

# Elements of Programming Languages

## Lecture Notes: $L_{lf}$

### 1 Abstract Syntax

$$\begin{array}{l}
 \text{Expr} \ni e ::= n \in \mathbb{N} \mid e_1 + e_2 \mid e_1 \times e_2 \qquad L_{\text{Arith}} \\
 \qquad \qquad \mid b \in \mathbb{B} \mid e_1 == e_2 \mid \text{if } e \text{ then } e_1 \text{ else } e_2 \quad L_{\text{If}} \\
 \\
 \text{Type} \ni \tau ::= \text{int} \qquad L_{\text{Arith}} \\
 \qquad \qquad \mid \text{bool} \qquad L_{\text{If}} \\
 \\
 \text{Value} \ni v ::= n \in \mathbb{N} \qquad L_{\text{Arith}} \\
 \qquad \qquad \mid b \in \mathbb{B} \qquad L_{\text{If}}
 \end{array}$$

### 2 Evaluation

$e \Downarrow v$  for  $L_{\text{Arith}}$

$$\frac{}{v \Downarrow v} \qquad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1 + e_2 \Downarrow v_1 +_{\mathbb{N}} v_2} \qquad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1 \times e_2 \Downarrow v_1 \times_{\mathbb{N}} v_2}$$

$e \Downarrow v$  for  $L_{\text{If}}$

$$\frac{e_1 \Downarrow v \quad e_2 \Downarrow v}{e_1 == e_2 \Downarrow \text{true}} \qquad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2 \quad v_1 \neq v_2}{e_1 == e_2 \Downarrow \text{false}} \\
 \frac{e \Downarrow \text{true} \quad e_1 \Downarrow v_1}{\text{if } e \text{ then } e_1 \text{ else } e_2 \Downarrow v_1} \qquad \frac{e \Downarrow \text{false} \quad e_2 \Downarrow v_2}{\text{if } e \text{ then } e_1 \text{ else } e_2 \Downarrow v_2}$$

### 3 Types

$\vdash e : \tau$  for  $L_{\text{Arith}}$

$$\frac{}{\vdash n : \text{int}} \qquad \frac{\vdash e_1 : \text{int} \quad \vdash e_2 : \text{int}}{\vdash e_1 + e_2 : \text{int}} \qquad \frac{\vdash e_1 : \text{int} \quad \vdash e_2 : \text{int}}{\vdash e_1 \times e_2 : \text{int}}$$

$\vdash e : \tau$  for  $L_{\text{If}}$

$$\frac{}{\vdash b : \mathbf{bool}} \quad \frac{\vdash e_1 : \tau \quad \vdash e_2 : \tau}{\vdash e_1 == e_2 : \mathbf{bool}} \quad \frac{\vdash e : \mathbf{bool} \quad \vdash e_1 : \tau \quad \vdash e_2 : \tau}{\vdash \text{if } e \text{ then } e_1 \text{ else } e_2 : \tau}$$