Formal Modeling in Cognitive Science 1 (2005–2006)

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Tutorial 6: Combinatorics, Basic Probability, Bayes' Theorem

Week 7 (20–24 February 2006)

1. **Combinatorics**

   (a) Assume that all monosyllabic words of English consist of exactly three letters. The first letter can be a consonant or a vowel, the second letter has to be a vowel, and the third letter has to be a consonant. Let the set of vowel be {a, e, i o, u}, the set of consonants the complement of this set.

      i. How many three-letter words are there if a given letter can occur more than once?
      ii. How many three-letter words are there if a given letter can occur only once?

   (b) For a psychological experiment, you want to recruit subjects from a class of 20 students. The subjects have to participate in the experiment in pairs.

      i. In how many different ways can you choose 10 ordered pairs of subjects?
      ii. In how many different ways can you choose 10 unordered pairs of subjects?

2. **Basic Probability**

   (a) Let A and B be two events with $P(A) = 0.59$ and $P(B) = 0.30$ and $P(A \cap B) = 0.21$. Compute the following probabilities:

      i. $P(A \cup B)$
      ii. $P(A \cap \bar{B})$
      iii. $P(\bar{A} \cup \bar{B})$
      iv. $P(\bar{A} \cap \bar{B})$

   (b) In linguistics, we are often interested in $P(w_{n+1}|w_n)$, the probability that a word w_{n+1} occurs given that the previous word is w_n. ($P(w_{n+1}|w_n)$ is sometimes called a *transitional probability*.)

      i. Assume you know the transitional probabilities $P(\mathit{spotted}|\mathit{the})$, $P(\mathit{dog}|\mathit{spotted})$, and $P(\mathit{the})$. What's the probability of the sequence *the spotted dog*?
      ii. Assume that you know that the word *amok* can follow the words *run*, *running*, and *ran*, which occur with the probabilities $P(\mathit{run}) = 0.5$, $P(\mathit{running}) = 0.25$, and $P(\mathit{ran}) = 0.25$. You also know that the transitional probabilities $P(\mathit{amok}|\mathit{run}) = 0.3$, $P(\mathit{amok}|\mathit{running}) = 0.2$, and $P(\mathit{amok}|\mathit{ran}) = 0.1$. What is the overall probability of seeing *amok*, i.e., $P(\mathit{amok})$?

   (c) A balanced die is tossed twice. Let A be the event that an even number comes up on the first toss, B be the event that an even number comes up on the second toss, and C the event that both tosses result in the same number. Which of the events A, B, and C dependent, which ones are independent?

3. **Bayes' Theorem**

   Assume that the prevalence of the disease *ritengitis* in the general population 1 in 500. Fever is a symptom of ritengitis, which is present in 3 in 10 cases. Fever is also a symptom of the disease *mesiopathy*, present in 7 in 10 cases. The prevalence of *mesiopathy* is 1 in 100. If a patient presents with fever, what is the probability that they have ritengitis?