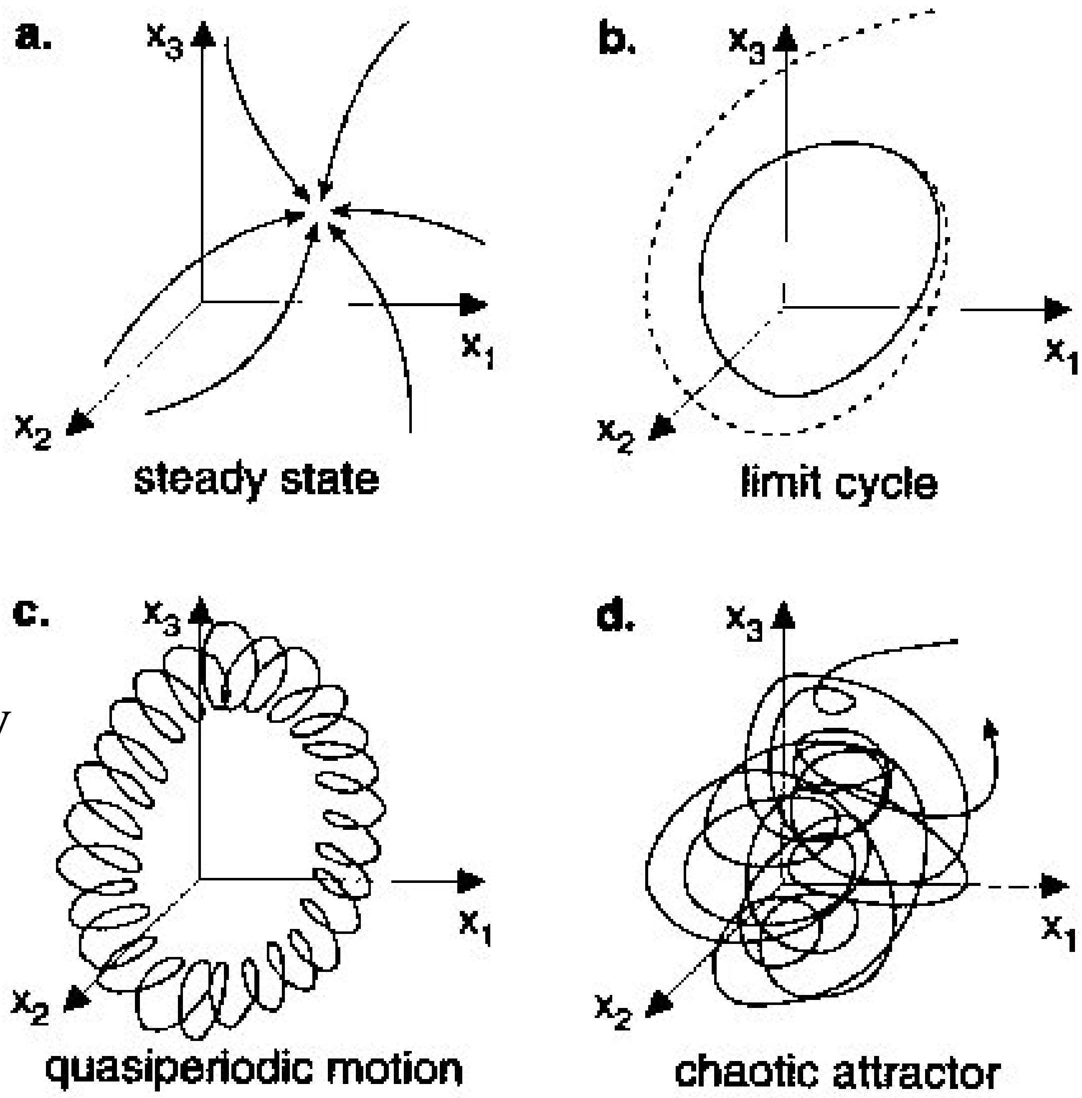
C is predicted result of adding A and B, D is measured result

Other kinds of attractors:

Periodic motion – system follows a repeated trajectory

Chaotic – system stays in the same region but doesn't repeat predictably

Limit cycles can be used for generating rhythmic behaviours



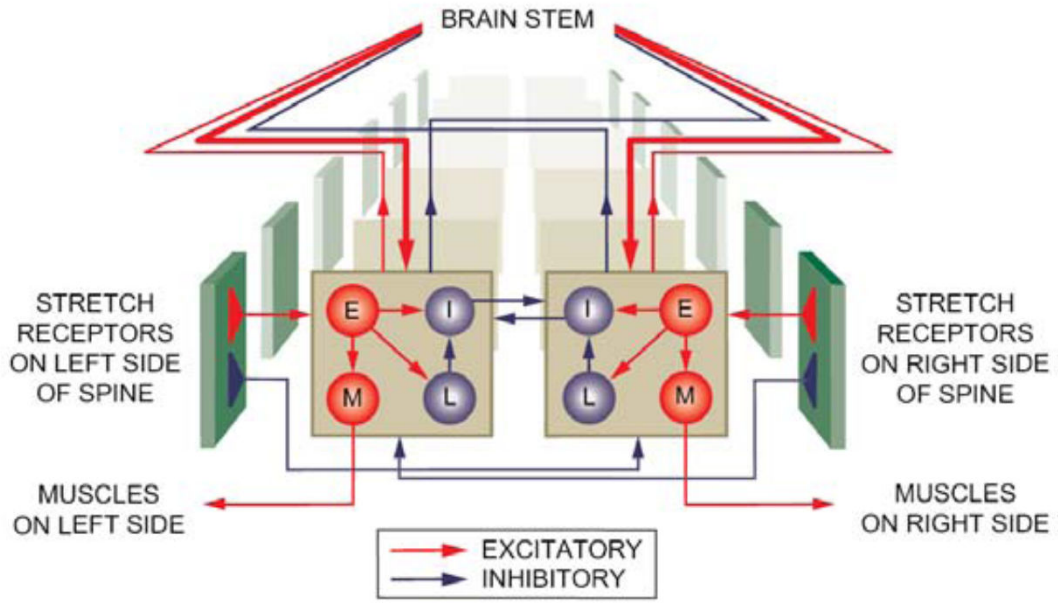
Example: Central pattern generators

• Many rhythmic behaviours in animals (e.g. breathing, chewing, walking, swimming, flying) are produced by intrinsic oscillators

• Small networks of neurons produce regular alternating burst patterns

• These can be coupled and modulated in various ways to produce co-ordinated behaviour

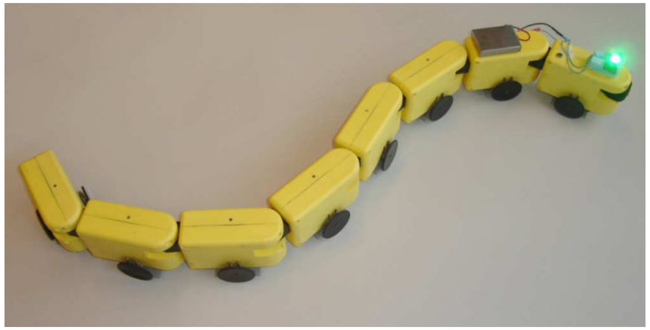
• Lamprey swimming is a well studied example

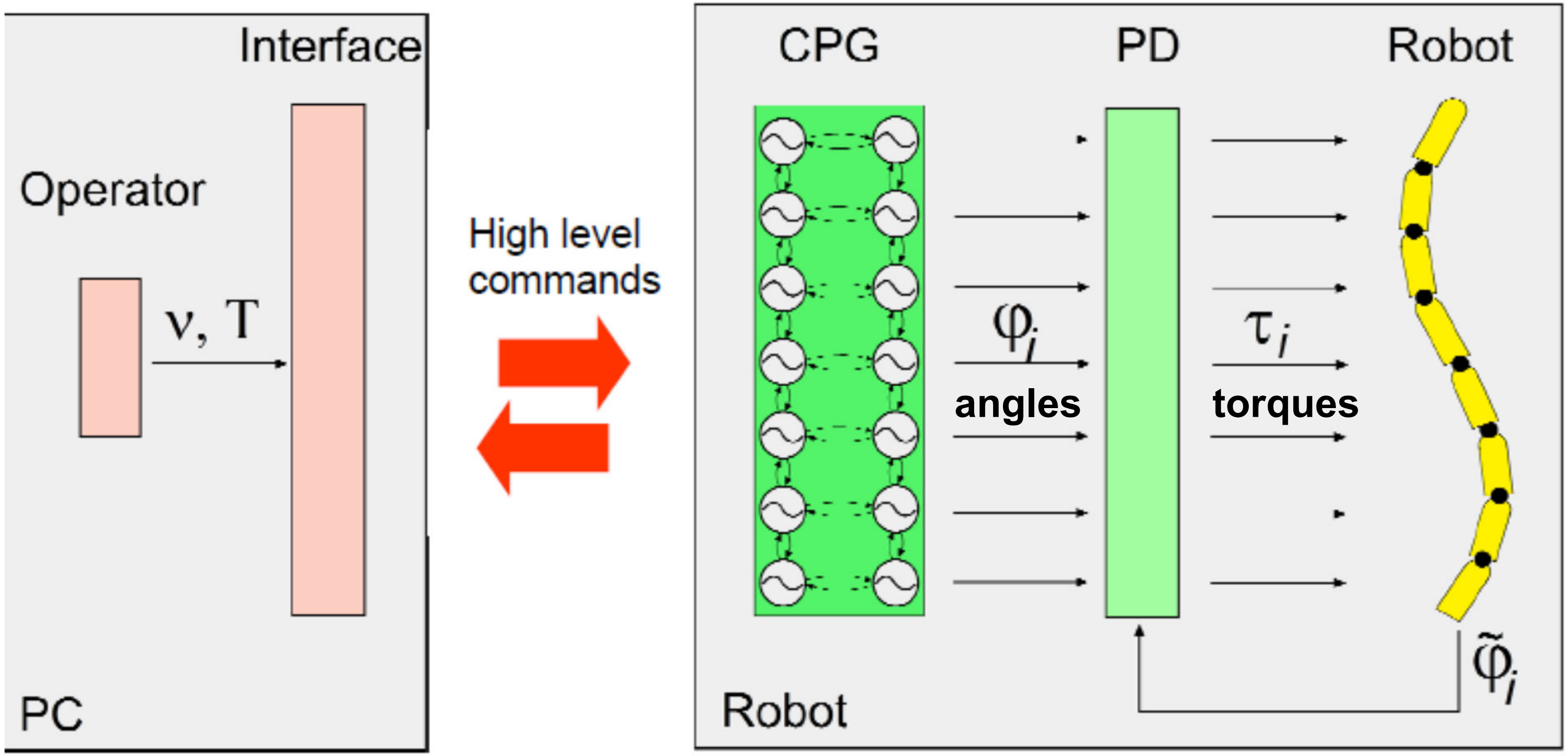


(Grillner et al, Sci. Am. 1996)

Crespi & Ijspeert (2008)

Lamprey-inspired robot



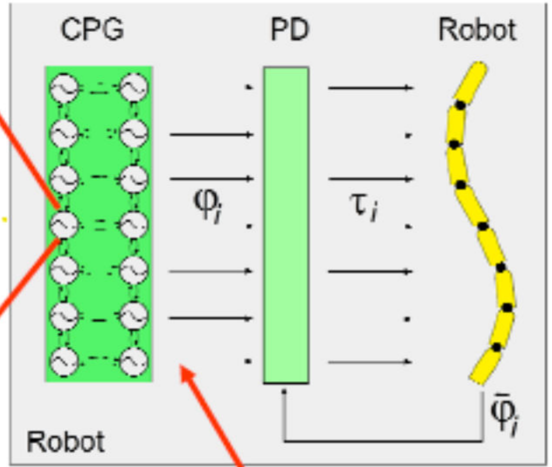


Crespi & Ijspeert (2008)

Phase $$\dot{\theta}_i = 2\pi \nu_i + \sum_j \left(w_{ij} \sin(\theta_j - \theta_i - \phi_{ij}) \right)$$

Amplitude $$\ddot{r}_i = a_i \left(\frac{a_i}{4} (R_i - r_i) - \dot{r}_i \right)$$

Output $$x_i = r_i (1 + \cos(\theta_i))$$



v_i intrinsic frequency, R_i intrinsic amplitude, a_i positive constant

w_{ij} and ϕ_{ij} determine coupling

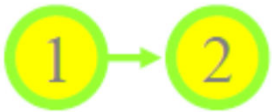
An isolated oscillator converges to:

$$x_i^{\infty}(t) = R_i (1 + \cos(2\pi \nu_i t + \theta_0))$$

Setpoints: $\boxed{\varphi_i = x_i - x_{N+i}}$

Left - right

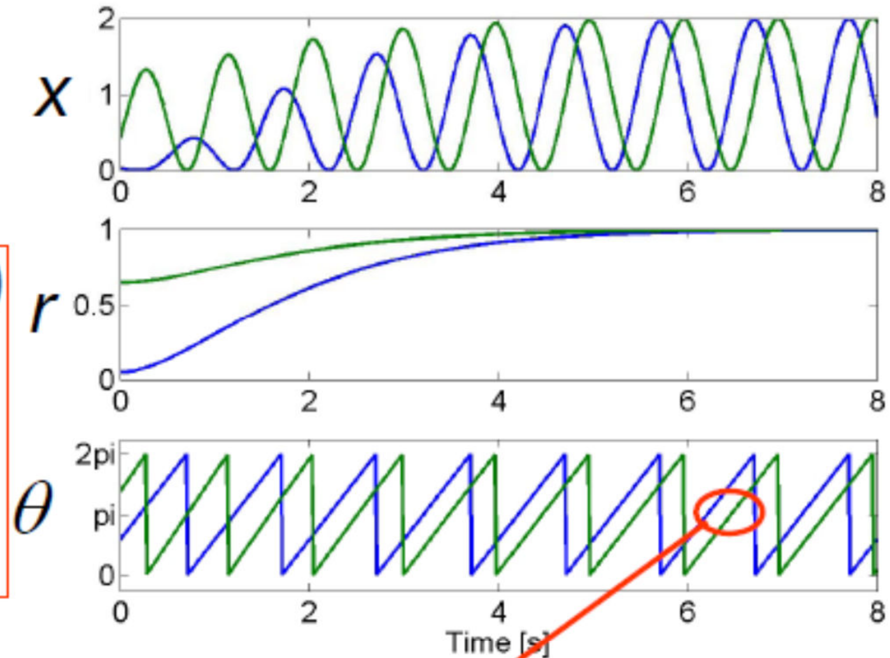
Crespi & Ijspeert (2008)



$$\dot{\theta}_i = 2\pi\nu_i + \sum_j \left(w_{ij}\sin(\theta_j - \theta_i - \phi_{ij})\right)$$

$$\ddot{r}_i = a_i\left(\frac{a_i}{4}(R_i - r_i) - \dot{r}_i\right)$$

$$x_i = r_i(1 + \cos(\theta_i))$$

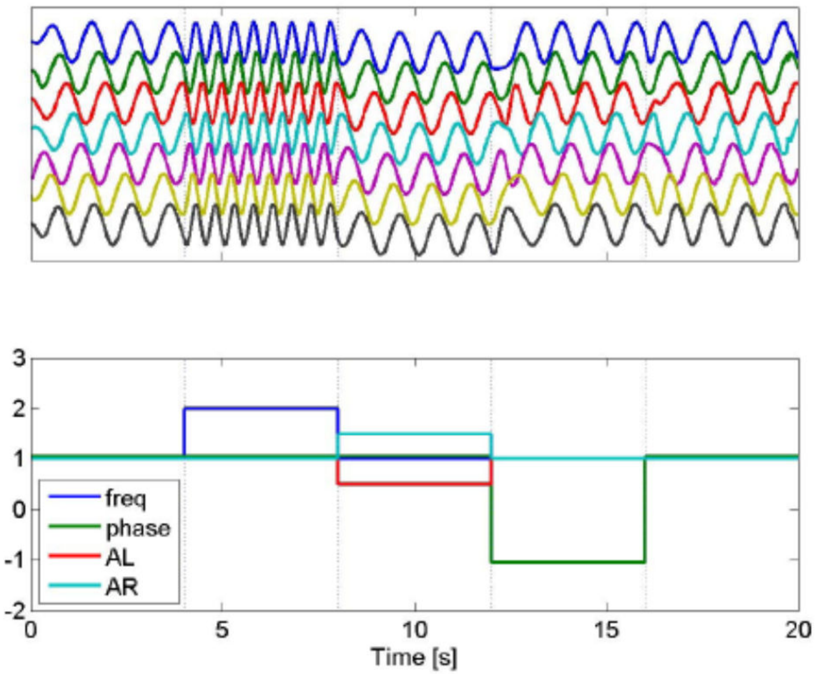


The phase difference $\phi = \theta_1 - \theta_2$ between two oscillators converges to

$$\phi_\infty = \arcsin\left(\frac{2\pi(\nu_1 - \nu_2)}{R_1 w_{21}}\right) - \phi_{21}$$

Crespi & Ijspeert (2008)

• Step changes in the control parameters (frequency, phase, left or right amplitude) results in smooth transition to different oscillation patterns and resulting robot motion



• In some cases, smooth change to control parameter may produce a sharp transition (a bifurcation) in the dynamics to produce new pattern (e.g. gaits)

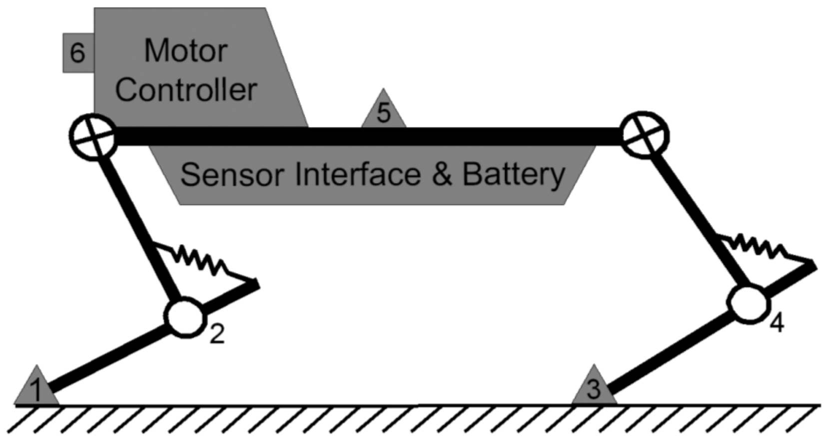
Useful properties of CPGs for robot control (see Ijspeert, 2008)

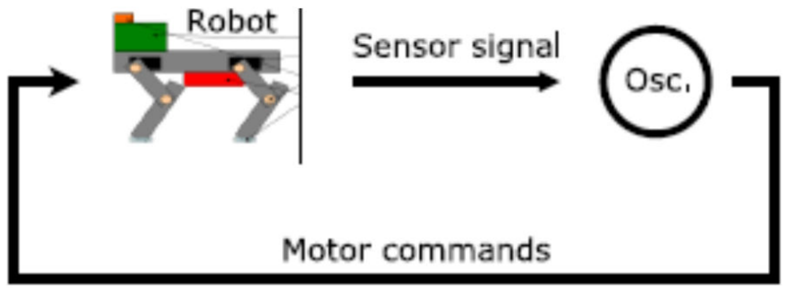
- CPG produces limit cycle behaviour and is thus robust to perturbation
- Very suited to distributed control, e.g. robots made up of variable number of modules
- Reduce dimensionality of the control problem as do not have to calculate for each actuator: can specify speed/direction/gait and dynamics solves the rest
- Introducing coupling from sensors can automatically entrain the dynamics to the robot's body/environment constraints, e.g., resistance of water vs. air
- Makes a good substrate for applying learning and optimisation methods.

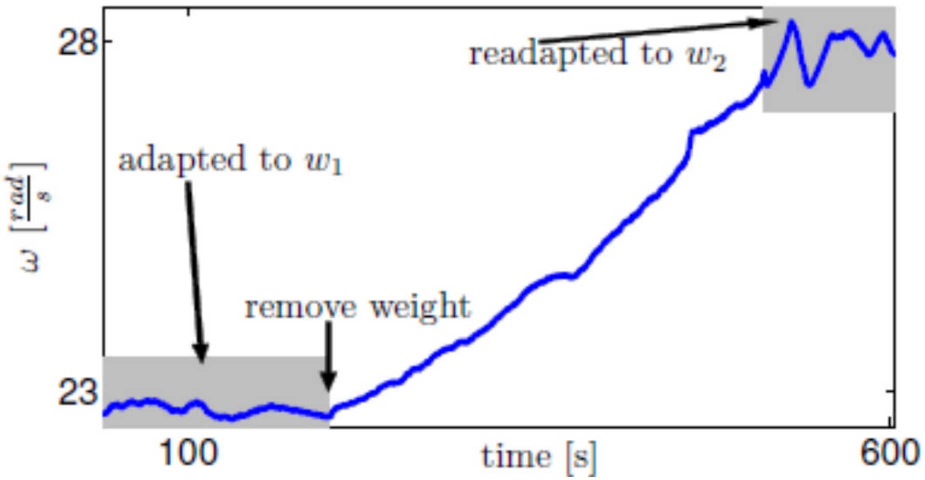
Entraining oscillators to the resonant frequency of the robot's dynamics

- E.g. 'Puppy' robot with actuated hip joint and passive spring knee joints
- Adaptive frequency oscillator uses sensor feedback to adjust control signal to match natural resonance
- Can immediately adapt to changes, such as > 20% weight difference

(Buchli et al., 2006)







A general framework for using dynamics in robot control? (Schaal *et. al* 2007)

- Idea: compose behaviour from sets of dynamic movement primitives:

$$\tau\dot{z} = \alpha_z(\beta_z(g-y)-z)+f$$

$$\tau\dot{y} = z$$

- For $f=0$, these dynamics describe a simple spring-damper system with parameters α_z, β_z, so that g *(=goal)* is a point attractor
- To obtain arbitrary trajectories to g, f is specified as follows:

$$f(x,g,y_o) = \frac{\sum \psi_i w_i x}{\sum \psi_i}(g-y_o), \quad \psi_i = e^{-h_i(x-c_i)^2}, \quad \tau\dot{x} = -\alpha_x x$$

- 'canonical' system variable x represents time passing, but in more flexible form, e.g. allows easy scaling in time, 'stopping' time etc.
- 'output' system f is a weighted composition from a set ψ_i of **basis functions** (like predefined force fields); could also be set of oscillators
- Control problem is then to find the weights w_i – can apply learning methods

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