

Formal Modeling in Cognitive Science 1 (2005–2006)

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Tutorial 7: Random Variables and Probability Distributions

Week 8 (27 February–3 March, 2006)

1. Probability Distributions and Probability Densities

(a) For each of the following, determine whether the given function can serve as the probability distribution for a random variable with the given range.

i. $f(x) = \frac{x-2}{5}$ for $x = 1, 2, 3, 4, 5$

ii. $f(x) = \frac{x^2}{30}$ for $x = 0, 1, 2, 3, 4$

iii. $f(x) = \frac{1}{5}$ for $x = 0, 1, 2, 3, 4, 5$

(b) Find the cumulative distribution $F(x)$ for the random variable with the probability distribution:

$$f(x) = \frac{x}{15} \text{ for } x = 1, 2, 3, 4, 5$$

(c) The probability density function of the random variable X is given by:

$$f(x) = \begin{cases} \frac{1}{8}(x+1) & \text{for } 2 \leq x < 4 \\ 0 & \text{elsewhere} \end{cases}$$

i. Find $f(2 < X < 3)$.

ii. Find the cumulative distribution $F(x)$.

2. Joint and Marginal Distributions

Two psychometric tests are administered to a group of 10 experimental subjects. The first test is a standardized aptitude test used for university admissions. It returns a score between 0 and 100. Let X be the random variable denoting the aptitude test score. The second psychometric test is a memory test. It measures the digit span, i.e., the number of digits in a sequence that a subject is able to remember before they make a mistake. Let Y be the random variable that denotes the digit span, ranging from 1 to 5. The results of the experiment are given in the following table:

Subject	1	2	3	4	5	6	7	8	9	10
x	60	50	40	50	60	30	30	20	30	50
y	5	3	3	3	3	3	3	4	4	4

(a) Compute the distributions of X and Y .

(b) Compute the joint distribution of X and Y .

(c) Compute the marginal distributions of X and Y .

(d) Are X and Y independent?

(e) Compute the conditional distributions of X given $Y = 3$.