Question 4 World Health Council

No medical teams	1	2	3
0	0	0	0
1	45	20	50
2	70	45	70
3	90	75	80
4	105	110	100
5	120	150	130

three interrelated decisions => three stages

x_n $(n=1,2,3)$ # teams to allocate to country n (stage n)

s - state - # of team still available

$p_i(x_i)$ - effectiveness for allocating x_i to i

choose x_1, x_2, x_3, s.t.

$$\max \sum_{i=1}^{3} p_i(x_i)$$

$$\text{s.} \quad \sum_{i=1}^{3} x_i = 5, \quad \forall i, x_i \geq 0$$

$$f_n(s, x_n) = p_n(x_n) + \max \sum_{i=n+1}^{3} p_i(x_i)$$

$$\sum_{i=n}^{3} x_i = s$$

$$f_n^*(s) = \max_{x_n = 0..s} f_n(s, x_n)$$

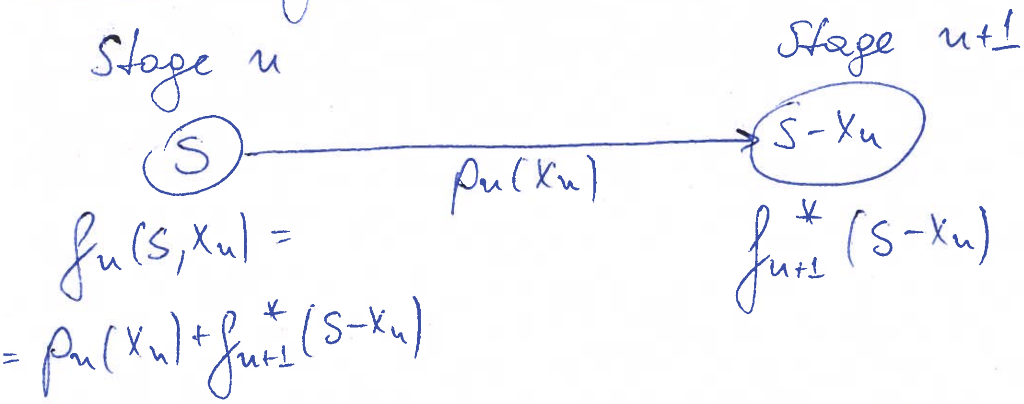
$$\Rightarrow f_n(s, x_n) = p_n(x_n) + f_{n+1}^*(s - x_n)$$

$$[f_4^* = 0]$$

$$f_n^*(s) = \max_{x_n = 0 \dots s} \{ p_n(x_n) + f_{n+1}^*(s - x_n) \}, \quad n = 1, 2$$

for $n = 3$, $f_3^*(s) = \max\limits_{x_3 = 0, \dots, s} p_3(x_3)$.

Structure of the problem:



Using Dynamic programming we work through the problem backwards by using the recursive relationship we formulated above:

Stage $n = 3$:

s	$f_3^*(s)$	x_3^*
0	0	0
1	50	1
2	70	2
3	80	3
4	100	4
5	130	5

Stage $n=2$:

$s \backslash x_n$	$f_2(s,x_n)=p_2(x_2)+f_3^*(s-x_2)$						$f_2^*(s)$	x_2^*
	0	1	2	3	4	5		
0	0						0	0
1	50	20					50	0
2	70	70	45				70	0,1
3	80	90	95	75			95	2
4	100	100	115	125	110		125	3
5	130	120	125	145	160	150	160	4

Stage $n=1$:

$s \backslash x_n$	$f_1(s,x_1)=p_1(x_1)+f_2^*(s-x_1)$						$f_1^*(s)$	x_1^*
	0	1	2	3	4	5		
5	160	170	165	160	155	120	170	1

$\Rightarrow$ Best strategy is:

$x_1^*=1$; $s=5-1=4$; $x_2^*=3$; $s=4-3=1$; $x_3^*=1$

Problem 2 : Scientists on Mars

* Goal - optimise $\min \prod_{i=1}^{3} p_i(x_i)$,

s.t. $\sum_{i=1}^{3} x_i = 2$, x_i are all nonnegative integers

$$f_n(S, x_n) = p_n(x_n) \times \text{minimum} \prod_{i=n+1}^{3} p_i(x_i),$$

s.t. $\sum_{i=n}^{3} x_i = S$, x_i are nonnegative integers,

for $n = 1, 2, 3$.

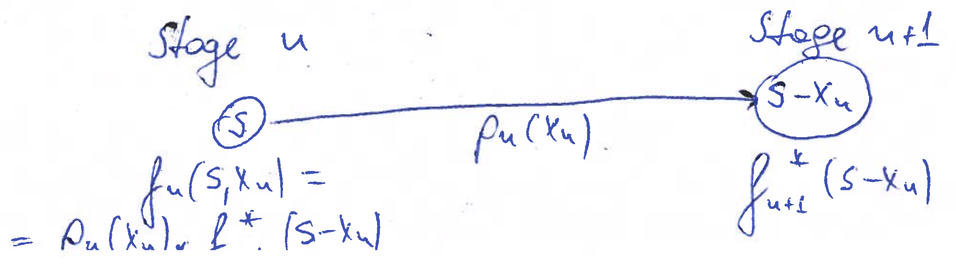
Thus $f_n^*(S) = \min_{x_n \le S} f_n(S, x_n)$.

$\Rightarrow$ We know $f_n(S, x_n) = p_n(x_n) f_{n+1}^*(S - x_n)$

$$\Rightarrow f_n^*(S) = \min_{x_n \le S} \{ p_n(x_n) \times f_{n+1}^*(S - x_n) \}, \text{ for } n = 1, 2$$

for $n = 3$, $f_3^*(S) = \min_{x_3 \le S} p_3(x_3)$

Structure of the problem:



Using Dynamic programming we work through the problem backwords:

Stage $n = 3$:

s	$f_3^*(s)$	x_3^*
0	0.8	0
1	0.5	1
2	0.3	2

Stage $n = 2$:

$s \backslash x_n$	$f_2(s,x_2) = p_2(x_2) \cdot f_3^*(s-x_2)$: 0	1	2	$f_2^*(s)$	x_2^*
0	0.48			0.48	0
1	0.30	0.32		0.30	0
2	0.18	0.20	0.16	0.16	2

Stage $n = 1$:

$s \backslash x_4$	$f_1(s,x_1) = p_1(x_1) \cdot f_2^*(s-x_1)$: 0	1	2	$f_1^*(s)$	x_1^*
2	0.064	0.06	0.072	0.06	1

$\Rightarrow$ The best strategy is $x_1^* = 1$, $x_2^* = 0$, $x_3^* = 1$

Problem 3: Carnival Planning

Weather Report unavailable

$$\underset{uwr}{U}(a_1) = \sum_{\omega} U(a_1,\omega)P(\omega) = 0.1\times(-15000) + 0.3\times(-5000) + 0.6\times 5000$$
$$= \underline{3000}$$

$$\underset{uwr}{U}(a_2) = \sum_{\omega} U(a_2,\omega)P(\omega) = -1000$$

$\Rightarrow$ If we don't take the report, the best we could do is a \$3000 profit (expected)

Weather Report available

* Subtract 1000 from all utility values to reflect the cost of the weather report.
* We have $P(f|\omega)$ and $P(\omega)$. Use Bayes' rule to calculate $P(\omega|f)$:

$$P(\omega|f) = \frac{P(f|\omega)P(\omega)}{P(f)} = \frac{P(f|\omega)P(\omega)}{\sum_{\omega} P(f|\omega)P(\omega)}$$

* Using that formula we get:

$P(\omega\|f)$	$\omega=R$	$\omega=C$	$\omega=S$	$P(f)$
$f=R$	0.368	0.316	0.316	0.19
$f=C$	0.062	0.563	0.375	0.32
$f=S$	0.020	0.123	0.875	0.49

$* P(f) = \sum_{\omega} P(f|\omega)P(\omega)$

* Using these distributions we can calculate the expected utility of taking any of the two actions, given a weather forecast.
* Using the marginal distribution we can calculate the expected utility of taking the weather forecast at all, assuming we act optimally after that.

If we get a forecast for rain, for example, these are the expected utilities of the two possible actions:

$$U(a_1, f=r) = \sum_{\omega} P(\omega|f)\, U(\omega, a_1) =$$

$$= 0.368 \times (-16000) + 0.316 \times (-6000) + 0.316 \times (8000) = -4840$$

$$U(a_2, f=r) = \sum_{\omega} P(\omega|f)\, U(\omega, a_2) = \underline{-2000}$$

$\Rightarrow$ for $f=r$, $a_r^* = a_2$

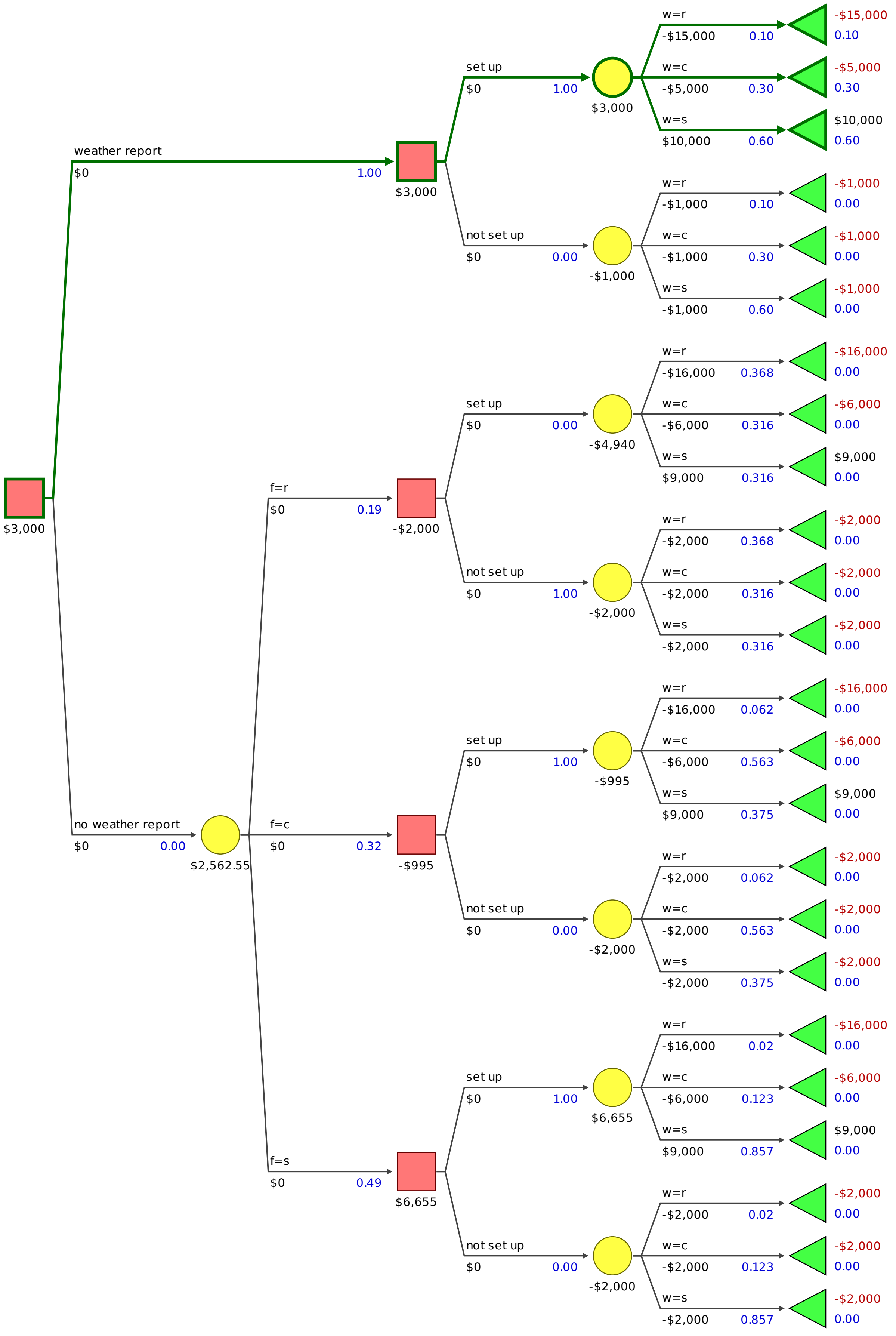
f	a_f^*	$U(a^*, f)$	$P(f)$
r	a_2	-2000	0.19
c	a_1	-885	0.32
s	a_1	6655	0.49

$$\Rightarrow U_{wr} = \sum_f P(f)\, U(a_f^*, f) = 0.19 \times (-2000) + 0.32 \times (-885) + 0.49 \times 6655 = 2563$$

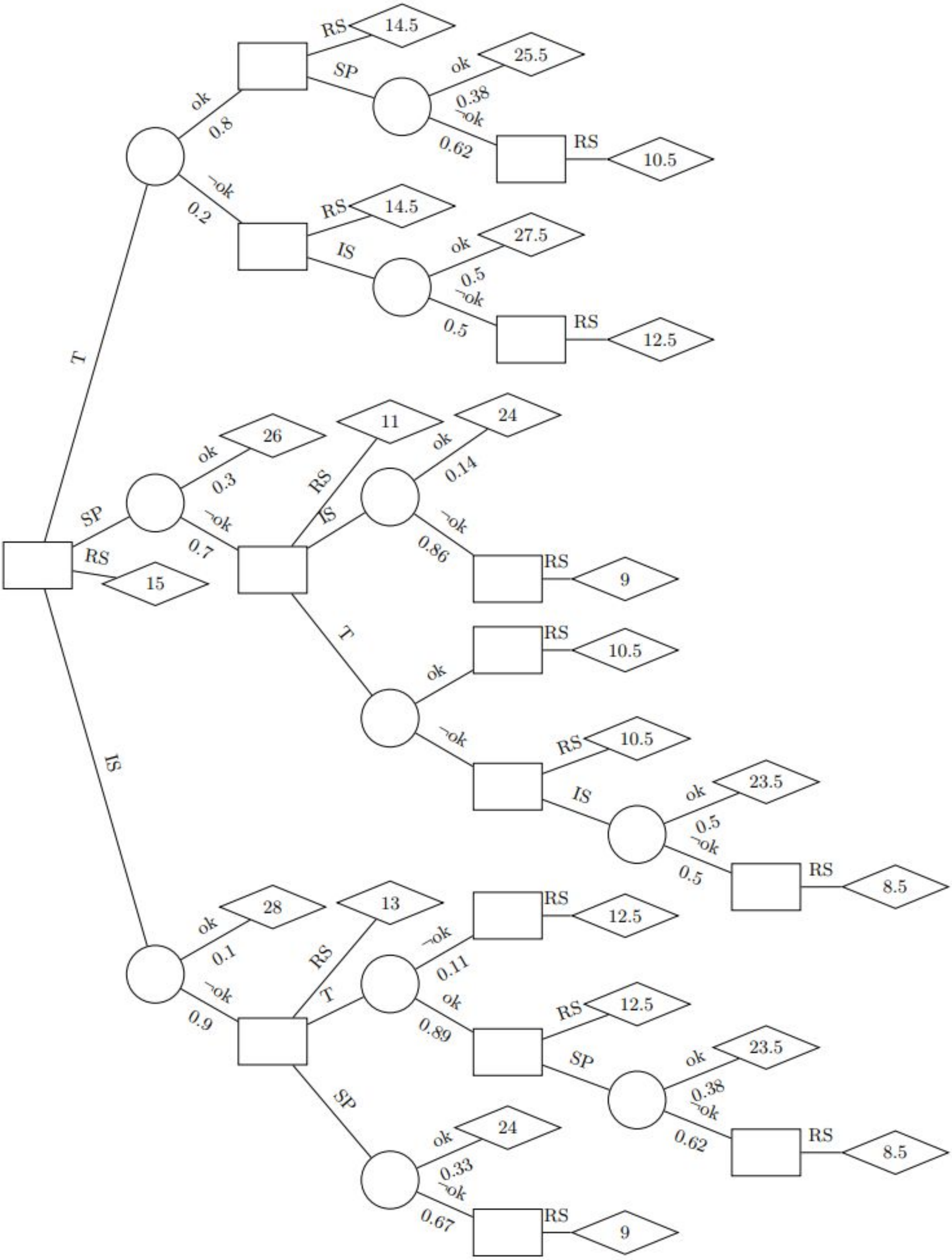
Remember $U_{nwr} = \underline{\underline{3000}}$

$\Rightarrow$ In expectation it is better not to take the weather report and set up directly.

Problem 3 : Carnival Example Decision Tree



<u>Problem 4:</u>
Full decision tree



Problem 4:

Result after solving the decision tree with average-out and fold-back algorithm