# EPL Exam Review Session

Simon Fowler
University of Edinburgh

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# Today's Session

- → Two hours (but longer if you like)
- → Plan: Few words to start us off, then questions from you
- → I have slides working through two types of questions:
  - → "Is this substitution correct?"
  - → "Is this system sound?"
- → ...but I've prepared all three exams, so we can go through any of them on the board

# Exam Information

- **Your exam**:
  - **Time**: Tuesday, 16th May 2017
  - **Location**: Patersons Land - 1.26 (Holyrood)
  - (Be sure to check closer to the time – these sometimes change!)
- **Exam format**:
  - Two hours
  - Question 1 is compulsory, then you have a choice between questions 2 and 3.
- **Revision Exercises**:
  - Three papers:
    - Mock exam (on EPL course page)
    - 2015/16 exam
    - 2015/16 resit exam
  - Tutorial questions

# 15/16 Resit Exam, Question 1(b)

Consider the following substitutions:

- $(\lambda x.x\ y)[x/y] = \lambda z.z\ x$
- $(\lambda x.\lambda y.(x, y, z))[(y, z)/x] = \lambda x.\lambda y.((y, z), y, z)$
- $(\lambda x.x + ((\lambda y.y)\ z))[y/z] = \lambda x.x + ((\lambda y.y)\ y)$
- $(\lambda x.x + ((\lambda y.y)\ z))[x/z] = \lambda x.x + ((\lambda y.y)\ x)$

For each one, explain whether the substitution has been performed correctly or not. If not, give the correct answer for the right-hand side.

[8 marks]

# 15/16 Resit Exam, Question 1(b)

$$(\lambda x.x\ y)[x/y] = \lambda z.z\ x$$

# 15/16 Resit Exam, Question 1(b)

$$(\lambda x.x\ y)[x/y] = \lambda z.z\ x$$

This is correct.

→ Substituting $x$ for $y$ naïvely would result in $\lambda x.x\ x$. Here, $x$ would be captured by the $\lambda x$ binder, changing the meaning of the program.

→ Instead, it is always safe to perform substitution by choosing fresh variables for the binders, and then performing the substitution:

  → $(\lambda z.z\ y)[x/y] = (\lambda z.z\ x)$

# 15/16 Resit Exam, Question 1(b)

$$(\lambda x.\lambda y.(x, y, z))[(y, z)/x] = \lambda x.\lambda y.((y, z), y, z)$$

# 15/16 Resit Exam, Question 1(b)

$$(\lambda x.\lambda y.(x, y, z))[(y, z)/x] = \lambda x.\lambda y.((y, z), y, z)$$

- → This is incorrect.
- → Whereas the $y$ in $(y, z)$ was free before the substitution, $y$ has been captured by the $\lambda y$ afterwards.
- → To correct the substitution, freshen the binders beforehand:

$$(\lambda a.\lambda b.(a, b, z))[(y, z)/x] = \lambda a.\lambda b.(a, b, z)$$

# 15/16 Resit Exam, Question 1(b)

$$(\lambda x.x + ((\lambda y.y)\ z))[y/z] = \lambda x.x + ((\lambda y.y)\ y)$$

# 15/16 Resit Exam, Question 1(b)

$$(\lambda x.x + ((\lambda y.y)\ z))[y/z] = \lambda x.x + ((\lambda y.y)\ y)$$

- $\rightarrow$ This is correct.
- $\rightarrow$ $z$ is not in the scope f the $\lambda y$ binder, so $y$ is not captured when it is substituted.

# 15/16 Resit Exam, Question 1(b)

$$(\lambda x.x + ((\lambda y.y)\, z))[x/z] = \lambda x.x + ((\lambda y.y)\, x)$$

→ This is incorrect.

→ $z$ is in the scope of $\lambda x$ before the substitution, so $x$ is captured by the binder.

→ As ever, this can be solved by freshening the binder before substituting:

$$(\lambda a.a + ((\lambda y.y)\, z)[x/z] = \lambda a.a + ((\lambda y.y)\, x)$$

# 15/16 Resit Paper: 2(d)

“Type soundness is often proved using two properties, called preservation and progress”. Define the preservation property.

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"Type soundness is often proved using two properties, called preservation and progress". Define the preservation property.

- **Preservation**: Typing is preserved under reduction.
  - More formally, if $\Gamma \vdash e : \tau$ and $e \mapsto e'$, then $\Gamma \vdash e' : \tau$.
- **Progress**: A well-typed term is either a value, or can take a reduction step (evaluation doesn't get "stuck")
  - More formally, if $\Gamma \vdash e : \tau$, then either $e$ is a value $v$, or there exists some $e'$ such that $e \mapsto e'$.
- **Soundness**: A system is sound if it satisfies preservation and progress.

These seem to come up a lot – they're worth knowing!

# 15/16 Resit Paper: 2($e$)

Consider the following rules which we might add to handle random number generation to a language that already has basic arithmetic:

$$\boxed{e \mapsto e'}$$

$$\frac{e \mapsto e'}{\texttt{randInt}(e) \mapsto \texttt{randInt}(e')} \qquad \frac{0 \leq n < v}{\texttt{randInt}(v) \mapsto n} \qquad \frac{v \leq 0}{\texttt{randInt}(v) \mapsto 0}$$

$$\boxed{\Gamma \vdash e : \tau}$$

$$\frac{\Gamma \vdash e : \texttt{int}}{\Gamma \vdash \texttt{randInt}(e) : \texttt{int}}$$

Is this system sound? Briefly explain why or why not.

# 15/16 Resit Paper: 2(*e*)

$$\boxed{e \mapsto e'}$$

$$\frac{e \mapsto e'}{\texttt{randInt}(e) \mapsto \texttt{randInt}(e')} \qquad \frac{0 \leq n < v}{\texttt{randInt}(v) \mapsto n} \qquad \frac{v \leq 0}{\texttt{randInt}(v) \mapsto 0}$$

$$\boxed{\Gamma \vdash e : \tau}$$

$$\frac{\Gamma \vdash e : \texttt{int}}{\Gamma \vdash \texttt{randInt}(e) : \texttt{int}}$$

Does the system satisfy preservation? If something reduces, does it have the same type?

→ Yes: the type is `int` before and after reduction.

Does the system satisfy progress? Can we always reduce?

→ Yes: if `randInt` is evaluating a value, then all values accounted for by the last two rules. If evaluating a subexpression, we can assume it takes a step, and thus conclude with the first rule.

# 15/16 Resit Paper: 2(*e*)

$$\boxed{e \mapsto e'}$$

$$\frac{e \mapsto e'}{\texttt{randInt}(e) \mapsto \texttt{randInt}(e')} \qquad \frac{0 \leq n < v}{\texttt{randInt}(v) \mapsto n} \qquad \frac{v \leq 0}{\texttt{randInt}(v) \mapsto 0}$$

$$\boxed{\Gamma \vdash e : \tau}$$

$$\frac{\Gamma \vdash e : \texttt{int}}{\Gamma \vdash \texttt{randInt}(e) : \texttt{int}}$$

How would we prove this formally?

- → Preservation: by induction on $e \mapsto e'$.
- → Progress: by induction on $\Gamma \vdash e : \tau$.

# 15/16 Paper: Question 2(c)

$$\boxed{e \mapsto e'}$$

$$\frac{e_1 \mapsto e_1'}{e_1 \div e_2 \mapsto e_1' \div e_2} \qquad \frac{e_2 \mapsto e_2'}{v_1 \div e_2 \mapsto v_1 \div e_2'} \qquad \frac{v_2 \neq 0}{v_1 \div v_2 \mapsto \mathit{fdiv}(v_1, v_2)}$$

$$\boxed{\Gamma \vdash e : \tau}$$

$$\frac{c \text{ is a floating-point constant}}{\Gamma \vdash c : \texttt{float}} \qquad \frac{\Gamma \vdash e_1 : \texttt{float} \quad \Gamma \vdash e_2 : \texttt{float}}{\Gamma \vdash e_1 \div e_2 : \texttt{float}}$$

Is this system sound?

# 15/16 Paper: Question 2(c)

$$\boxed{e \mapsto e'}$$

$$\frac{e_1 \mapsto e_1'}{e_1 \div e_2 \mapsto e_1' \div e_2} \qquad \frac{e_2 \mapsto e_2'}{v_1 \div e_2 \mapsto v_1 \div e_2'} \qquad \frac{v_2 \neq 0}{v_1 \div v_2 \mapsto \mathit{fdiv}(v_1, v_2)}$$

$$\boxed{\Gamma \vdash e : \tau}$$

$$\frac{c \text{ is a floating-point constant}}{\Gamma \vdash c : \texttt{float}} \qquad \frac{\Gamma \vdash e_1 : \texttt{float} \quad \Gamma \vdash e_2 : \texttt{float}}{\Gamma \vdash e_1 \div e_2 : \texttt{float}}$$

Is this system sound?

- No.
- Preservation holds: if we take a reduction step, we still end up with a float.
- Progress does not hold: we cannot reduce $v_1 \div 0$ since no rules match, yet $v_1 \div 0$ is not a value.