Formal Modeling in Cognitive Science 1 (2005–2006)

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Tutorial 9: Entropy; Mutual Information

Week 10 (13–17 March, 2006)

1. **Relationship between Entropy and Mutual Information**

   Let X and Y be two non-independent random variables. You know the entropies $H(X)$ and $H(Y)$ and the conditional entropies $H(X|Y)$ and $H(Y|X)$.

   (a) What is the mutual information of X with itself?

   (b) What is the joint entropy $H(X,Y)$, and what would it be if the random variables X and Y were independent?

   (c) Give an alternative expression for $I(X;Y)$ in terms of the joint entropy $H(X,Y)$ and the entropies $H(X)$ and $H(Y)$.

2. **Computing Entropy and Mutual Information**

   Let X and Y be a random variables over the sample space $S = \{a,b,c,d\}$. The joint distribution of these two random variables is as follows:

		x			
		a	b	c	d
	a	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
y	b	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	0
	c	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{16}$	0
	d	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{16}$	0

   (a) Write down the marginal distribution for X and compute the entropy $H(X)$.

   (b) Write down the marginal distribution for Y and compute the entropy $H(Y)$.

   (c) What is the joint entropy $H(X,Y)$ of the two random variables?

   (d) What is the conditional entropy $H(Y|X)$?

   (e) What is the mutual information $I(X;Y)$ between the two random variables?

3. **Twenty Questions**

   Consider a variant of the game Twenty Questions in which you have to guess which one of seven horses won a race. The probability distribution over winning horses is as follows:

horse	1	2	3	4	5	6	7
prob. of winning	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

   (a) Assuming an optimal strategy, what is the minimum number of yes/no questions that you need to ask in order to find out which horse won?

   (b) What is the expected number of questions if you adopt the following strategy: first ask about the horse with the highest probability, then about the horse with the second highest probability, etc.?