Formal Verification

Lecture 1: Introduction to Model Checking and Temporal Logic

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¹Acknowledgement: Adapted from original material by Paul Jackson, including some additions by Bob Atkey.
Create a *formal model* of some system of interest
- Hardware
- Communication protocol
- Software, esp. concurrent software
Formal Verification (in a nutshell)

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  - Communication protocol
  - Software, esp. concurrent software

- Describe formally a *specification* that we desire the model to satisfy
Formal Verification (in a nutshell)

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  - Hardware
  - Communication protocol
  - Software, esp. concurrent software

- Describe formally a *specification* that we desire the model to satisfy

- Check the model satisfies the specification
  - theorem proving (usually interactive but not necessarily)
  - Model checking
Introduction to Model Checking

- Specifications as Formulas, Programs as Models
- Programs are abstracted as Finite State Machines
- Formulas are in Temporal Logic
Interpretation $|=\text{ Formula}$

The relationship between interpretations $M$ and formulas $\phi$:

$$M |= \phi$$

We say $M$ models $\phi$.

Questions we can ask:
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We say $M$ **models** $\phi$.

Questions we can ask:

1. For a fixed $\phi$, is $M \models \phi$ true for all $M$?
   - **Validity of $\phi$**
   - This can be done via proof in a theorem prover e.g. Isabelle.
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   - “Theory discovery”/“Learning from Data”/“Generalisation”
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4. For a fixed $M$ and $P$, is it the case that $M \models \phi$?
   - Model Checking
Model Checking

At a high level, many tasks can be rephrased as model checking.

<table>
<thead>
<tr>
<th>“Interpretations” $M$</th>
<th>“Formulas” $\phi$</th>
<th>Task</th>
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<tbody>
<tr>
<td>sequences of tokens</td>
<td>grammars</td>
<td>parsing</td>
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<td>database tables</td>
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<td>audio data</td>
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<td>finite state machines</td>
<td>temporal logic</td>
<td>specification checking</td>
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Details differ widely, but question of “is this data consistent with this statement? (and to what degree?)” is extremely common.

Historically, “Model Checking” usually refers to the last one. This is the one we will cover over the next few lectures.
Uses of Model Checking

Model Checking has been used to:

▶ Check Microsoft Windows device drivers for bugs
  ▶ The “Static Driver Verifier” tool

▶ The SPIN tool (http://spinroot.com):
  ▶ http://spinroot.com/spin/success.html
  ▶ Flood control barrier control software
  ▶ Call processing software at Lucent
  ▶ Parts of Mars Science Laboratory, Deep Space 1, Cassini, the Mars Exploration Rovers, Deep Impact
  ▶ ...

▶ PEPA (Performance Evaluation Process Algebra)
  http://www.dcs.ed.ac.uk/pepa/
  ▶ Multiprocessor systems
  ▶ Biological systems

▶ ...

Model Checking – Models

A model of some system has:

- A finite set of states
- A subset of states considered as the initial states
- A transition relation which, given a state, describes all states that can be reached “in one time step”.

Good for

- Software, sequential and concurrent
- Digital hardware
- Communication protocols

Refinements of this setup can handle: Infinite state spaces, Continuous state spaces, Continuous time, Probabilistic Transitions. Good for hybrid (i.e., discrete and continuous) and control systems.
Model Checking – Models

Models are *always* abstractions of reality.
Model Checking – Models

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- We must choose what to model and what not to model

In the words of The Cure's 'Pictures of You':

I've been looking so long at these pictures of you
That I almost believe that they're real
I've been living so long with my pictures of you
That I almost believe that the pictures are All I can feel

Do not do this: the pictures are not real.
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» Use Temporal Logic
Model Checking – Specifications

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▶ Use Temporal Logic

Specifications are built from:

1. Primitive properties of individual states
   e.g., “is on”, “is off”, “is active”, “is reading”;

2. propositional connectives \( \land, \lor, \neg, \rightarrow \);  

3. and temporal connectives: e.g.,

   At all times, the system is not simultaneously reading and writing.
   If a request signal is asserted at some time, a corresponding grant signal will be asserted within 10 time units.
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- **Use Temporal Logic**

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3. **and temporal connectives**: e.g.,
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The exact set of temporal connectives differs across temporal logics. Logics can differ in how they treat time:

- **Linear time vs. Branching time**

These differ in reasoning about non-determinism.
Non-determinism

In general, system descriptions are *non-deterministic*.

A system is *non-deterministic* when, from some state there are *multiple* alternative next states to which the system could transition.

Non-determinism is good for:

- Modelling alternative inputs to the system from its environment (*External non-determinism*)
- Under-specifying the model, allowing it to capture many possible system implementations (*Internal non-determinism*)
Linear vs. Branching Time

- **Linear Time**
  - Considers paths (sequences of states)
  - If system is non-deterministic, many paths for each initial state
  - Questions of the form:
    - For all paths, does some path property hold?
    - Does there exist a path such that some path property holds?

- **Branching Time**
  - Considers tree of possible future states from each initial state
  - If system is non-deterministic from some state, tree forks
  - Questions can become more complex, e.g.,
    - For all states reachable from an initial state, does there exist an onwards path to a state satisfying some property?
  - Most-basic branching-time logic (CTL) is complementary to most-basic linear-time logic (LTL).
  - Richer branching-time logic (CTL+) incorporates CTL and LTL.
Linear vs. Branching Time

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A Taste of LTL – Syntax

LTL = Linear(-time) Temporal Logic

Assume some set $Atom$ of atomic propositions

Syntax of LTL formulas $\phi$:

$$\phi ::= p | \neg \phi | \phi \lor \phi | \phi \land \phi | \phi \rightarrow \phi | X\phi | F\phi | G\phi | \phi U \psi$$

where $p \in Atom$.

Pronunciation:

- $X\phi$ — neXt $\phi$
- $F\phi$ — Future $\phi$
- $G\phi$ — Globally $\phi$
- $\phi U \psi$ — $\phi$ Until $\psi$

Other common connectives: $W$ (weak until), $R$ (release).

Precedence high-to-low: $(X, F, G, \neg), (U), (\land, \lor), \rightarrow$
LTL formulas are evaluated at a position \( i \) along a path \( \pi \) through the system (a path is a sequence of states connected by transitions)

- An atomic \( p \) holds if \( p \) is true for the state at position \( i \).
- The propositional connectives \( \neg, \land, \lor, \rightarrow \) have their usual meanings.
- Meaning of LTL connectives:
  - \( X\phi \) holds if \( \phi \) holds at the next position;
  - \( F\phi \) holds if there exists a future position where \( \phi \) holds;
  - \( G\phi \) holds if, for all future positions, \( \phi \) holds;
  - \( \phi U\psi \) holds if there is a future position where \( \psi \) holds, and \( \phi \) holds for all positions prior to that.

This will be made more formal in the next lecture.
A Taste of LTL – Examples

1. $G\text{ invariant}$
   
   $\text{invariant}$ is true for all future positions

2. $G (\text{read} \land \text{write})$
   
   In all future positions, it is not the case that $\text{read}$ and $\text{write}$

3. $G (\text{request} \rightarrow \exists F \text{ grant})$
   
   At every position in the future, a $\text{request}$ implies that there exists a future point where $\text{grant}$ holds.

4. $G (\text{request} \rightarrow \neg (\text{request} \lor \text{grant}))$
   
   At every position in the future, a $\text{request}$ implies that there exists a future point where $\text{grant}$ holds, and $\text{request}$ holds up until that point.

5. $G F \text{ enabled}$
   
   In all future positions, there is a future position where $\text{enabled}$ holds.

6. $F G \text{ enabled}$
   
   There is a future position, from which all future positions have $\text{enabled}$ holding.
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Summary

- Introduction to Model Checking (H&R 3.1, 3.2)
  - The Model Checking problem
  - Informal introduction to LTL

- Next time:
  - Formal introduction to LTL.