PRACTICE QUESTIONS – NOT FOR USE IN EXAM

Module Title: FORMAL MODELLING IN COGNITIVE SCIENCE 1
Exam Diet (Dec/May/Aug): MAY DIET 2005

Brief notes on answers:

1. (a) A categorial syllogism is an argument of two premises, each containing pairs drawn from the three terms: A, B and C, arranged in one of the four 'figures': ABBC, BACB, ABCB, BABC. Each premise has one of the four quantifiers All, Some, No, Some not. A conclusion to such a syllogism must contain one of the same four quantifiers and the two end-terms A and C. From this specification it follows that there are 64 distinct pairs of syllogistic premises. If we only consider models in which there are at least some As, Bs and Cs, then there are 27 pairs of premises with valid expressible conclusions. An example would be:

   All B are A. Some B are not C. has the valid conclusion that *Some A are not C.* An example of a syllogism without a valid conclusion would be *All A are B. Some B are not C.*

   (b) A credulous reasoning task is one in which the hearer attempts to construct the speaker's intended model of the premises. In the so-called 'immediate inference' task, subjects are given single syllogistic premises, and asked whether other single premises follow from them, or not. For example, a subject might be given *All A are B* and asked whether *All B are A* follows, whether its negation follows, or whether it could be either true or false on different occasions.

   In the results from this task, there are many patterns of inference that subjects make which are probably interpretable as credulous conclusions. For example, in the case used to introduce the task above, the inference is invalid in classical logic (which demands the third answer), but could be valid by closed world reasoning in a credulous logic. If the database contains no other conditional with B as its conclusion, then the only way for a thing to be B is for it to be A.

   Another commonly observed example of credulous reasoning from this task is when subjects conclude from *Some A are B* that *Some A are not B.* This pattern of reasoning was called an implicature by Grice. It appears to rest on reasoning that the speaker would have said that *All A are B* if that were the case. This is not a pattern that is valid in the default logic described in the course (no existential quantifiers were presented in that logic), but it is a general case of credulous reasoning in an attempt to derive a more specific 'intended' model of the premise than classical logic will afford.

   (c) If subjects have a credulous interpretation of the task, they will frequently draw conclusions to problems which do not have classically valid conclusions. This will have the effect of lowering their scores on these no-valid-conclusion problems, since the experimenter typically adopts a classical definition of validity.

   Aristotle's two meta-principles of the syllogism state that:

   (i) Problems with valid conclusions always have at least one universal premise.

   (ii) Problems with valid conclusions always have at least one positive premise.

   The syllogism *All A are B. Some B are not C* is an example which passes both Aristotle's principles and so is a candidate for having a valid conclusion but in fact it does not have one. Such syllogisms are observed to be particularly hard to recognise as no valid conclusion problems.

It is not clear whether adoption of credulous interpretations for the premises would make such problems **particularly** hard to recognise. It would make them hard to recognise because credulous interpretation provides them with valid conclusions (eg. by reversing the conditional premise in this example) but this is also true for some problems which fail Aristotle's principles.

2. Wason's selection task consists of the following materials and instructions:

> Below is depicted a set of four cards, of which you can see only the exposed face but not the hidden back. On each card, there is a number on one of its sides and a letter on the other.
>
> Also below there is a rule which applies only to the four cards. Your task is to decide which if any of these four cards you *must* turn in order to decide if the rule is true. Don't turn unnecessary cards. Tick the cards you want to turn.
>
> **Rule:** *If there is a vowel on one side, then there is an even number on the other side.*
>
> **Cards:**
>
> | A | K | 4 | 7 |
> |---|---|---|---|

Wason believed that there was a unique correct answer to this problem which was to choose to turn the A and the 7 cards. He observed that in fact only about 5-10% of his subjects adopted this response and so concluded that subjects were irrational and illogical. He explained their choices mainly in terms of a failure to seek falsifying evidence for the rule.

Wason's claims suffer from several problems. The most fundamental is the issue whether there is a single right answer, since there are several interpretations of his rule which should lead to different choices. Wason assumed that his subject ought to adopt a classical material implication interpretation of the conditional, even though it is widely accepted that this is not the usual interpretation of such natural language conditionals. It is indeed very odd to describe as a 'rule' something which is only intended to apply to a closed set of cases.

The most likely initial interpretation of such a conditional is as a 'robust' law-like conditional which may have exceptions. If this is the interpretation a subject adopts, then their logically correct response (however unlikely they might be to give such a response) would be to deny the possibility of the task, since no amount of cases can establish the truth of such a rule without also providing evidence about what counts as an exception and as a counterexample.

So one might reply that Wason got his task wrong because he specified only one 'correct' answer where there are several. But one might also ask whether Wason was right in his explanations for subjects' behaviour and about whether there is evidence about the actual interpretations that subjects do adopt. For example, it is possible that no subjects actually do adopt this robust law-like interpretation of the conditional and so Wason may have been right that subjects actually adopt a material implication reading, and only fail to turn A and 7 because they fail to attempt falsification.

On the second issue first, the best evidence that robustness of interpretation might be a problem is that subjects find Stenning & van Lamblagen's 'two-rule' task easier considerably easier (at least a much higher proportion adopt a material reading). This task asks for selection of cards to reveal which one of two rules is true and which false. The task therefore distracts attention from robustness of either rule and focuses on choice of which one fits the data. It is hard to explain the result of this task in any other terms, because, on the face of it the task is considerably more complicated.

Taking the first issue second, was Wason right about falsification? Again there are two sub-issues. One is whether he (and Popper) was right that falsification is in fact the right inductive strategy here. In fact, falsification can be a bad strategy if the 'negative-sets' are very much larger than the positive ones. But even if they are accepted to be equal and falsification is therefore a reasonable strategy, it is still questionable whether subjects are in fact failing to falsify. Subjects often explain that if they turn over A, then it will either verify or falsify the rule, so they cannot be dismissed as not finding falsifications relevant.

3. (a) Eigenvalues 0 and 2, with eigenvectors $(1, -1)$ and $(1, 1)$.
   (b) This is a projection matrix.
   (c) Det $= 0$, hence not invertable.
   (d) $x_1 + x_2 = 3$, this is a line.

4. (a) $f(x) = \frac{1}{2} + c\,\exp(-2x)$, where c is an arbitrary constant.
   (b) $f(x) = \frac{1}{2} + \frac{1}{2}\exp(-2x)$.
   (c) $2xe^{x^2}/(1 + e^{x^2})^2$.
   (d) The derivative is only zero when $x = 0$. The function value is 0.5 at $x = 0$. It's a minimum.

5. (a) (i). The marginal distributions are:

		x 1	2	3	$f(y)$
	2	0.3	0.1	0	0.4
y	4	0.3	0	0	0.3
	6	0.1	0.1	0.1	0.3
$f(x)$		0.7	0.2	0.1	

(ii). The conditional distribution can be computed as: $f(y|x) = \frac{f(x,y)}{f(x)}$. Hence $f(y|x = 1)$ is:

		$x = 1$
	2	$\frac{3}{7}$
y	4	$\frac{3}{7}$
	6	$\frac{1}{7}$

(iii). Using Bayes' Theorem, we can compute $f(y|x)$ as:

$$f(y|x) = \frac{f(y)f(x|y)}{f(x)}$$

(iv). The entropy of X is:

$$
\begin{aligned}
H(X) &= -\sum_{x \in X} f(x) \log f(x) \\
&= -(0.7 \log 0.7 + 0.2 \log 0.2 + 0.1 \log 0.1) \\
&= 0.36 + 0.46 + 0.33 = 1.15
\end{aligned}
$$

[In the actual exam, the values will be so simple that all solutions can be calculated with pencil and paper.]

(b) Chebyshev's Theorem is as follows:
If μ and σ are the mean and the standard deviation of a random variable X, and $\sigma \neq 0$, then for any positive constant k:

$$P(|x - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

In other words, the probability that X will take on a value within k standard deviations of the mean is at least $1 - \frac{1}{k^2}$.
This theorem can be used in cognitive science to estimate the spread of an empirical distribution. For example, if we are dealing with reaction time data (or eye-movement data), and we know the mean and the standard deviation of the data, then we know that at least $1 - \frac{1}{2^2} = 75\%$ of the data points fall within two standard deviations of the mean.

6. (a) The total number of word in the language is $85 + 10 + 5 = 100$. The number of possible four-word sequences is therefore 100^4.

(b) For the first pattern, the number of possible sentences is $85 \cdot 10 \cdot 84 \cdot 83$, for the second pattern it is $85 \cdot 10 \cdot 5 \cdot 84$.

(c) (i). $P(N) = 0.85$, $P(V) = 0.1$, $P(P) = 0.05$.

(ii). The number of bits required to transmit a single value of X is given by the entropy $H(X)$:

$$
\begin{aligned}
H(X) &= -\sum_{x \in X} f(x) \log f(x) \\
&= -(0.85 \log 0.85 + 0.1 \log 0.1 + 0.05 \log 0.05) \\
&= 0.20 + 0.33 + 0.22 = 0.75
\end{aligned}
$$

[In the actual exam, the values will be so simple that all solutions can be calculated with pencil and paper.]
If four independent values of X are to be transmitted, the entropy of the sequence is $4 \cdot 0.75 = 3$.

(d) If a sequence of n values of X is to be transmitted, then the entropy of the sequence is $H(X_1, \ldots, X_n)$. If the values of X are independent (as assumed in 6(c.ii)), then $H(X_1, \ldots, X_n) = H(X_1) + \ldots H(X_n)$. However, in reality these values will not be independent. For example, after a verb V, the only possible values are N and P (assuming the sentence templates in 6(b)). This means that the true entropy of a sequence of four values should be computed as:

$$H(X_1, \ldots, X_n) = H(X_1) + H(X_2|X_1) + \ldots + H(X_n|X_1, \ldots, X_{n-1})$$

Where $H(X_2|X_1)$ is the conditional entropy of X_2 given X_1, etc.