

*PRACTICE QUESTIONS – NOT FOR USE IN EXAM*

UNIVERSITY OF EDINBURGH

COLLEGE OF SCIENCE AND ENGINEERING

SCHOOL OF INFORMATICS

**FORMAL MODELLING IN COGNITIVE SCIENCE 1**

**Thursday 1 April 2004**

**00:00 to 00:00**

Convener: H Thompson  
External Examiner: J Barnden

**INSTRUCTIONS TO CANDIDATES**

**ANSWER A TOTAL OF THREE QUESTIONS** as follows:

either **QUESTION 1** or **QUESTION 2** and  
either **QUESTION 3** or **QUESTION 4** and  
either **QUESTION 5** or **QUESTION 6**.

**The total of marks that can be obtained is 100.**

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1. **Logic** [33 marks]
- (a) What is a categorial syllogism? [7 marks]
  - (b) Cite some evidence that many undergraduate subjects interpret the task of solving them as a credulous task. [19 marks]
  - (c) Might this explain why problems without valid classical logic are observed to be harder, especially for problems which are not ruled out from having conclusions by Aristotle's two metalogical principles? [7 marks]

**2. Logic**

[33 marks]

Was Wason right about his selection task?

**3. Mathematics for Neural and Connectionist Modelling**

[33 marks]

Given a matrix  $M = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ .

- (a) Calculate the eigenvalues and eigenvectors. [6 marks]
- (b) Sketch the eigenvectors. What does the matrix describe? Interpret the results. [6 marks]
- (c) Calculate the determinant of the matrix. Is the matrix invertible? [9 marks]
- (d) Solve  $M.x = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ . What does the solution describe? [12 marks]

**4. Mathematics for Neural and Connectionist Modelling**

[33 marks]

Given the differential equation  $df(x)/dx = -2f(x) + 1$ .

- (a) What is the solution to this equation? [7 marks]
- (b) Given that  $f(0) = 1$ , what is now the solution? [6 marks]
- (c) Given  $g(y) = 1/(1 + \exp -y)$  and  $h(x) = x^2$ . Calculate  $dg(h(x))/dx$ . [10 marks]
- (d) Where is the derivative  $dg(h(x))/dx$  zero? It is a maximum or a minimum? [10 marks]

**5. Probability and Information Theory**

[34 marks]

(a) Given two random variables  $X$  and  $Y$  with the following joint probability distribution  $f(x, y)$ :

		$x$		
		1	2	3
$y$	2	0.3	0.1	0
	4	0.3	0	0
	6	0.1	0.1	0.1

- i. Compute the marginal distributions for  $X$  and  $Y$ .[8 marks]
  - ii. Compute the conditional distribution  $f(y|x = 1)$ .[5 marks]
  - iii. Give an alternative way of computing  $f(y|x)$ ?[5 marks]
  - iv. Compute the Entropy of  $X$ .[6 marks]
- (b) State Chebyshev’s theorem and explain its relevance. Also mention possible applications in cognitive science. [10 marks]

**6. Probability and Information Theory**

[34 marks]

A controlled language is defined as a subset of a natural language (e.g., English) with restricted grammar and vocabulary. Controlled languages are used to standardize text production, e.g., for technical documents that have to be translated into a large number of languages.

Assume a controlled language that only contains nouns, verbs, and prepositions. This language has a vocabulary consisting of 85 nouns (N), 10 verbs (V), and 5 prepositions (P).

- (a) Compute the number of four-word sequences that are possible in this controlled language (word can be repeated).
- (b) Now assume the grammar of the language is also controlled. Only sentences with the following structures are allowed:

N V N N  
N V P N

Compute the number of sentences that are possible given that words cannot be repeated in a sentence.

- (c)
  - i. Assume that all words in the language are equally likely. Compute the probabilities  $P(N)$ ,  $P(V)$ , and  $P(P)$ .
  - ii. Now assume a random variable  $X = \{N, V, P\}$  and the probabilities you computed in the previous question. What is the number of bits required on average to transmit a sequence of four values of this random variable? (Assume that the values in the sequence are independent.)
- (d) What is conditional entropy? Describe an application in the controlled language scenario.