Empirical Methods in Natural Language Processing Lecture 15 Machine translation (II): Word-based models and the EM algorithm

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Lexical translation

- How to translate a word → look up in dictionary

  Haus — *house, building, home, household, shell.*

- *Multiple translations*
  - some more frequent than others
  - for instance: *house*, and *building* most common
  - special cases: *Haus* of a *snail* is its *shell*

- Note: During all the lectures, we will translate from a foreign language into English

Collect statistics

- Look at a *parallel corpus* (German text along with English translation)

Translation of *Haus*	**Count**
house	8,000
building	1,600
home	200
household	150
shell	50

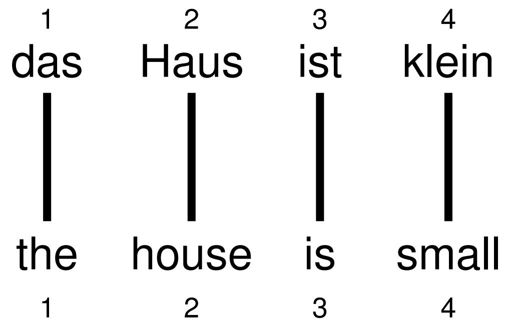
Estimate translation probabilities

- *Maximum likelihood estimation*

$$p_f(e) = \begin{cases} 0.8 & \text{if } e = \textit{house}, \\ 0.16 & \text{if } e = \textit{building}, \\ 0.02 & \text{if } e = \textit{home}, \\ 0.015 & \text{if } e = \textit{household}, \\ 0.005 & \text{if } e = \textit{shell}. \end{cases}$$

Alignment

- In a parallel text (or when we translate), we **align** words in one language with the words in the other



- Word *positions* are numbered 1–4

Alignment function

- Formalizing *alignment* with an **alignment function**

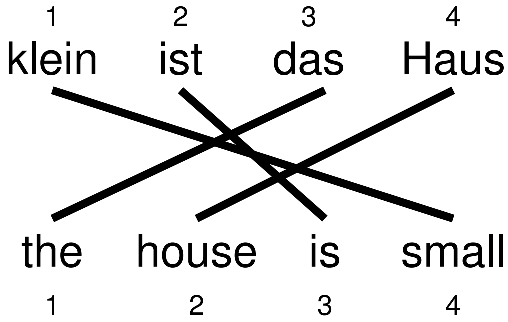
- Mapping an English target word at position i to a German source word at position j with a function $a : i \to j$

- Example

$$a : \{1 \to 1, 2 \to 2, 3 \to 3, 4 \to 4\}$$

Reordering

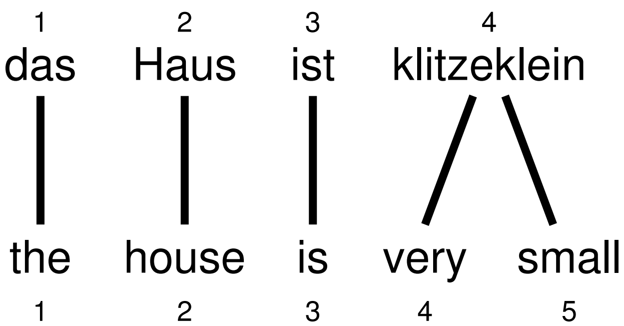
- Words may be **reordered** during translation



$$a : \{1 \rightarrow 3, 2 \rightarrow 4, 3 \rightarrow 2, 4 \rightarrow 1\}$$

One-to-many translation

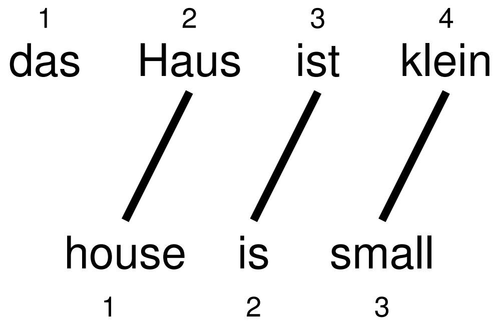
- A source word may translate into **multiple** target words



$$a : \{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 4\}$$

Dropping words

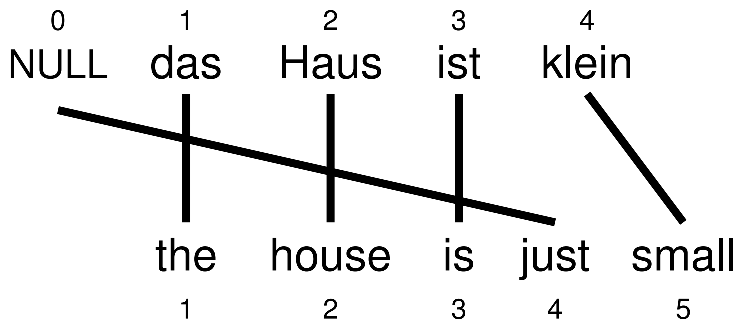
- Words may be **dropped** when translated
  - The German article *das* is dropped



$$a : \{1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 4\}$$

Inserting words

- Words may be **added** during translation
  - The English *just* does not have an equivalent in German
  - We still need to map it to something: special NULL token



$$a : \{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 0, 5 \rightarrow 4\}$$

IBM Model 1

- *Generative model*: break up translation process into smaller steps
  - **IBM Model 1** only uses *lexical translation*
- Translation probability
  - for a foreign sentence $\mathbf{f} = (f_1, ..., f_{l_f})$ of length l_f
  - to an English sentence $\mathbf{e} = (e_1, ..., e_{l_e})$ of length l_e
  - with an alignment of each English word e_j to a foreign word f_i according to the alignment function $a : j \rightarrow i$

$$p(\mathbf{e}, a|\mathbf{f}) = \frac{\epsilon}{(l_f + 1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})$$

  - parameter ϵ is a *normalization constant*

Example

das

e	$t(e\|f)$
the	0.7
that	0.15
which	0.075
who	0.05
this	0.025

Haus

e	$t(e\|f)$
house	0.8
building	0.16
home	0.02
household	0.015
shell	0.005

ist

e	$t(e\|f)$
is	0.8
's	0.16
exists	0.02
has	0.015
are	0.005

klein

e	$t(e\|f)$
small	0.4
little	0.4
short	0.1
minor	0.06
petty	0.04

$$\begin{aligned} p(e, a|f) &= \frac{\epsilon}{4^3} \times t(\text{the}|\text{das}) \times t(\text{house}|\text{Haus}) \times t(\text{is}|\text{ist}) \times t(\text{small}|\text{klein}) \\ &= \frac{\epsilon}{4^3} \times 0.7 \times 0.8 \times 0.8 \times 0.4 \\ &= 0.0028\epsilon \end{aligned}$$

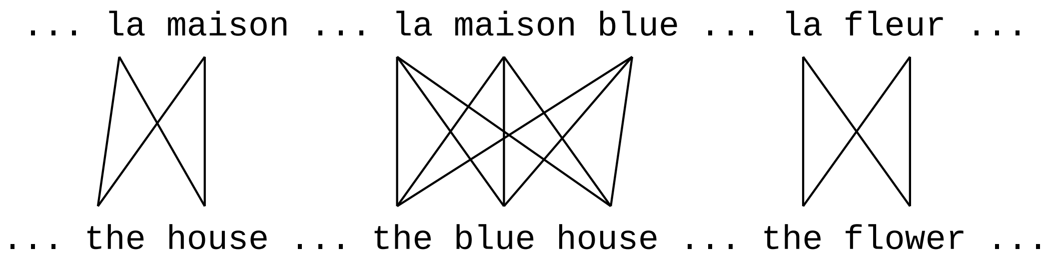
Learning lexical translation models

- We would like to *estimate* the lexical translation probabilities $t(e|f)$ from a parallel corpus
- ... but we do not have the alignments
- **Chicken and egg problem**
  - if we had the *alignments*,
    $\rightarrow$ we could estimate the *parameters* of our generative model
  - if we had the *parameters*,
    $\rightarrow$ we could estimate the *alignments*

EM algorithm

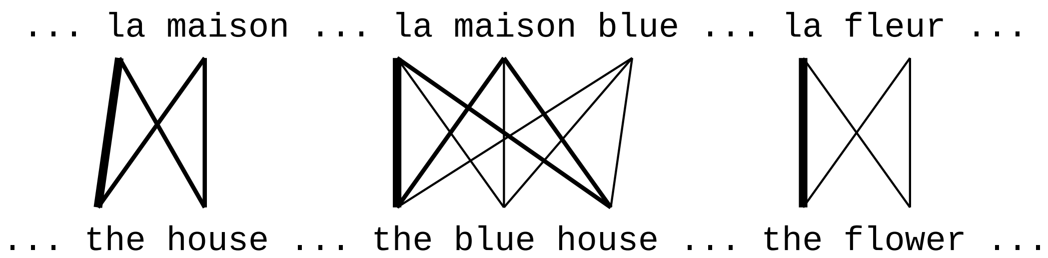
- **Incomplete data**
  - if we had *complete data*, would could estimate *model*
  - if we had *model*, we could fill in the *gaps in the data*
- **Expectation Maximization (EM)** in a nutshell
  - initialize model parameters (e.g. uniform)
  - assign probabilities to the missing data
  - estimate model parameters from completed data
  - iterate

EM algorithm



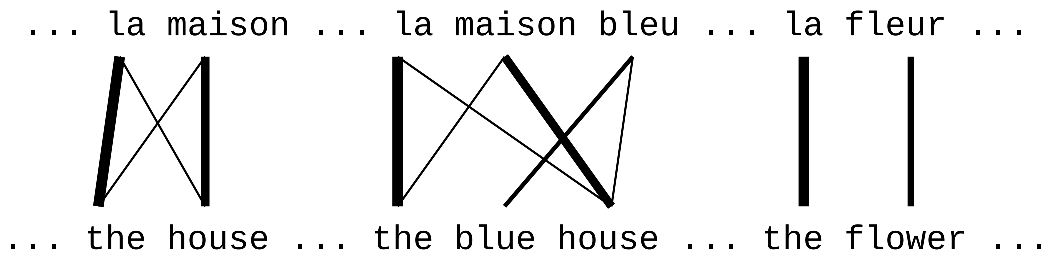
- Initial step: all alignments equally likely
- Model learns that, e.g., *la* is often aligned with *the*

EM algorithm



- After one iteration
- Alignments, e.g., between *la* and *the* are more likely

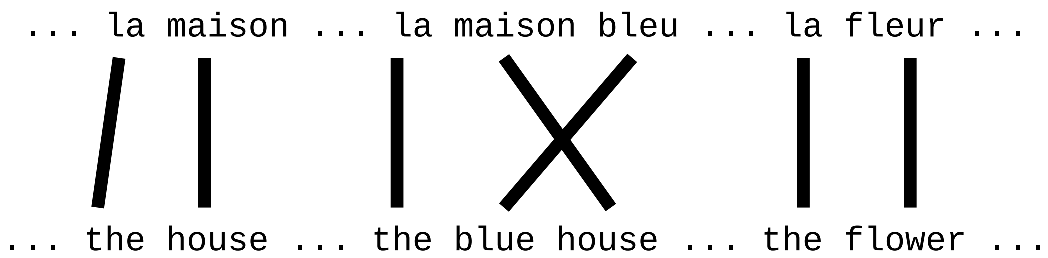
EM algorithm



- After another iteration

- It becomes apparent that alignments, e.g., between *fleur* and *flower* are more likely (**pigeon hole principle**)

EM algorithm

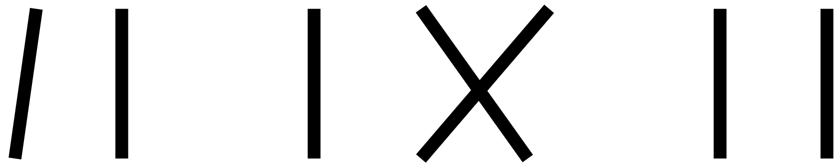


- Convergence

- Inherent hidden structure revealed by EM

EM algorithm

... la maison ... la maison bleu ... la fleur ...



... the house ... the blue house ... the flower ...

p(la|the) = 0.453
p(le|the) = 0.334
p(maison|house) = 0.876
p(bleu|blue) = 0.563
...

- Parameter estimation from the aligned corpus

IBM Model 1 and EM

- EM Algorithm consists of two steps
- **Expectation-Step**: Apply model to the data
  - parts of the model are hidden (here: alignments)
  - using the model, assign probabilities to possible values
- **Maximization-Step**: Estimate model from data
  - take assign values as fact
  - collect counts (weighted by probabilities)
  - estimate model from counts
- Iterate these steps until **convergence**

IBM Model 1 and EM

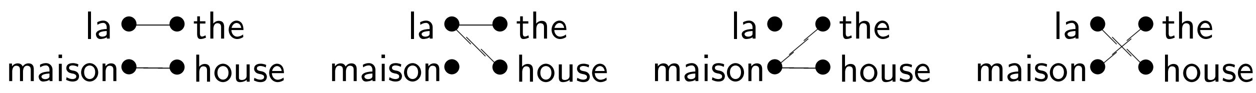
- We need to be able to compute:
  - Expectation-Step: probability of alignments
  - Maximization-Step: count collection

IBM Model 1 and EM

- **Probabilities**

$$p(\text{the}|\text{la}) = 0.7 \qquad p(\text{house}|\text{la}) = 0.05$$
$$p(\text{the}|\text{maison}) = 0.1 \qquad p(\text{house}|\text{maison}) = 0.8$$

- **Alignments**



$$p(\mathbf{e}, a|\mathbf{f}) = 0.56 \quad p(\mathbf{e}, a|\mathbf{f}) = 0.035 \quad p(\mathbf{e}, a|\mathbf{f}) = 0.08 \quad p(\mathbf{e}, a|\mathbf{f}) = 0.005$$

$$p(a|\mathbf{e}, \mathbf{f}) = 0.824 \quad p(a|\mathbf{e}, \mathbf{f}) = 0.052 \quad p(a|\mathbf{e}, \mathbf{f}) = 0.118 \quad p(a|\mathbf{e}, \mathbf{f}) = 0.007$$

- **Counts**

$$c(\text{the}|\text{la}) = 0.824 + 0.052 \qquad c(\text{house}|\text{la}) = 0.052 + 0.007$$
$$c(\text{the}|\text{maison}) = 0.118 + 0.007 \qquad c(\text{house}|\text{maison}) = 0.824 + 0.118$$

IBM Model 1 and EM: Expectation Step

- We need to compute $p(a|\mathbf{e},\mathbf{f})$
- Applying the *chain rule*:

$$p(a|\mathbf{e},\mathbf{f}) = \frac{p(\mathbf{e},a|\mathbf{f})}{p(\mathbf{e}|\mathbf{f})}$$

- We already have the formula for $p(\mathbf{e},\mathbf{a}|\mathbf{f})$ (definition of Model 1)

IBM Model 1 and EM: Expectation Step

- We need to compute $p(\mathbf{e}|\mathbf{f})$

$$p(\mathbf{e}|\mathbf{f}) = \sum_a p(\mathbf{e},a|\mathbf{f})$$
$$= \sum_{a(1)=0}^{l_f} ... \sum_{a(l_e)=0}^{l_f} p(\mathbf{e},a|\mathbf{f})$$
$$= \sum_{a(1)=0}^{l_f} ... \sum_{a(l_e)=0}^{l_f} \frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})$$

IBM Model 1 and EM: Expectation Step

$$p(\mathbf{e}|\mathbf{f}) = \sum_{a(1)=0}^{l_f} ... \sum_{a(l_e)=0}^{l_f} \frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})$$

$$= \frac{\epsilon}{(l_f+1)^{l_e}} \sum_{a(1)=0}^{l_f} ... \sum_{a(l_e)=0}^{l_f} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})$$

$$= \frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} \sum_{i=0}^{l_f} t(e_j|f_i)$$

- Note the trick in the last line
  - removes the need for an *exponential* number of products

→ this makes IBM Model 1 estimation **tractable**

The trick

(case $l_e = l_f = 2$)

$$\sum_{a(1)=0}^{2} \sum_{a(2)=0}^{2} = \frac{\epsilon}{3^2} \prod_{j=1}^{2} t(e_j|f_{a(j)}) =$$

$$= t(e_1|f_0)\ t(e_2|f_0) + t(e_1|f_0)\ t(e_2|f_1) + t(e_1|f_0)\ t(e_2|f_2) +$$
$$+ t(e_1|f_1)\ t(e_2|f_0) + t(e_1|f_1)\ t(e_2|f_1) + t(e_1|f_1)\ t(e_2|f_2) +$$
$$+ t(e_1|f_2)\ t(e_2|f_0) + t(e_1|f_2)\ t(e_2|f_1) + t(e_1|f_2)\ t(e_2|f_2) =$$
$$= t(e_1|f_0)\ (t(e_2|f_0) + t(e_2|f_1) + t(e_2|f_2)) +$$
$$+ t(e_1|f_1)\ (t(e_2|f_1) + t(e_2|f_1) + t(e_2|f_2)) +$$
$$+ t(e_1|f_2)\ (t(e_2|f_2) + t(e_2|f_1) + t(e_2|f_2)) =$$
$$= (t(e_1|f_0) + t(e_1|f_1) + t(e_1|f_2))\ (t(e_2|f_2) + t(e_2|f_1) + t(e_2|f_2))$$

IBM Model 1 and EM: Expectation Step

- Combine what we have:

$$\begin{aligned} p(\mathbf{a}|\mathbf{e},\mathbf{f}) &= p(\mathbf{e},\mathbf{a}|\mathbf{f})/p(\mathbf{e}|\mathbf{f}) \\ &= \frac{\frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} t(e_j|f_{a(j)})}{\frac{\epsilon}{(l_f+1)^{l_e}} \prod_{j=1}^{l_e} \sum_{i=0}^{l_f} t(e_j|f_i)} \\ &= \prod_{j=1}^{l_e} \frac{t(e_j|f_{a(j)})}{\sum_{i=0}^{l_f} t(e_j|f_i)} \end{aligned}$$

IBM Model 1 and EM: Maximization Step

- Now we have to *collect counts*
- Evidence from a sentence pair **e,f** that word e is a translation of word f:

$$c(e|f;\mathbf{e},\mathbf{f}) = \sum_a p(a|\mathbf{e},\mathbf{f}) \sum_{j=1}^{l_e} \delta(e,e_j)\delta(f,f_{a(j)})$$

- With the same simplication as before:

$$c(e|f;\mathbf{e},\mathbf{f}) = \frac{t(e|f)}{\sum_{i=0}^{l_f} t(e|f_i)} \sum_{j=1}^{l_e} \delta(e,e_j) \sum_{i=0}^{l_f} \delta(f,f_i)$$

IBM Model 1 and EM: Maximization Step

- After collecting these counts over a corpus, we can estimate the model:

$$t(e|f;\mathbf{e},\mathbf{f}) = \frac{\sum_{(\mathbf{e},\mathbf{f})} c(e|f;\mathbf{e},\mathbf{f}))}{\sum_f \sum_{(\mathbf{e},\mathbf{f})} c(e|f;\mathbf{e},\mathbf{f}))}$$

IBM Model 1 and EM: Pseudocode

```
initialize t(e|f) uniformly
do until convergence
  set count(e|f) to 0 for all e,f
  set total(f) to 0 for all f
  for all sentence pairs (e_s,f_s)
    for all words e in e_s
      total_s(e) = 0
      for all words f in f_s
        total_s(e) += t(e|f)
    for all words e in e_s
      for all words f in f_s
        count(e|f) += t(e|f) / total_s(e)
        total(f)   += t(e|f) / total_s(e)
  for all f
    for all e
      t(e|f) = count(e|f) / total(f)
```

Higher IBM Models

IBM Model 1	lexical translation
IBM Model 2	adds absolute **reordering model**
IBM Model 3	adds **fertility model**
IBM Model 4	relative reordering model
IBM Model 5	fixes **deficiency**

- Only IBM Model 1 has *global maximum*
  - training of a higher IBM model builds on previous model
- Compuationally biggest change in Model 3
  - trick to simplify estimation does not work anymore
  - → *exhaustive* count collection becomes computationally too expensive
  - **sampling** over high probability alignments is used instead

IBM Model 4