Elements of Programming Languages
Tutorial 6: Classes, subtyping, and comprehensions
Week 8 (November 7–11, 2016)

Starred exercises are more challenging. Please try all unstarred exercises before the tutorial meeting.

1. Imperative programming
   Write evaluation derivations for the following imperative programs, starting with the environment $\sigma = [x = 3, y = 4]$.
   (a) $y := x + x$
   (b) if $x == y$ then $x := x + 1$ else $y := y + 2$
   (c) (*) while $x < y$ do $x := x + 1$

2. Covariant and contravariant type parameters
   In Scala, a type parameter in a definition can be marked covariant by prefixing it with + and contravariant by prefixing it with −.
   Consider the following Scala code:

   ```scala
   abstract class Super
   class Sub1(n: Int) extends Super
   class Sub2(b: Boolean) extends Super
   class Box1[+A] // covariant
   class Box2[−A] // contravariant
   
   def g1(x: Box1[Super]) = x
   def g2(x: Box1[Sub1]) = x
   def h1(x: Box2[Super]) = x
   def h2(x: Box2[Sub1]) = x
   ```

   Suppose that $A$ is replaced with one of the types Any, Nothing, Super, Sub1, or Sub2. For which values of $A$ do the following calls typecheck:
   - $g1(new Box1[\text{A}])$
   - $g2(new Box1[\text{A}])$
   - $h1(new Box2[\text{A}])$
   - $h2(new Box2[\text{A}])$

   (It may help to draw a matrix with rows labeled by the function names and columns by the five possible types for $A$). You can type all of these expressions into Scala to find this out. What is the pattern?
3. Parameterized traits

Traits can also be parameterized by types. The built-in trait `Ordered[T]` is an example:

```scala
trait Ordered[T] {
  def compare(that: T): Int
  def < (that: T): Boolean = ???
  def <= (that: T): Boolean = ???
}
```

Here, the type parameter `T` is needed to name the type of other elements to which `this` will be compared. The `this.compare(that)` operation returns a negative integer if `this` is less than `that`, zero if they are equal and a positive integer if `this` is greater than `that`.

Based on this specification, fill in the `???” regions in the above code snippet with code that defines standard comparison operators such as `<` in terms of `compare`. Define the remaining operations `>`, `>=`, `!=`, `==`.

4. List comprehensions

Using the desugaring rules for list comprehensions described in Lecture 11, give the resulting list and convert the following list comprehension expressions to plain Scala code.

(a) `for (x <- List(1,2,3)) yield (x + 1)`
(b) `for (x <- List(1,2,3); if (x % 2 == 0)) yield (x / 2)`
(c) `(* for (x <- List(1,2,3); y <- List(1,2,3); if (x < y)) yield (x, y)`

5. (⋆) Covariant lists

Covariance and subtyping allows us to define lists more cleanly:

```scala
abstract class List[+A]
case object Nil extends List[Nothing]
case class Cons[+A](head: A, tail: List[A]) extends List[A]
```

Now `Nil` is a case object, so it doesn’t need a parameter list, and it extends `List[Nothing]`, so it doesn’t need a type parameter. (Case objects with only a type parameter and no value parameters are not allowed in Scala.) This is much closer to the way lists are actually defined in Scala.

(a) Define a list expression that has type `List[Any]` and would not type-check in the absence of subtyping.

(b) Define a `List[A]` member `append` that allows lists of different types as arguments, provided the two element types have a common supertype. (Hint: Scala type parameters can be given both lower and upper type bounds, e.g. `def foo[A, B >: A, C <: A]` says that `foo` has three type parameters, `A, B` which must be a supertype of `A, and C which must be a subtype of `A.`)