

Elements of Programming Languages

Lecture Notes: L_{Let}

1 Abstract Syntax

$$\begin{aligned}
 \text{Expr } \ni e & ::= n \in \mathbb{N} \mid e_1 + e_2 \mid e_1 \times e_2 && L_{\text{Arith}} \\
 & \mid b \in \mathbb{B} \mid e_1 == e_2 \mid \text{if } e \text{ then } e_1 \text{ else } e_2 && L_{\text{If}} \\
 & \mid x \mid \text{let } x = e_1 \text{ in } e_2 && L_{\text{Let}} \\
 \\
 \text{Type } \ni \tau & ::= \text{int} && L_{\text{Arith}} \\
 & \mid \text{bool} && L_{\text{If}} \\
 \\
 \Gamma & ::= x_1 : \tau_1, \dots, x_n : \tau_n \\
 \\
 \text{Value } \ni v & ::= n \in \mathbb{N} && L_{\text{Arith}} \\
 & \mid b \in \mathbb{B} && L_{\text{If}}
 \end{aligned}$$

1.1 Fresh variables

In the following, \oplus stands for any binary operator.

$x \# e$

$$\begin{aligned}
 & \frac{n \in \mathbb{N}}{x \# n} \quad \frac{x \# e_1 \quad x \# e_2}{x \# e_1 \oplus e_2} \\
 & \frac{b \in \mathbb{B}}{x \# b} \quad \frac{x \# e \quad x \# e_1 \quad x \# e_2}{x \# \text{if } e \text{ then } e_1 \text{ else } e_2} \\
 & \frac{x \neq y}{x \# y} \quad \frac{x \# e_2}{x \# \text{let } x = e_1 \text{ in } e_2} \quad \frac{x \neq y \quad x \# e_1 \quad x \# e_2}{x \# \text{let } y = e_1 \text{ in } e_2}
 \end{aligned}$$

1.2 Substitution

$$\begin{aligned}
 n[e/x] & = n \\
 (e_1 \oplus e_2)[e/x] & = e_1[e/x] \oplus e_2[e/x] \\
 b[e/x] & = b \\
 (\text{if } e_0 \text{ then } e_1 \text{ else } e_2)[e/x] & = \text{if } (e_0[e/x]) \text{ then } (e_1[e/x]) \text{ else } (e_2[e/x]) \\
 x[e/x] & = e \\
 y[e/x] & = y \quad (x \neq y) \\
 (\text{let } y = e_1 \text{ in } e_2)[e/x] & = \text{let } y = e_1[e/x] \text{ in } e_2[e/x] \\
 & \quad (\text{where } y \notin FV(e))
 \end{aligned}$$

2 Evaluation

$e \Downarrow v$ for L_{Arith}

$$\frac{}{v \Downarrow v} \quad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1 + e_2 \Downarrow v_1 +_{\mathbb{N}} v_2} \quad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1 \times e_2 \Downarrow v_1 \times_{\mathbb{N}} v_2}$$

$e \Downarrow v$ for L_{If}

$$\frac{e_1 \Downarrow v \quad e_2 \Downarrow v}{e_1 == e_2 \Downarrow \mathbf{true}} \quad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2 \quad v_1 \neq v_2}{e_1 == e_2 \Downarrow \mathbf{false}}$$
$$\frac{e \Downarrow \mathbf{true} \quad e_1 \Downarrow v_1}{\mathbf{if } e \mathbf{ then } e_1 \mathbf{ else } e_2 \Downarrow v_1} \quad \frac{e \Downarrow \mathbf{false} \quad e_2 \Downarrow v_2}{\mathbf{if } e \mathbf{ then } e_1 \mathbf{ else } e_2 \Downarrow v_2}$$

$e \Downarrow v$ for L_{Let}

$$\frac{e_1 \Downarrow v_1 \quad e_2[v_1/x] \Downarrow v_2}{\mathbf{let } x = e_1 \mathbf{ in } e_2 \Downarrow v_2}$$

3 Types

$\boxed{\Gamma \vdash e : \tau}$ for L_{Arith}

$$\frac{}{\Gamma \vdash n : \text{int}} \quad \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}} \quad \frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 \times e_2 : \text{int}}$$

$\boxed{\Gamma \vdash e : \tau}$ for L_{If}

$$\frac{}{\Gamma \vdash b : \text{bool}} \quad \frac{\Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1 == e_2 : \text{bool}} \quad \frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash e_1 : \tau \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash \text{if } e \text{ then } e_1 \text{ else } e_2 : \tau}$$

$\boxed{\Gamma \vdash e : \tau}$ for L_{Let}

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau} \quad \frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_2}$$