



THE UNIVERSITY *of* EDINBURGH  
**informatics**

# Embedded Systems

## Lecture 11: Worst-Case Execution Time

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# Overview

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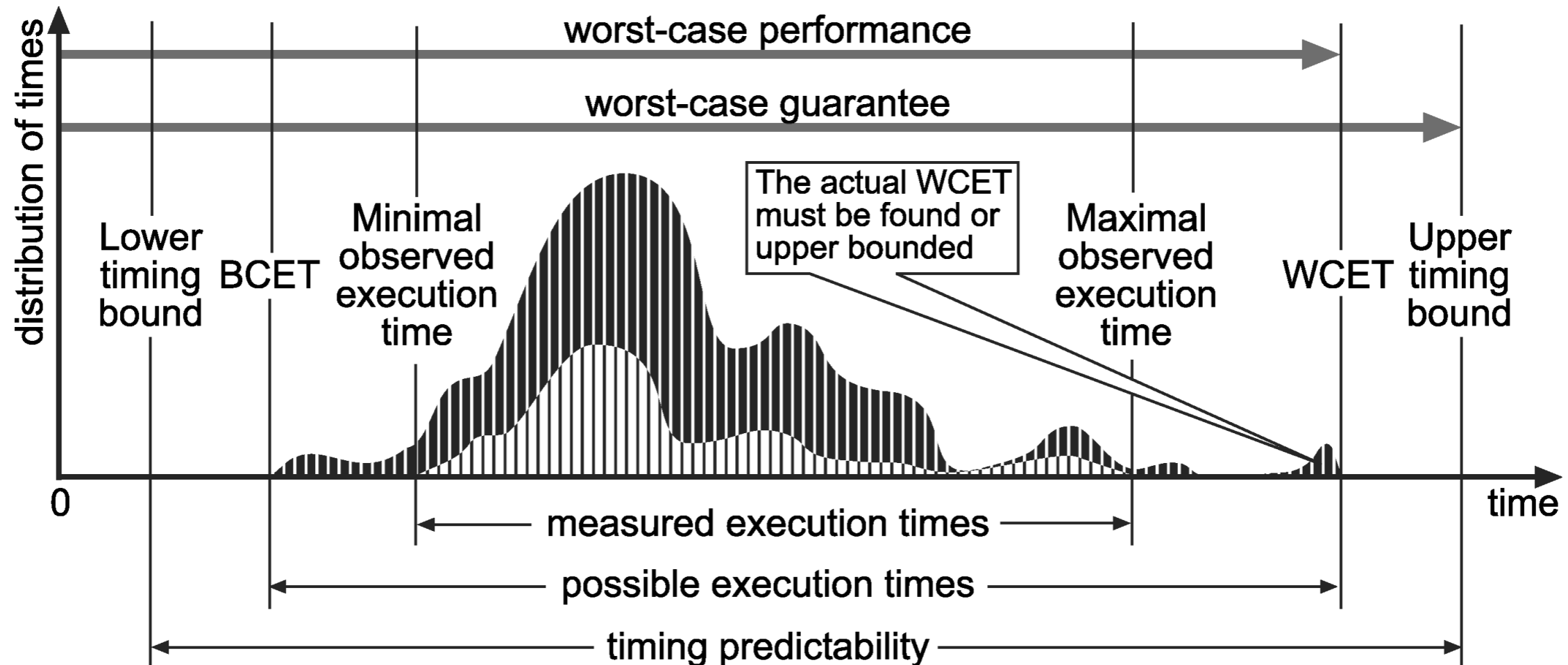
- Motivation
- Worst-Case Execution Time Analysis
  - Types of Execution Times
  - Measuring vs. Analysing
  - Flow Analysis
  - Low-Level Analysis
  - Calculation

# Motivation: Characteristics of Real-Time Systems

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- Concurrent control of separate system components
- Reactive behaviour
- **Guaranteed response times**
- Interaction with special purpose hardware
- Maintenance usually difficult
- Harsh environment
- Constrained resources
- Often cross-development
- Large and complex
- Often have to be extremely dependable

# What is *the* “Execution Time” of a program?



[Wilhelm+08]

- WCET:** Worst-Case Execution Time
- BCET:** Best-Case Execution Time
- ACET:** Average-Case Execution Time

The WCET/BCET is the longest/shortest execution time possible for a program.  
Must consider all possible inputs—including perhaps inputs that violate specification.

# Why may we care about the WCET?

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- We are interested in WCET to. . .
  - perform schedulability analysis
  - ensure meeting deadlines
  - assess resource needs for real-time systems
- **WCET accuracy may be safety-critical!**

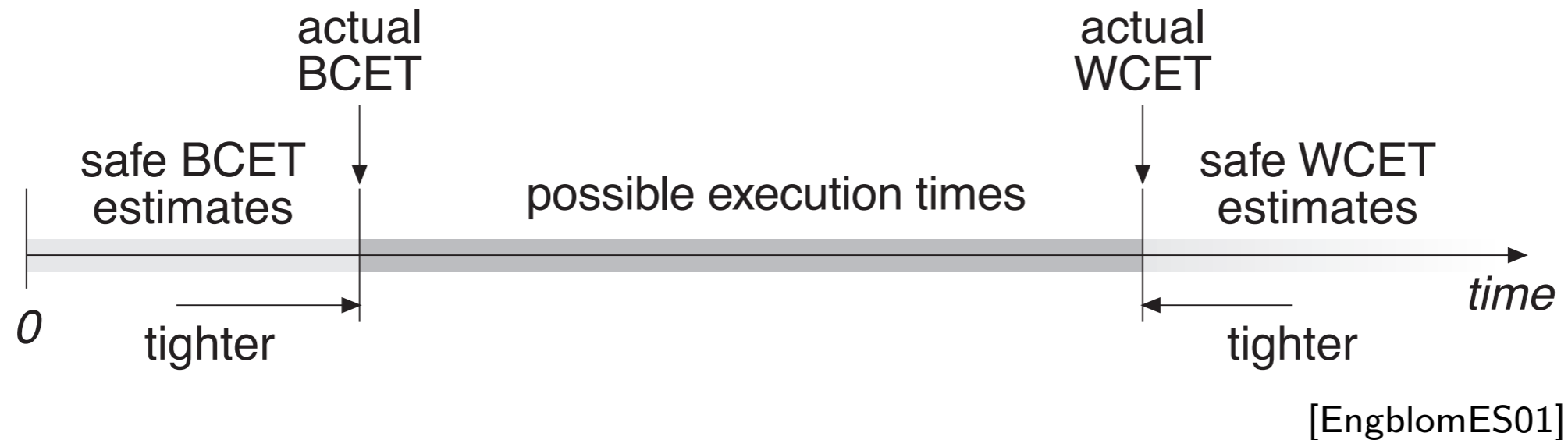
# And why may we care about the BCET?

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- We are interested in BCET to. . .
  - benchmark hardware
  - assess code quality
  - assess resource needs for non/soft real-time systems
  - ensure meeting livelines (new starting points)

# What is *the* “Execution Time” of a program?

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- Approaches for approximating WCET or BCET
  - **Measuring:** Measure run time of program on target hardware
  - **Analysis:** Compute estimate of run time, based on program analysis and model of target hardware
  - **Hybrid:** Combine measurements with program analysis

# Measuring WCET/BCET

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- Execution time may depend on program inputs
  - In this case, quality of measurements depends on judicious choice of inputs
- Execution time may depend on execution context (cache content, state of pipeline, ...)
- Typically need to add safety margin to best/worst result measured
- **Extensive testing/measurement still common practice**



# Measuring Program Run Times

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- Call OS timing routines
  - Account for cost of calls to timing routines themselves
- Access hardware timers directly
- Use external hardware
  - Oscilloscope, Logic analyser
- Count emulator cycles
- High water marking
  - Continuously record max execution times
  - Standard feature of RTOSs
  - May include this in shipped products
  - Read at service intervals

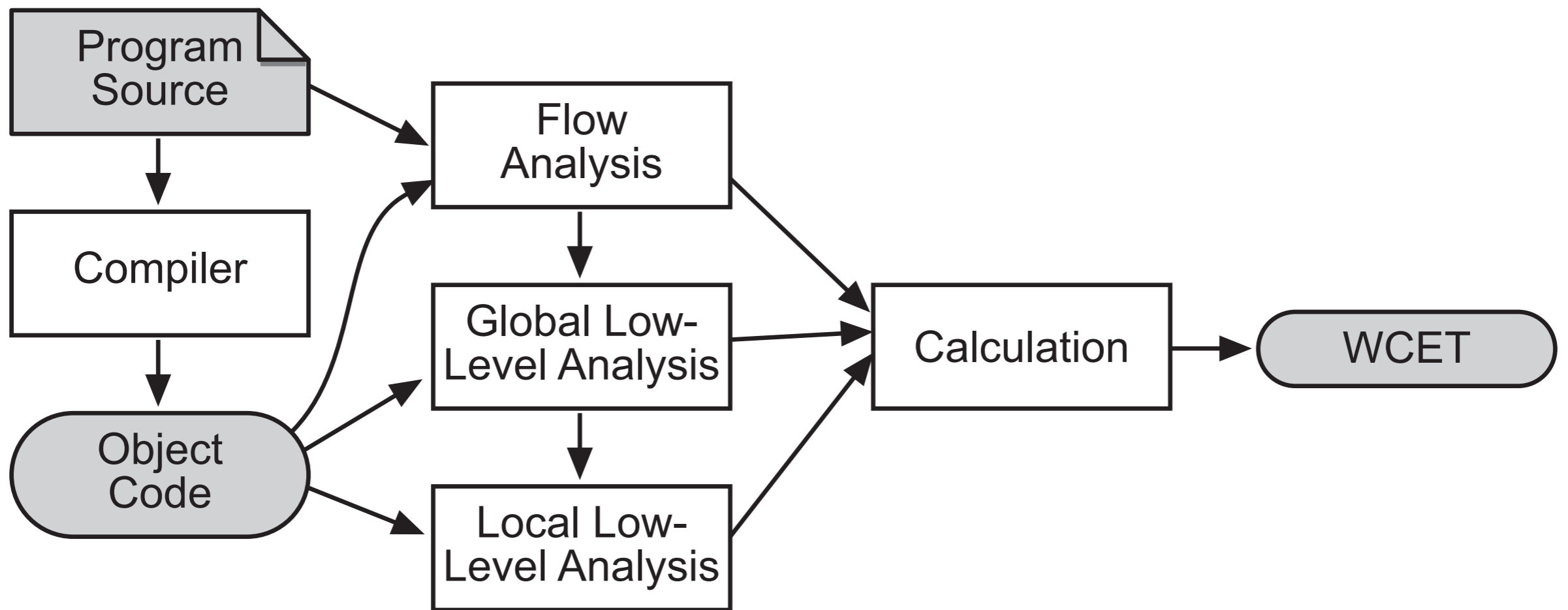
# Analysing WCET/BCET

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- Instead of measuring execution times, compute them
- **Advantages**
  - Can ensure safety of result
  - Saves testing effort
- **Disadvantages**
  - Try to be as tight as possible—may not always succeed
  - Typically requires extensive analysis effort
- Accuracy depends on
  - Complexity of hardware
  - Program structure
  - Quality of hardware model
  - Program analysis capabilities

# Analysing WCET/BCET

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[EngblomES01]

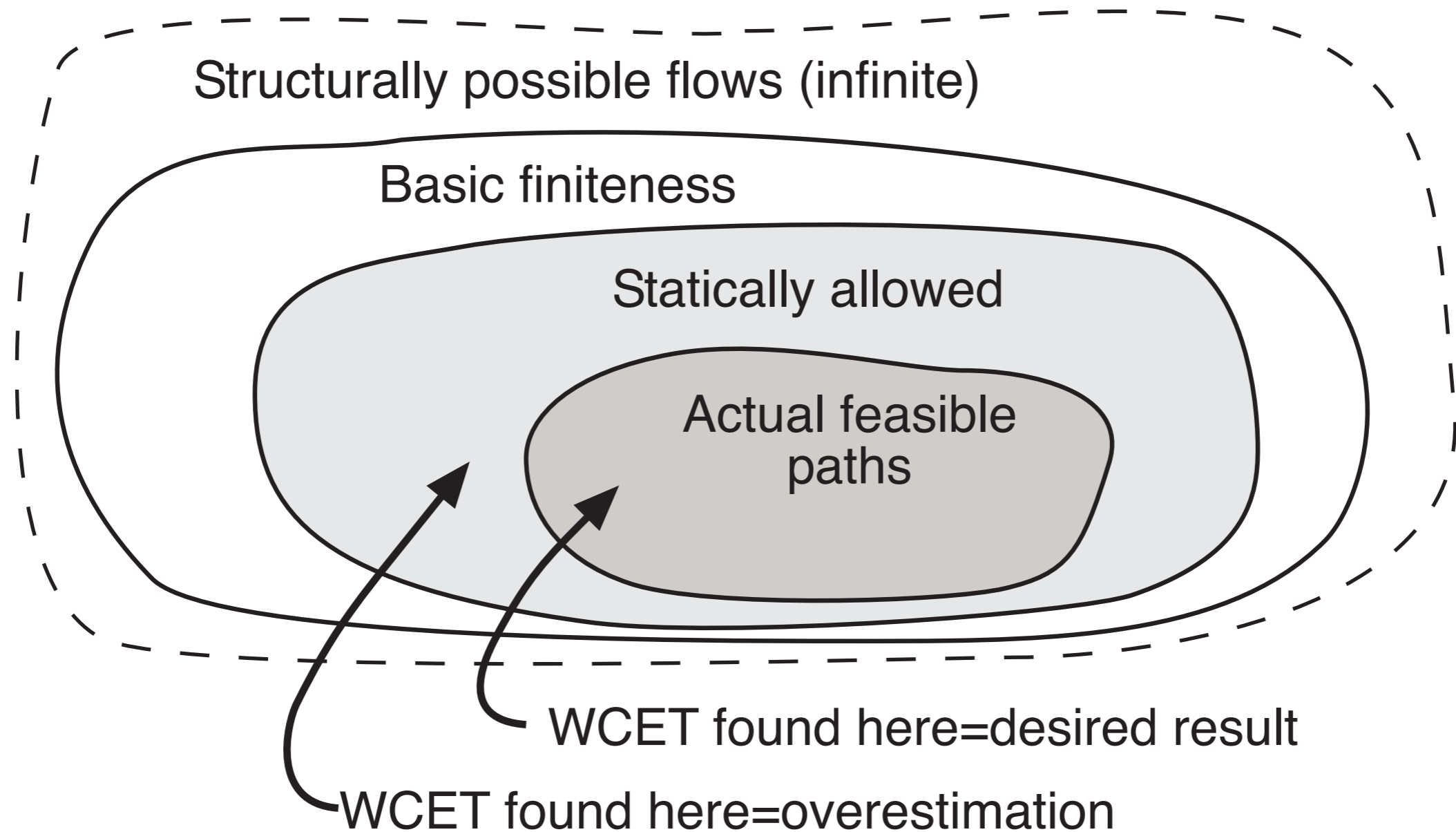
# Flow Analysis

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- Analyse dynamic behaviour of program
  - Number of loop iterations, Recursion depth, Input dependences, Infeasible paths, Function instances, ...
- Get information from
  - Static Analysis
  - Manual Annotation
- Analysis level
  - Object code
  - Source code (may need non-trivial mapping to object code)

# Flow Analysis

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# Flow Analysis

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- The set of **structurally possible** flows for a program, i.e. those given by the structure of the program, is usually infinite, since e.g. loops can be taken an arbitrary number of times
- The executions are made finite by bounding all loops with some upper limit on the number of executions (**basic finiteness**)
- Adding even more information, e. g. about the input data, allows the set of executions to be narrowed down further, to a set of **statically allowed** paths. This is the “optimal” outcome of the flow analysis.

# Flow Analysis

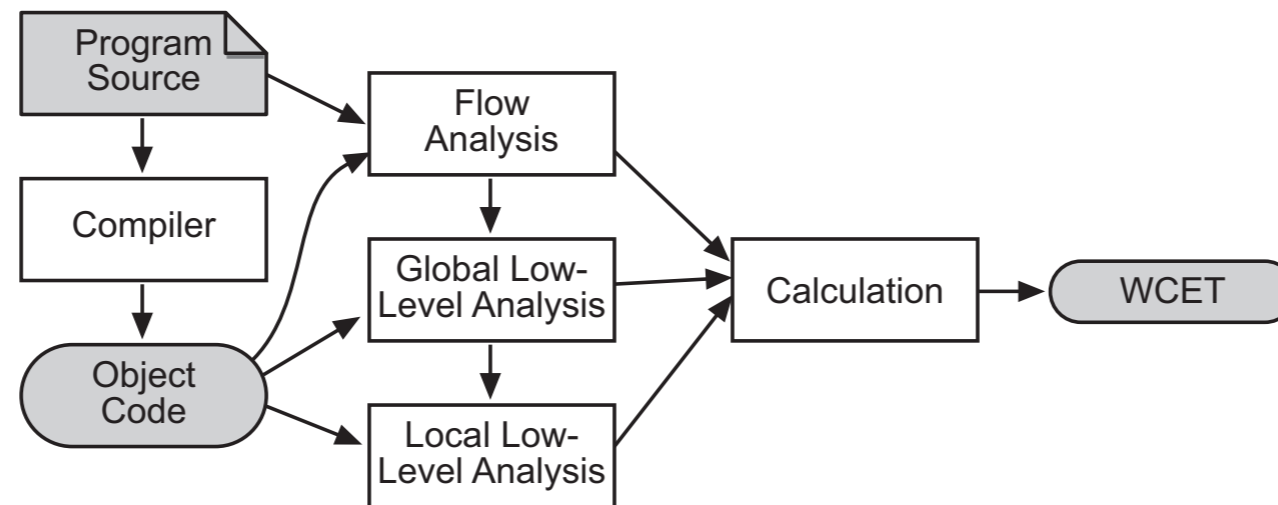
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```
    const int max = 100;
    foo (float x) {
A:     for(i = 1; i <= max; i++) {
B:         if (x > 5)
C:             x = x * 2;
           else
D:             x = x + 2;
E:         if (x < 0)
F:             b[i] = a[i];
G:         bar (i)
           }}
```

- **Loop bounds:** Easy to find in this example; in general, very difficult to determine
- **Infeasible paths:** Can we exclude a path, based on data analysis?  
A-B-C-E-F-G is infeasible—since if  $x > 5$ , it is not possible that  $x * 2 < 0$ .  
*Well, really? What about integer overflows? Must be sure that these do not happen in the example...*

# Low-Level Analysis

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[EngblomES01]

- Determine execution time for program parts
- Account for hardware effects (pipeline, caches...)
- Work on object code
- Exact analysis generally not possible

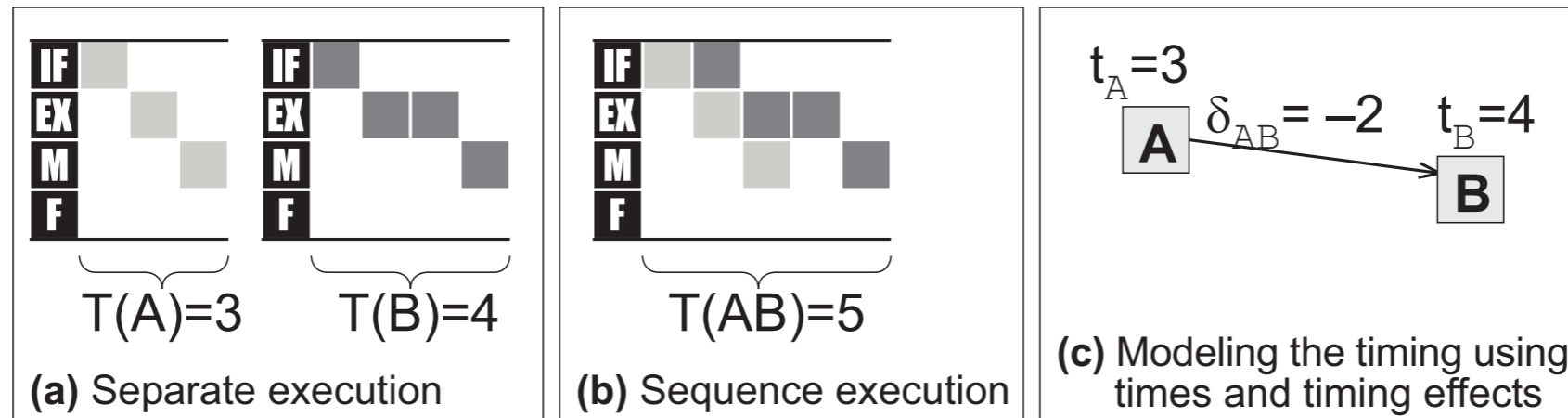


# Low-Level Analysis

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- Global Low-Level Analysis
  - Considers execution time effects of machine features that reach across entire program
  - Instruction/data caches, branch predictors, translation lookaside buffers (TLBs)
- Local Low-Level Analysis
  - Considers machine features that affect single instruction & its neighbours
  - Scalar/superscalar pipelines

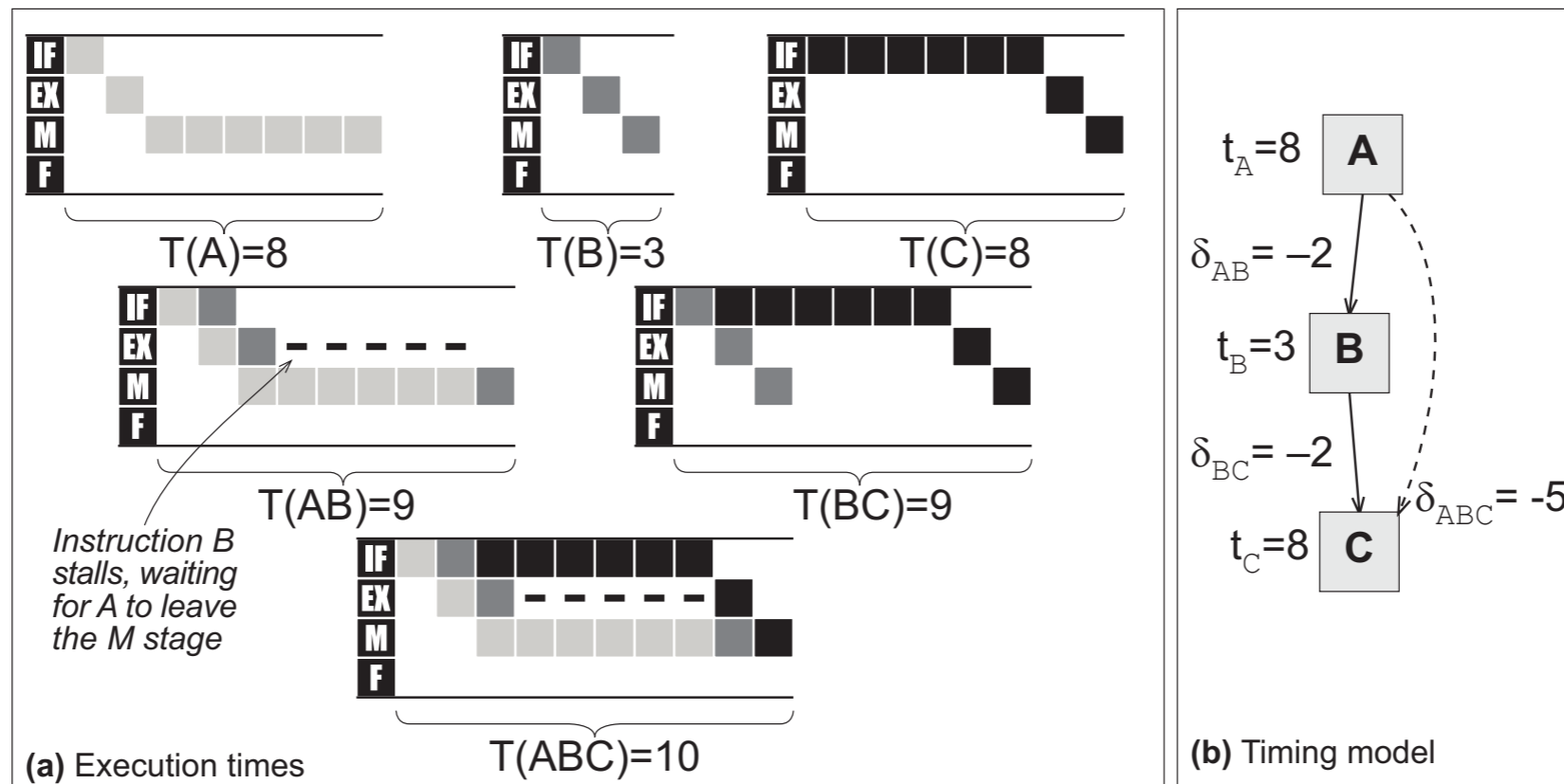
# Local Low-Level Analysis - Pipelining



[EngblomJ02]

- Pipeline effect of two successive instructions
- Pipeline overlap reduces overall computation time by  $\delta = -2$

# Local Low-Level Analysis - Pipelining



[EngblomJ02]

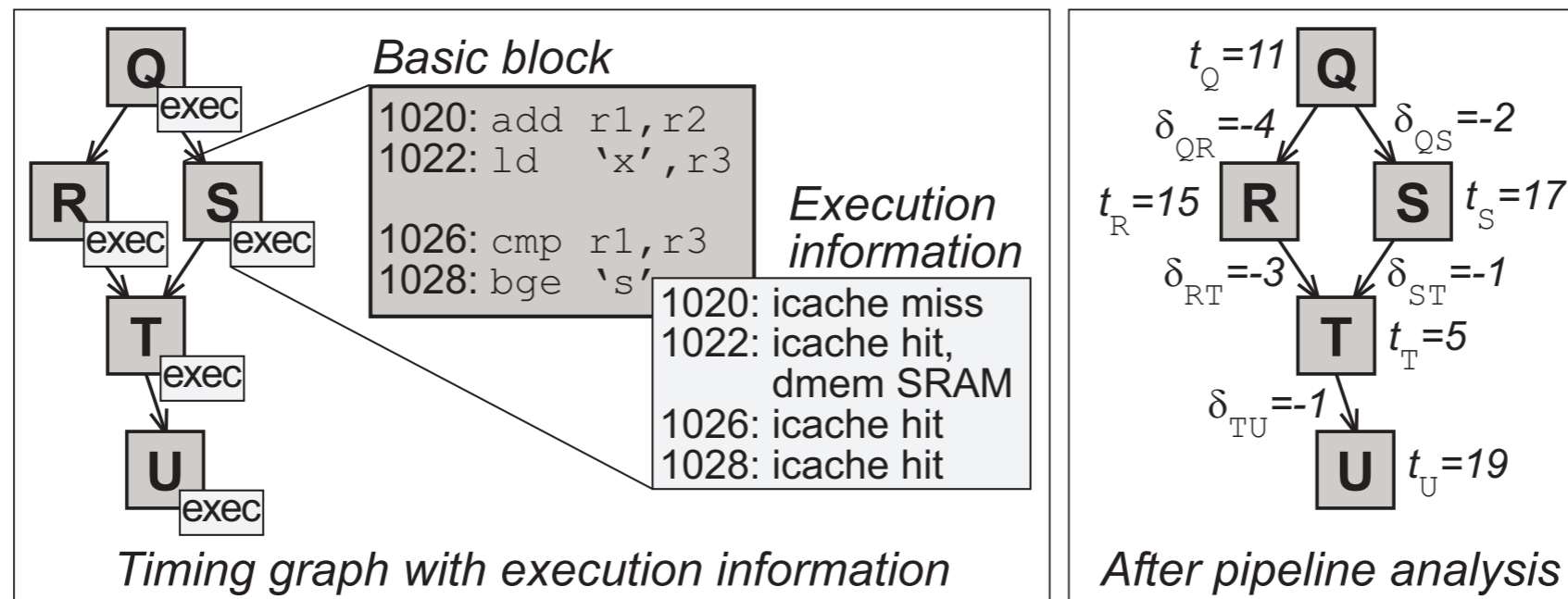
- Pipelining effect of three successive instructions
- Reduction of combining three instructions can be larger than sum of savings when combining them pair-wise!

# Global Low-Level Analysis - Caches

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- Instruction Caches
  - Predictable from control flow
- Data Caches
  - No simple way to predict accesses
  - Very difficult analysis problem
- Unified Caches
  - Very pessimistic as a result of combining instructions & data

# Global Low-Level Analysis - Caches

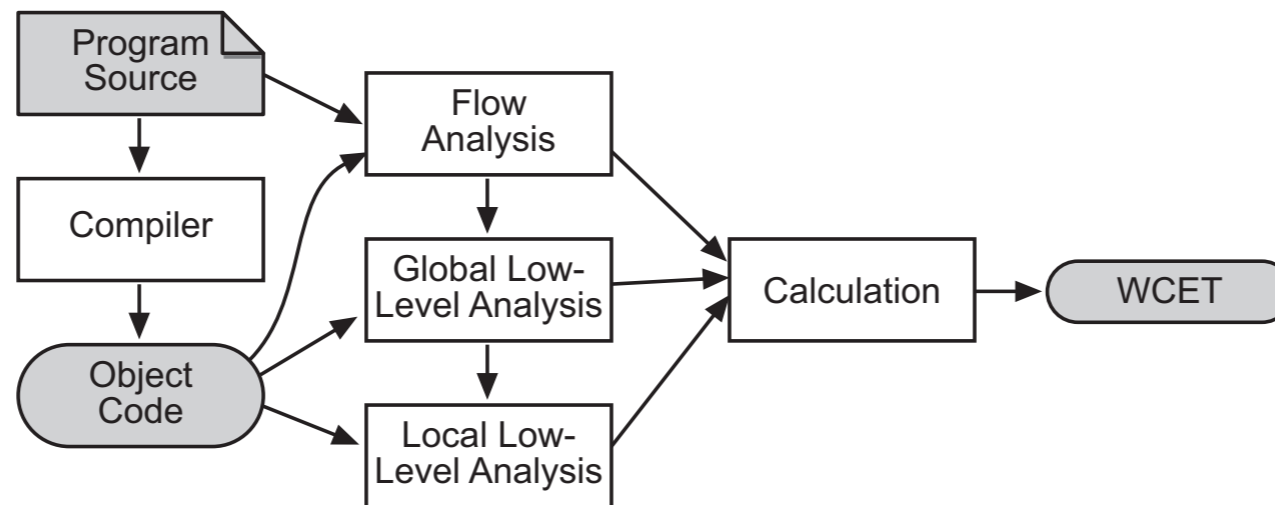


[StappertEE01]

- May split loops to differentiate between first and successive loop iterations
- Must combine with pipelining effects

# WCET Calculation

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[EngblomES01]

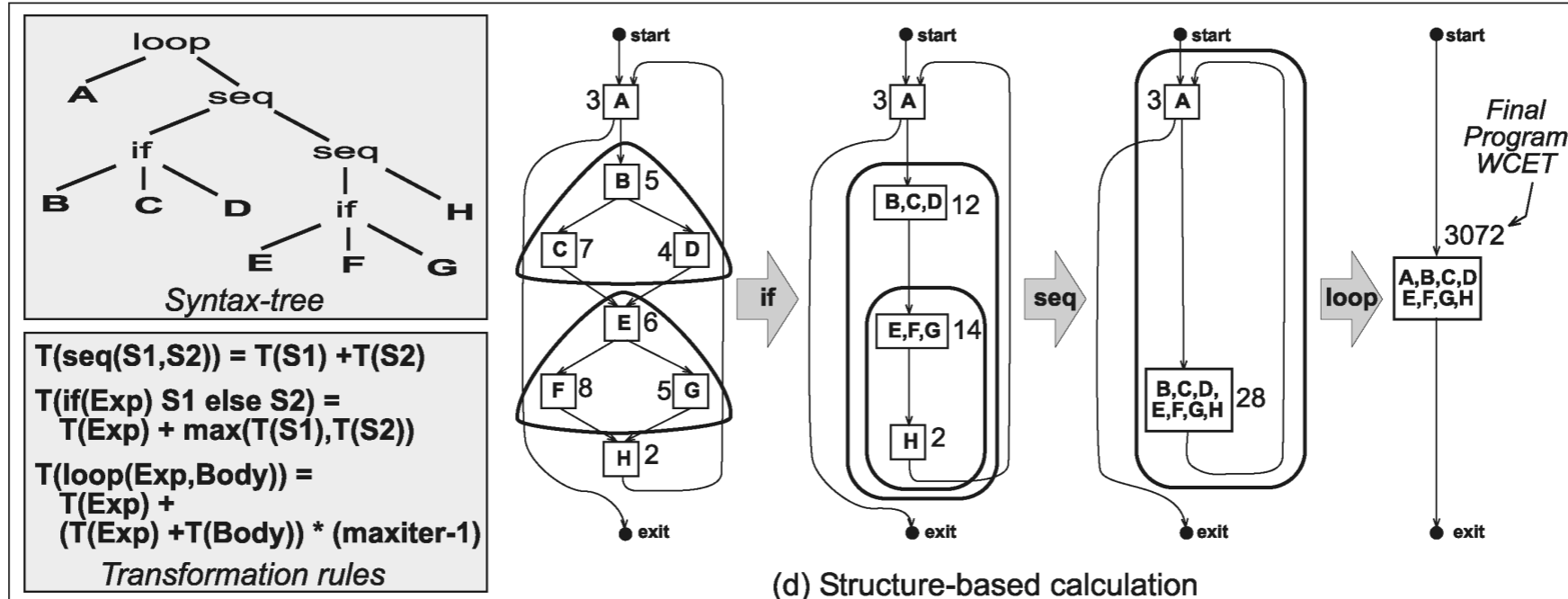
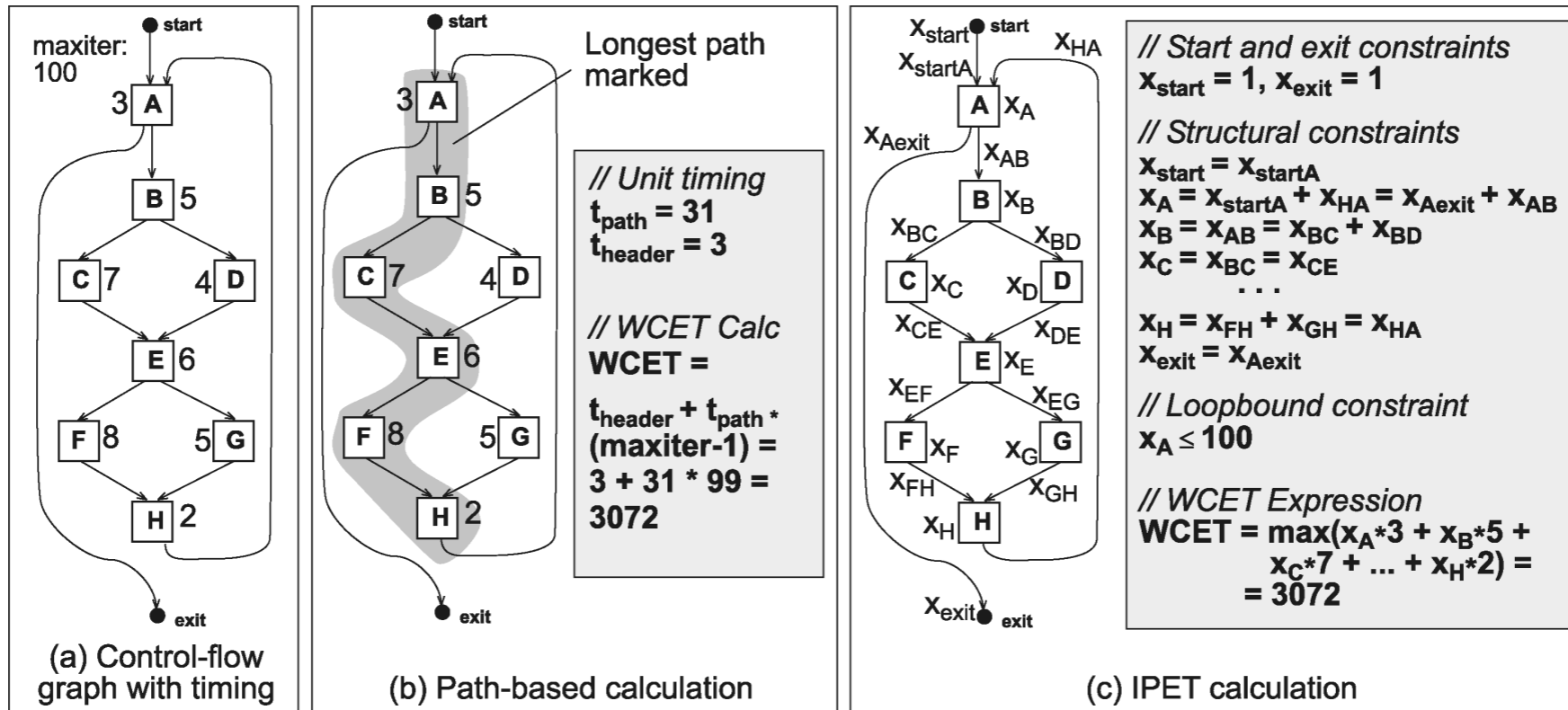
- Task: Find the path that results in the longest execution time
- Several approaches in use
- Properties of approaches
  - Program flow allowed
  - Object code structure (optimisations?)
  - Pipeline effect modelling
  - Solution complexity

# WCET Calculation

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- Path-based
- Constraint-based  
Implicit Path Enumeration Technique - IPET
- Structure-based

# WCET Calculation





# Path-Based Bound Calculation

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- Upper bound for a task is determined by computing bounds for different paths in the task, searching for the overall path with the longest execution time.
- Defining feature is that possible execution paths are represented explicitly.
- Natural within a single loop iteration, but problems with flow information extending across loop nesting levels.
- Number of paths is exponential in the number of branch points.
- Possibly requiring heuristic search methods.

# Implicit Path Enumeration

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- Program flow and basic block execution time bounds are combined into sets of arithmetic constraints.
- Each basic block and program flow edge in the task is given a time coefficient, expressing the upper bound of the contribution of that entity to the total execution time every time it is executed.

# Structure-based Bound Calculation

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- Upper bound is calculated in a bottom-up traversal of the syntax tree of the task combining bounds computed for constituents of statements according to combination rules for that type of statement.
- Not every control flow can be expressed through the syntax tree
- Assumes straight-forward correspondence between source structures and the target program
  - Not easily admitting code optimisations
- In general, not possible to add additional flow information (as in IPET).

# Summary

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- Motivation
- Worst-Case Execution Time Analysis
  - Types of Execution Times
  - Measuring vs. Analysing
  - Flow Analysis
  - Low-Level Analysis
  - WCET Calculation

# Preview

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- Real-Time Operating Systems
- MQX