Formal Modeling in Cognitive Science 1 (2005–2006)

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Tutorial 8: Expectation and Variance; Special Distributions

Week 9 (6–10 March, 2006)

1. **Expectation and Variance**

   (a) For the discrete random variable X with the following probability distribution:

   $$f(x) = \frac{|x-2|}{7} \text{ for } x = -1, 0, 1, 2, 3$$

   determine $E(X)$ and $\mathrm{var}(X)$. Now assume the functions $g(X) = 3X + 2$ and $h(X) = X^2$ and determine $E(g(X))$ and $E(h(X))$.

   (b) In Chebyshev's theorem, which form does the inequality take for $k = 1, 2, 3, 4$?

2. **Covariance**

   The *covariance* of two random variables X and Y with the joint distribution $f(x,y)$ is defined as:

   $$\mathrm{cov}(X,Y) = E((X-\mu_X)(Y-\mu_Y)) = \sum_x \sum_y (x-\mu_X)(y-\mu_Y) \cdot f(x,y)$$

   where μ_X and μ_Y are the means of X and Y.

   Assume that X and Y have the following joint distribution:

(x,y)	0	1	2
0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{12}$
1	$\frac{2}{9}$	$\frac{1}{6}$	0
2	$\frac{1}{36}$	0	0

   (a) Compute the marginal distributions of X and Y.

   (b) Use the marginal distributions to compute μ_X and μ_Y.

   (c) Now compute the covariance of X and Y.

3. **Special Distributions**

   (a) A scientist claims that 1 in 10 car accidents are due to driver fatigue. Using the formula for the binomial distribution, compute the probability that at most 3 of 5 accidents that happen on a given day are due to driver fatigue.

   (b) In a reaction time experiment, the response latency in seconds is distributed according to the standard normal distribution. What is the probability that the reaction time is between 0 and 1 seconds?