Stochastic Games with Lossy Channels

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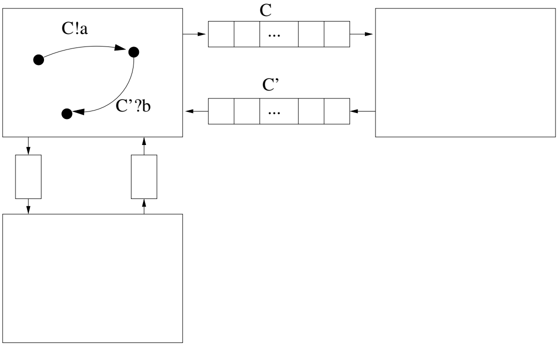
Outline

FIFO-Channel Systems

Finite automata which communicate with each other by

- asynchronous message passing
- communication channels with unbounded buffers
- FIFO: first-in first-out



Verification Questions

- Simple questions
  - Reachability
  - Termination (i.e., time-boundedness)
  - Space-boundedness
- Büchi-questions/LTL model checking
  - Does some infinite run visit F infinitely often?
    Classic: $s \models \exists\Box\Diamond F$.
    Probabilistic: $Prob(s \models \Box\Diamond F) > 0$.
  - Do almost all infinite runs visit F infinitely often?
    Classic: $s \models \forall\Box\Diamond F$.
    Probabilistic: $Prob(s \models \Box\Diamond F) = 1$.

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Perfect Communication Channels

- Channels can encode Turing-tape
- Simulate Turing machines [Brand & Zafiropoulo, 1983]
- Undecidable verification problems

Lossy Communication Channels

- Any message in transit can spontaneously be lost.
- Induces substring-ordering on channel-configurations $\longrightarrow$ Higman's Lemma; well-quasi-ordering.
- $Pre^*(s)$ upward-closed and effectively constructible.
- Reachability and termination problems decidable [Abdulla & Jonsson; Finkel et.al.], but not primitive recursive [Schnoebelen].
- However, space-boundedness and Büchi-problems still undecidable [Abdulla & Jonsson], even if one abstracts from message ordering (lossy counter machines) [Mayr].

Probabilistic Message Losses

In every step, every message in transit is lost with probability $\lambda > 0$, independently of other messages.

Control-transitions also made probabilistic, $p \xrightarrow{0.3} x, p \xrightarrow{0.7} y$.

$\Longrightarrow$ Markov chain.

- Beyond a certain limit (depending on λ) the total number of all messages is more likely to decrease than to increase.
- The infinite-state Markov chain has a finite attractor.
- Decisive: $Prob(s \models \Diamond(F \cup \tilde{F})) = 1$, where $\tilde{F} = \overline{Pre^*(F)}$.
- Büchi-problems decidable and $Prob(s \models \Box\Diamond F)$ can be effectively approximated up-to every error $\epsilon > 0$. [Rabinovich; Abdulla & Rabinovich; Abdulla & Ben Henda & Mayr].

Scheduler

Control-transitions (fully or partly) under the control of a scheduler, but message losses still probabilistic.
$\Longrightarrow$ Markov Decision Process (MDP)/1-player game.

- 1-player game with almost-sure Büchi-objective $Prob(s \models \Box\Diamond F) = 1$ is pure memoryless determined and decidable [Baier, Bertrand, Schnoebelen].
- 1-player game with positive Büchi-objective $Prob(s \models \Box\Diamond F) > 0$ requires infinite memory to win, and is undecidable [Baier, Bertrand, Schnoebelen].

2-Player Games with Almost-sure Büchi-Objective

Two player stochastic game

- Player 0 (Scheduler) tries to achieve $\mathit{Prob}(s \models \Box\Diamond F) = 1$
- Player 1 (Opponent) tries to frustrate this.
- Control-states either under control of some player, or probabilistic.
- Message losses still probabilistic.

Theorem 2-player stochastic games with almost sure Büchi objective on probabilistic lossy FIFO-channel systems are pure memoryless determined and decidable.

Intuition

Player 0 (the scheduler) must, against any opponent,

1. Drive the game towards F, i.e., so that F is reached with at least some probability $p > 0$.
   Probabilistic transitions work in favor of player 0 w.r.t. this goal.
   This is enough, since, in the long run the chance of avoiding F is then $(1 - p)^\infty = 0$, provided that uniformly $p > 0$.
2. Totally avoid configurations where goal 1 can no longer be achieved.
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Construction of Winning Sets

Define sequence $\{X_i\}_{i\in\mathbb{N}}$: $X_0 \subseteq X_1 \subseteq \cdots$ of sets of states which are winning for player 1 with a positive probability.
Auxiliary sequence $\{M_i\}_{i\in\mathbb{N}}$: $M_0 \supseteq M_1 \supseteq \cdots$.
Let $X_0 := \emptyset$, $M_0 := S$ and

$$M_{i+1} := \mathrm{Force}^0(\overline{X_i}, F) \qquad X_{i+1} := \mathrm{Force}^1(S, \overline{M_{i+1}})$$

For player $\sigma \in \{0, 1\}$, the set $\mathrm{Force}^\sigma(I, F)$ is where player σ can, with a positive probability, force the run to eventually reach F, while always remaining within I. These sets can be constructed as solutions of reachability games.

Construction of Winning Sets (Cont.)

Lemma $X_0 \subseteq \overline{M_1} \subseteq X_1 \subseteq \overline{M_2} \subseteq X_2 \subseteq \cdots$ and the sequence $\{X_i\}_{i \in \mathbb{N}}$ converges.

Note. These sets are not upward-closed themselves, but are derived from other upward-closed sets (due to the effect of message losses). These upward-closed sets converge by Higman's Lemma and cause convergence of $\{X_i\}_{i \in \mathbb{N}}$.

Finally, the winning states for player 0 are given by

$$W^0 := \bigcap_{i \geq 0} M_i = \overline{X_{limit}}$$

Crucial Requirements for Correctness

1. The infinite system has a finite attractor, i.e., any infinite game almost certainly visits a finite subset infinitely often.
2. The system is finitely branching, which limits the power of the opponent. For any state s, the solution of the reachability game from s to F is either zero or uniformly $\geq c_s > 0$ for a constant c_s.

The chance p of visiting F before re-visiting the finite attractor is uniformly bounded by the minimum of the finitely many constants c_s for the finitely many states s in the finite attractor. Thus uniformly $p > 0$.

Summary and Extensions

Theorem 2-player stochastic games with almost sure Büchi objective on probabilistic lossy FIFO-channel systems are pure memoryless determined and decidable.
This is the only computable question. Even for 1-player games it follows from the construction in [BBS]

- Optimal achievable value for $Prob(s \models \Box\Diamond F)$ doesn't exist.
- Games with limit sure Büchi objective ($Prob(s \models \Box\Diamond F) \geq 1 - \epsilon$, for any $\epsilon > 0$) on GPLCS require infinite memory and are undecidable.
- For every constant $k \in [0, 1)$ the question if the scheduler can achieve $Prob(s \models \Box\Diamond F) > k$ is undecidable.
- The optimal value for $Prob(s \models \Box\Diamond F)$ cannot be approximated.
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Open Questions

In the pure PLCS Markov chain case

- Is $\mathit{Prob}(s \models \Box\Diamond F) \geq 0.5$ decidable?
- Is $\mathit{Prob}(s \models \Box\Diamond F)$ always an algebraic number?

For Further Reading I

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