Elements of Programming Languages
Lecture Notes: $\mathsf{L}_{\mathsf{While}}$

1 Abstract Syntax

$$
\begin{array}{rcll}
\mathit{Expr} \ni e & ::= & n \in \mathbb{N} \mid e_1 + e_2 \mid e_1 - e_2 \mid e_1 \times e_2 & \text{Numbers} \\
 & \mid & b \in \mathbb{B} \mid e_1 == e_2 & \text{Booleans} \\
\mathit{Value} \ni v & ::= & n \mid b & \\
\mathit{Stmt} \ni s & ::= & \texttt{skip} \mid s_1; s_2 \mid x := e & \\
 & \mid & \texttt{if}\ e\ \texttt{then}\ s_1\ \texttt{else}\ s_2 \mid \texttt{while}\ e\ \texttt{do}\ s & \\
\mathit{Env} \ni \sigma & ::= & [x_1 = v_1, \ldots, x_n = v_n] &
\end{array}
$$

2 Evaluation

$\boxed{\sigma, e \Downarrow v}$

$$
\frac{n \in \mathbb{N}}{\sigma, n \Downarrow n}
\qquad
\frac{\sigma, e_1 \Downarrow v_1 \quad \sigma, e_2 \Downarrow v_2}{\sigma, e_1 + e_2 \Downarrow v_1 +_{\mathbb{N}} v_2}
\qquad
\frac{\sigma, e_1 \Downarrow v_1 \quad \sigma, e_2 \Downarrow v_2}{\sigma, e_1 - e_2 \Downarrow v_1 -_{\mathbb{N}} v_2}
\qquad
\frac{\sigma, e_1 \Downarrow v_1 \quad \sigma, e_2 \Downarrow v_2}{\sigma, e_1 * e_2 \Downarrow v_1 \times_{\mathbb{N}} v_2}
$$

$$
\frac{b \in \mathbb{B}}{\sigma, b \Downarrow b}
\qquad
\frac{\sigma, e_1 \Downarrow v \quad \sigma, e_2 \Downarrow v}{\sigma, e_1 == e_2 \Downarrow \texttt{true}}
\qquad
\frac{\sigma, e_1 \Downarrow v_1 \quad \sigma, e_2 \Downarrow v_2 \quad v_1 \neq v_2}{\sigma, e_1 == e_2 \Downarrow \texttt{false}}
$$

$\boxed{\sigma, s \Downarrow \sigma'}$

$$
\frac{}{\sigma, \texttt{skip} \Downarrow \sigma}
\qquad
\frac{\sigma, s_1 \Downarrow \sigma' \quad \sigma', s_2 \Downarrow \sigma''}{\sigma, s_1; s_2 \Downarrow \sigma''}
$$

$$
\frac{\sigma, e \Downarrow \texttt{true} \quad \sigma, s_1 \Downarrow \sigma'}{\sigma, \texttt{if}\ e\ \texttt{then}\ s_1\ \texttt{else}\ s_2 \Downarrow \sigma'}
\qquad
\frac{\sigma, e \Downarrow \texttt{false} \quad \sigma, s_2 \Downarrow \sigma'}{\sigma, \texttt{if}\ e\ \texttt{then}\ s_1\ \texttt{else}\ s_2 \Downarrow \sigma'}
$$

$$
\frac{\sigma, e \Downarrow \texttt{true} \quad \sigma, s \Downarrow \sigma' \quad \sigma', \texttt{while}\ e\ \texttt{do}\ s \Downarrow \sigma''}{\sigma, \texttt{while}\ e\ \texttt{do}\ s \Downarrow \sigma''}
$$

$$
\frac{\sigma, e \Downarrow \texttt{false}}{\sigma, \texttt{while}\ e\ \texttt{do}\ s \Downarrow \sigma}
\qquad
\frac{\sigma, e \Downarrow v}{\sigma, x := e \Downarrow \sigma[x := v]}
$$