Genetic Algorithms and Genetic Programming

Lecture 12: (3/11/09)

Ant Colony Optimization II

School of Informatics

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Overview: Remainder of the course

I. GA (1-7)
II. GP (8-10)
III. ACO (11-13): Ant colony optimization
IV. PSO (14-15): Particle swarm optimization and differential evolution
V. NC (16): Overview on DNA computing, Membrane computing, Molecular computing, Amorphous computing, Organic computing, ….
VI. Wrapping up: Metaheuristic search (17)

Not included:

artificial neural networks, quantum computing, cellular automata, artificial immune systems

Definition of a Combinatorial Optimization Problem (COP)

(S, Ω, f)

- S is a search space defined over a finite set of discrete decision variables
- Ω is a set of constraints among the variables
- f is an objective function to be minimized
- S is contained in $D=\{v^1,\ldots,v^d\}$
- S_Ω: set of solutions that satisfy all constraints
- Optimum (maximum) $s^* \in S_\Omega : f(s^*) \geq f(s) \text{ for all } s \in S_\Omega$
- Task: Find at least one optimum

GA/GP and ACO as COPs

	GA/GP	ACO
S	• bit strings/trees	• paths in a graph
Ω	• correctness (GP)	• e.g. non-intersecting
f	• fitness	• total path length
S_{Ω}	• correct programs	• e.g. non-intersecting paths
opt.	• fittest individual/ best program on fitness cases	• path of minimal length

Applying Ant Methods to Optimisation

What we need to set up an ACO

- *Problem representation* that allows the solution to be built up incrementally
- *Desirability heuristic* η to help in building up the solution
- *Constraints* that permit only feasible/valid solutions to be constructed
- *Pheromone update rule* incorporating quality of the solution
- *Probability rule* that is a function of desirability and pheromone strength

ACO algorithm (in brief)

- Set parameters, initialize pheromone trails
- SCHEDULE_ACTIVITIES
  - ConstructAntSolutions
  - DaemonActions {optional}
  - UpdatePheromones
- END_SCHEDULE_ACTIVITIES

ACO algorithm

Algorithm 1 The framework of a basic ACO algorithm

```
input: An instance P of a CO problem model 𝒫 = (𝒮, f, Ω).
InitializePheromoneValues(𝒯)
s_bs ← NULL                                                      init best-so-far solution
while termination conditions not met do
    S_iter ← ∅
    for j = 1, ..., n_a do                                       loop over ants
        s ← ConstructSolution(𝒯)
        if s is a valid solution then                            set of valid solutions
            s ← LocalSearch(s)        {optional}
            if (f(s) < f(s_bs)) or (s_bs = NULL) then s_bs ← s   update best-so-far
            S_iter ← S_iter ∪ {s}                                store valid solutions
        end if
    end for
    ApplyPheromoneUpdate(𝒯, S_iter, s_bs)
end while
output: The best-so-far solution s_bs
```

ACO Algorithms: Ant system (AS)

(Dorigo et al. 1991/92)

Probability rule

$$p\left(c_{ij} \mid s_k^p\right) = \begin{cases} \dfrac{\tau_{ij}^{\alpha}\eta_{ij}^{\beta}}{\sum\limits_{c_{im} \in N\left(s_k^p\right)} \tau_{im}^{\alpha}\eta_{im}^{\beta}} & \text{if } j \in N\left(s_k^p\right) \\ 0 & \text{otherwise} \end{cases}$$

c_{ij}: graph edge, s_k^p: partial solution of ant k,
N possible continuation of s_k^p, Transition $i \rightarrow j$

Pheromone update

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

Pheromone production

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{1}{L_k} & \text{if ant } k \text{ used edge } (i,j) \text{ in its tour} \\ 0 & \text{otherwise} \end{cases}$$

http://www.scholarpedia.org/article/Ant_colony_optimization

Ant colony system (ACS)

(Dorigo & Gambardella 1997)

Pseudorandom proportional rule:
(use probability rule with probability $1-q_0$, use maximum with probability q_0)

$$p\left(c_{ij} \mid s_k^p\right) = \begin{cases} \dfrac{\tau_{ij}^{\alpha}\eta_{ij}^{\beta}}{\sum_{c_{im}\in N\left(s_k^p\right)} \tau_{im}^{\alpha}\eta_{im}^{\beta}} & \text{if } j \in N\left(s_k^p\right) \\ 0 & \text{otherwise} \end{cases}$$

c_{ij}: graph edge, s_k^p: partial solution of ant k, N possible continuation of s_k^p, Transition $i \rightarrow j$

Local pheromone update (after each step by each ant)

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \rho\,\Delta\tau_0$$

and offline by the best ant

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \rho\,\Delta\tau_{ij}^{\text{best}}$$

Pheromone production (best ant only)

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{1}{L_k} & \text{if ant } k \text{ used edge } (i,j) \text{ in its tour} \\ 0 & \text{otherwise} \end{cases}$$

Max-Min Ant System (MMAS)

(i) Only the best ant adds pheromone trails (iteration best or best so far)

$$\tau_{ij} \leftarrow (1-\rho)\tau_{ij} + \rho\,\Delta\tau_{ij}^{\text{best}}$$

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{1}{L_k} & \text{if ant } k \text{ used edge } (i,j) \text{ in its tour} \\ 0 & \text{otherwise} \end{cases}$$

(ii) Minimum and maximum values of the pheromone are explicitly limited (by truncation): $\tau_{\min}, \quad \tau_{\max}$

Pseudorandom proportional rule:

$$p\left(c_{ij} \mid s_k^p\right) = \begin{cases} \dfrac{\tau_{ij}^{\alpha}\eta_{ij}^{\beta}}{\sum_{c_{im}\in N\left(s_k^p\right)} \tau_{im}^{\alpha}\eta_{im}^{\beta}} & \text{if } j \in N\left(s_k^p\right) \\ 0 & \text{otherwise} \end{cases}$$

Initialize by maximum $\tau_{\max} = \dfrac{1}{\rho L^*}$, L^* best so far or optimum (if known)

(minimum empirical)

Stützle and Hoos (2000)

ACO variants

ACO variant	Authors	Year
Elitist AS (EAS)	Dorigo	1992
Continuous ACO (CACO)	Bilchev and I.C. Parmee	1995
	Dorigo, Maniezzo, and Colorni	1996
Ant Colony System (ACS)	Dorigo and Gambardella	1997
Rank-based AS (RAS)	Bullnheimer, Hartl, and Strauss	1999
Max-Min Ant System (MMAS)	Stützle and Hoos	2000
Hyper-Cube Framework (HCF)	Blum and Dorigo	2004

C. Blum (2005) Ant colony optimization: Introduction and recent trends. *Physics of Life Reviews* **2**:4, 353-373.

Negative pheromones in real ants

Robinson EJ, Jackson DE, Holcombe M, Ratnieks FL (2005) Insect communication: 'no entry' signal in ant foraging. *Nature*. **438**:7067, 442.

Abstract: Forager ants lay attractive trail pheromones to guide nestmates to food, but the effectiveness of foraging networks might be improved if pheromones could also be used to repel foragers from unrewarding routes. Here we present empirical evidence for such a negative trail pheromone, deployed by Pharaoh's ants (Monomorium pharaonis) as a 'no entry' signal to mark an unrewarding foraging path. This finding constitutes another example of the sophisticated control mechanisms used in self-organized ant colonies.

Properties of ACO in a numerical experiment

Fig. 3. Evolution of best tour length (Oliver30). Typical run.

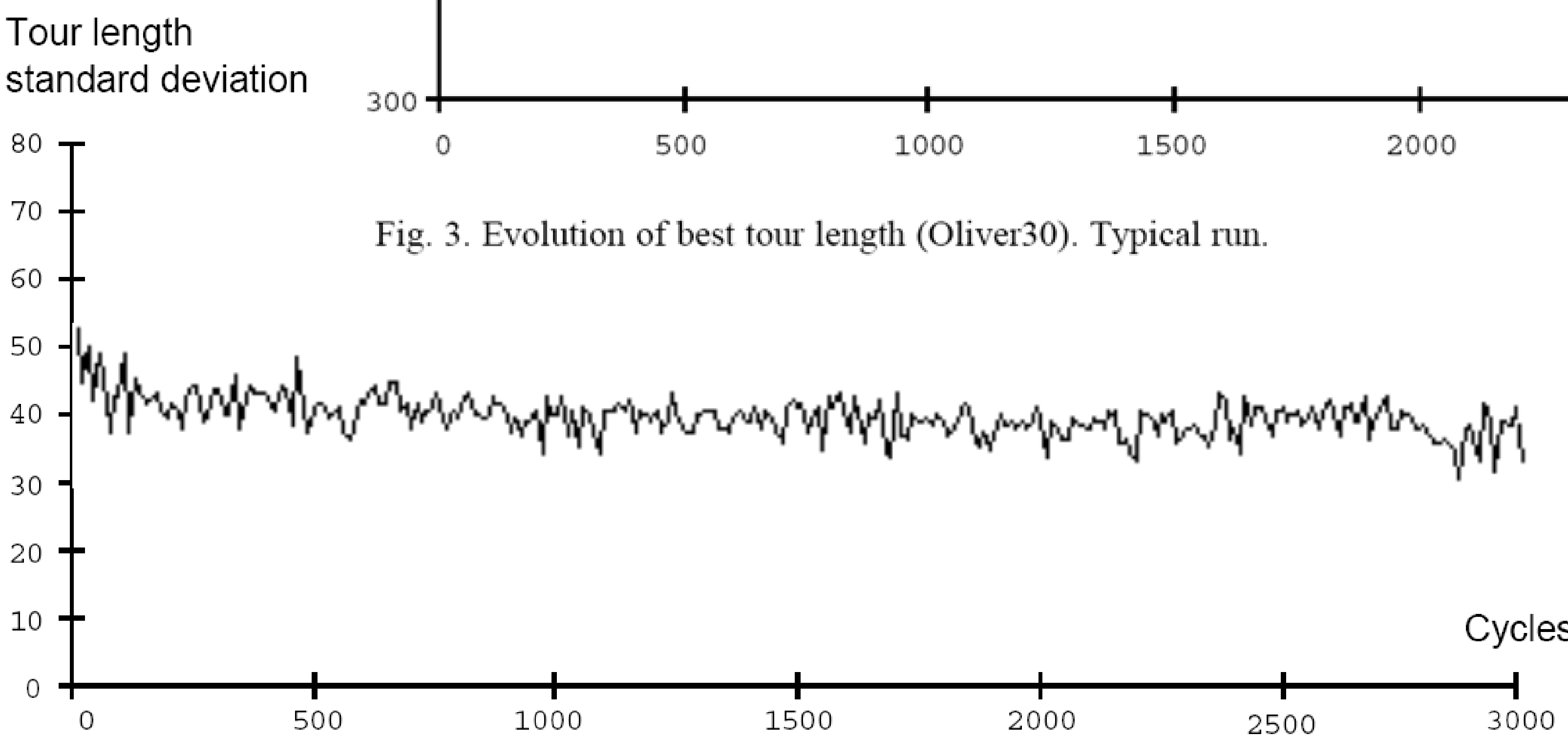


Fig. 4. Evolution of the standard deviation of the population's tour lengths (Oliver30). Typical run.

Dorigo et al.: Ant System: Optimization by a Colony of Cooperating Agents

Properties of ACO in a numerical experiment

Fig. 5. Evolution of the average node branching of the problem's graph (Oliver30). Typical run.

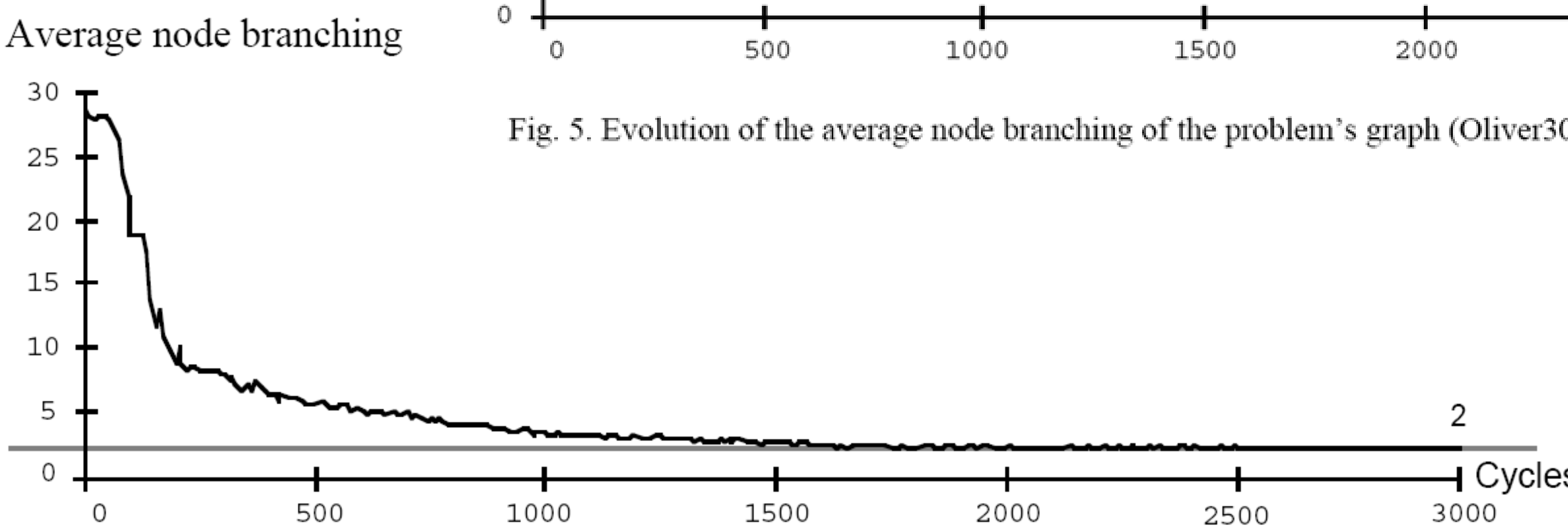


Fig. 7. Average node branching of a run going to stagnation behavior (Oliver30). Typical run obtained setting $\alpha=5$ and $\beta=2$.

Dorigo et al.: Ant System: Optimization by a Colony of Cooperating Agents
IEEE Transactions on Systems, Man, and Cybernetics–Part B, Vol.26, No.1, 1996, pp.1-13

Numerical experiment

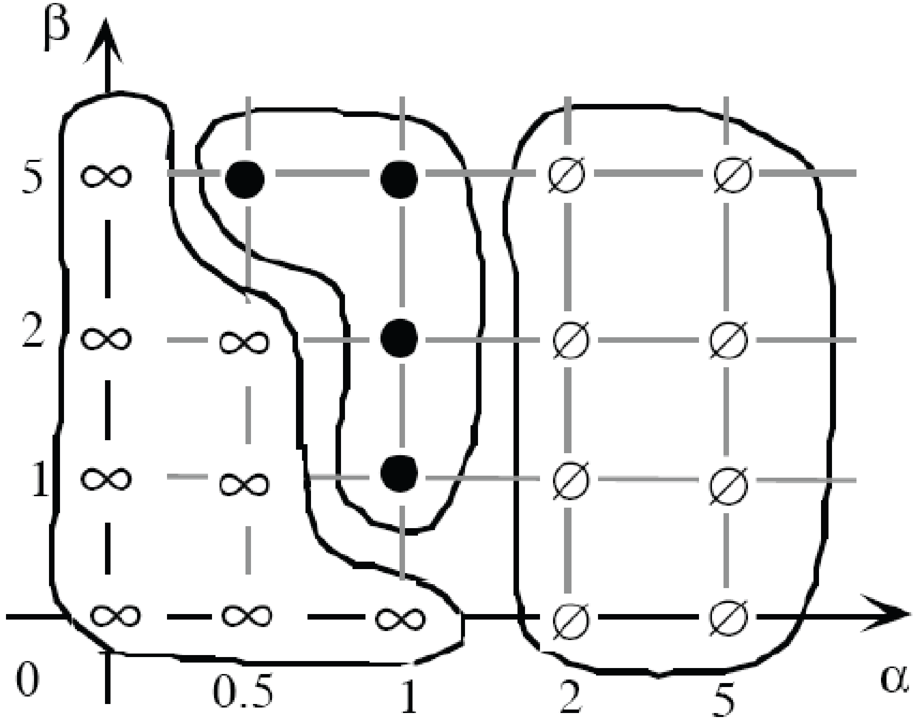


Fig. 8. *Ant-cycle* behavior for different combinations of α-β parameters.

● - The algorithm finds the best known solution without entering the stagnation behavior.
∞ - The algorithm doesn't find good solutions without entering the stagnation behavior.
∅ - The algorithm doesn't find good solutions and enters the stagnation behavior.

Dorigo et al.: Ant System: Optimization by a Colony of Cooperating Agents
IEEE Transactions on Systems, Man, and Cybernetics–Part B, Vol.26, No.1, 1996, pp.1-13

Theoretical results: Overview

- Convergence in probability of an ACO algorithm (Gutjahr 2000)
  (Theoretical bounds, but not very practical)
- Run-time analysis (short)
- Understanding ACO: Search biases (later)
- Relations to other optimization algorithms (later)

Convergence

- For simplified algorithms; bounds not very tight
- Given a lower bound for the pheromones the algorithm explores everywhere and must therefore find an optimal solution given sufficiently long time, i.e.
- **Theorem** (Dorigo & Stuetzle): Let $p^*(t)$ be the probability that ACO (best-so-far update and lower bound for pheromones) finds an optimal solution at least once within the first t iterations. Then, for an arbitrarily small $eps > 0$ and for a sufficiently large t it holds that:
  $p^*(t) = 1 - eps$ and asymptotically $\lim_{t\to\infty} p^*(t) = 1$.
- Logarithmically decreasing τ_{min} is also OK
- save convergence times can be very large!

Run-time analysis

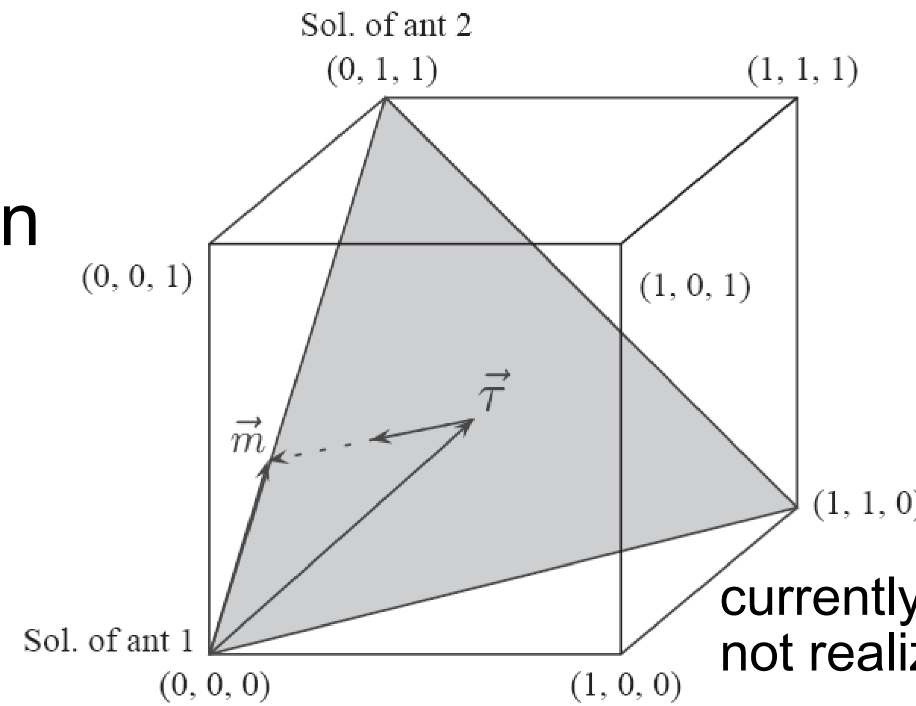
- For large evaporation rate (unrealistic!): identical to (1+1)-ES (i.e. weakly exponential)
- For small evaporation rates polynomial complexity can be achieved (for ONE-MAX)

Neumann & Witt (2007)

Search space: “Hyper-cube framework”

- Given a solution (path) $s=(s_1,\ldots,s_n)$
- the solution is a subsets of the edges E of a graph G=(N,E)
- Partitioning of E:
  if a link belongs to s: 1
  otherwise 0
- s can be represented by a binary vector with dimension n(n-1)/2 (!)
- Pheromones are updated in the span of the solutions
- Not an algorithms, but a framework which applies for several variants

another solution



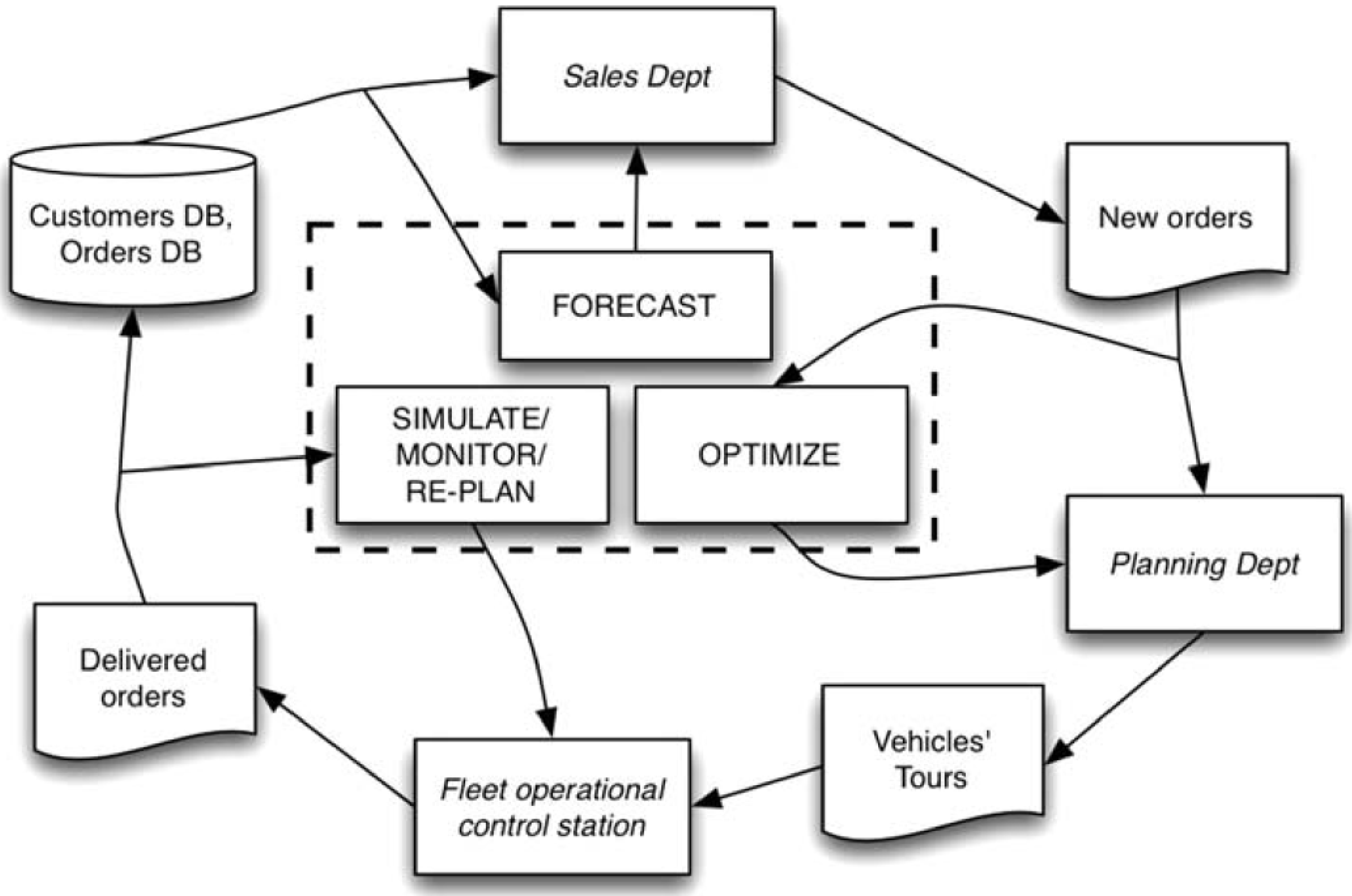
currently not realized

best solution

Relation to other algorithms

- Next time.

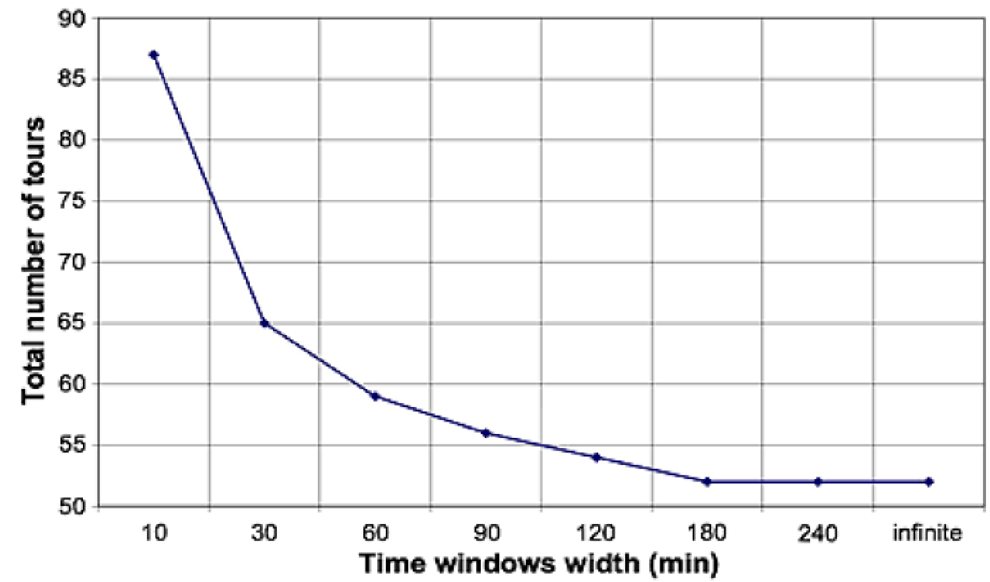
Vehicle routing problems



A.E. Rizzoli, · R. Montemanni · E. Lucibello · L.M. Gambardella (2007) Ant colony optimization for real-world vehicle routing problems Swarm Intelligence 1: 135–151

Vehicle routing problems

- E.g. distribute 52000 pallets to 6800 customers over a period of 20 days
- Dynamic problem: continuously incoming orders
- Strategic planning: Finding feasible tours is hard
- Computing time: 5 min (3h for human operators)
- More tours required for narrower arrival time window
- Implicit knowledge on traffic learned from human operators



	Human planner	AR-RegTW	AR-Free
Total number of tours	2056	1807	1614
Total km	147271	143983	126258
Average truck loading	76.91%	87.35%	97.81%

A.E. Rizzoli, · R. Montemanni · E. Lucibello · L.M. Gambardella (2007) **Ant colony optimization for real-world vehicle routing problems** Swarm Intelligence 1: 135–151

ACO Reading suggestions

General:

- M. Dorigo & K. Socha, An Introduction to Ant Colony Optimization: In T. F. Gonzalez, Approximation Algorithms and Metaheuristics, CRC Press, 2007. IridiaTr2006-010r003.pdf
- M. Dorigo, T. Stützle (2004) Ant Colony Optimization, MIT Press.

Theory:

- M. Dorigo, V. Maniezzo, A. Colorni (1996) Ant System: Optimization by a Colony of Cooperating Agents. *IEEE Transactions on Systems, Man, and Cybernetics* **B 26**:1, 1-13.
- M. Dorigo and C. Blum. Ant colony optimization theory: A survey. *Theoretical Computer Science*, 344(2–3):243–278, 2005.

Applications

- see proceedings of the ANTS conferences or the journal *Swarm Intelligence*

Ant-Robotics

Krieger MJ, Billeter JB, Keller L. (2000) Ant-like task allocation and recruitment in cooperative robots. *Nature* **406**:6799, 992-995.