FMCS 1 Practical 3: Dynamics of simple neuron populations

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1 Excitatory and inhibitory populations

We study the rate dynamics of two populations of neurons, an excitatory one group and an inhibitory one group. The populations are recurrently connected. The firing rates ν are given by the following differential equations:

$$\begin{aligned} \tau_e \frac{d\nu_e}{dt} &= -\nu_e + [M_{ee}\nu_e + M_{ei}\nu_i - \gamma_e]_+ \\ \tau_i \frac{d\nu_i}{dt} &= -\nu_i + [M_{ii}\nu_i + M_{ie}\nu_e - \gamma_i]_+ \end{aligned}$$

With the symbol $[x]_+$ we indicate rectification, i.e. $[x]_+ = x$ if $x > 0$ and $[x]_+ = 0$ if $x \leq 0$. Because firing rates can never be negative, this seems a decent thing to do. The subscript e (i) denote that the population is excitatory (inhibitory).

Here τ_e and τ_i are the time constants of the populations. The γ are the firing thresholds. Describe briefly for yourself the meaning of the different terms in these equations.

The connection weights between the populations are given by the M's. Set $M_{ee} = 1.25$, $M_{ie} = 1$, $M_{ii} = M_{ei} = -1$. Check that these signs of M make sense, namely connections from the excitatory population should increase activity, while those from the inhibitory population should decrease it. Furthermore set $\gamma_e = -10$Hz, $\gamma_i = 10$Hz. The negative threshold value for the excitatory population causes it to be active even without input. Alternatively, we could have provided external input to the excitatory population. Finally, set $\tau_e = 10$ms; the value of τ_i we keep as a variable.

In order to integrate these equation (i.e. follow the evolution in time), we use that $\frac{df(t)}{dt} = \lim_{\Delta t \to 0} \frac{1}{\Delta t}[f(t + \Delta t) - f(t)]$. Therefore, the equation $\tau \frac{df(t)}{dt} = -f(t) + g(t)$ gives that the value of f at the next time-step is $f(t + \Delta t) = (1 - \frac{\Delta t}{\tau})f(t) + \frac{\Delta t}{\tau} g(t)$. This way we can step through time. For a proper approximation one should have $\Delta t \ll \tau$.

a) Implement the differential equations in Matlab. Plot the activity, that is, ν_e and ν_i as it develops in time, for $\tau_i = 50$.

We can analyze this system more formally by making a linear approximation around the fixed point. Linear approximation means in this case that we stay away from the activity regime where the rectification acts. The stability matrix is

$$N = \begin{pmatrix} \frac{M_{ee}-1}{\tau_e} & \frac{M_{ei}}{\tau_e} \\ \frac{M_{ie}}{\tau_i} & \frac{M_{ii}-1}{\tau_i} \end{pmatrix}$$

The eigenvalues of this matrix determine the stability of the network around the fixed point. The Matlab function `eig(N)` gives the eigenvalues of a matrix N; with `real()` and `imag()` the real and imaginary parts of the eigenvalues can be extracted.

b) Explore the real and imaginary parts of the eigenvalues as a function of τ_i. There are four different regimes. Try $\tau_i = 10$, $\tau_i = 50$, $\tau_i = 100$, $\tau_i = 1000$. For each value plot the activities. If you have time you plot the real and imaginary parts versus τ_i.

c) Plot ν_i against ν_e in a phase plot. e.g. `plot(ve_array, vi_array,'*')` for the above choices of τ_i.