Bayesian Search for Missing Aircraft

20 April 2017

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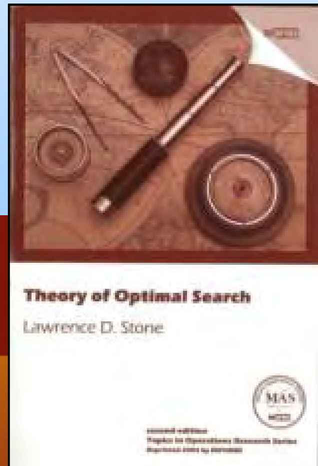
Bayesian search theory provides a principled and successful method for planning searches for lost aircraft and other objects

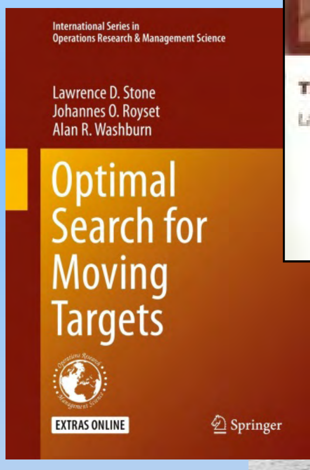
Outline

- Background
  - Successful applications of Bayesian search
- What do we mean by Bayesian search?
- Example: Air France AF447 Search Planning
- Question: How were Bayesian search planning methods applied to the Malaysia Airlines MH370 search?

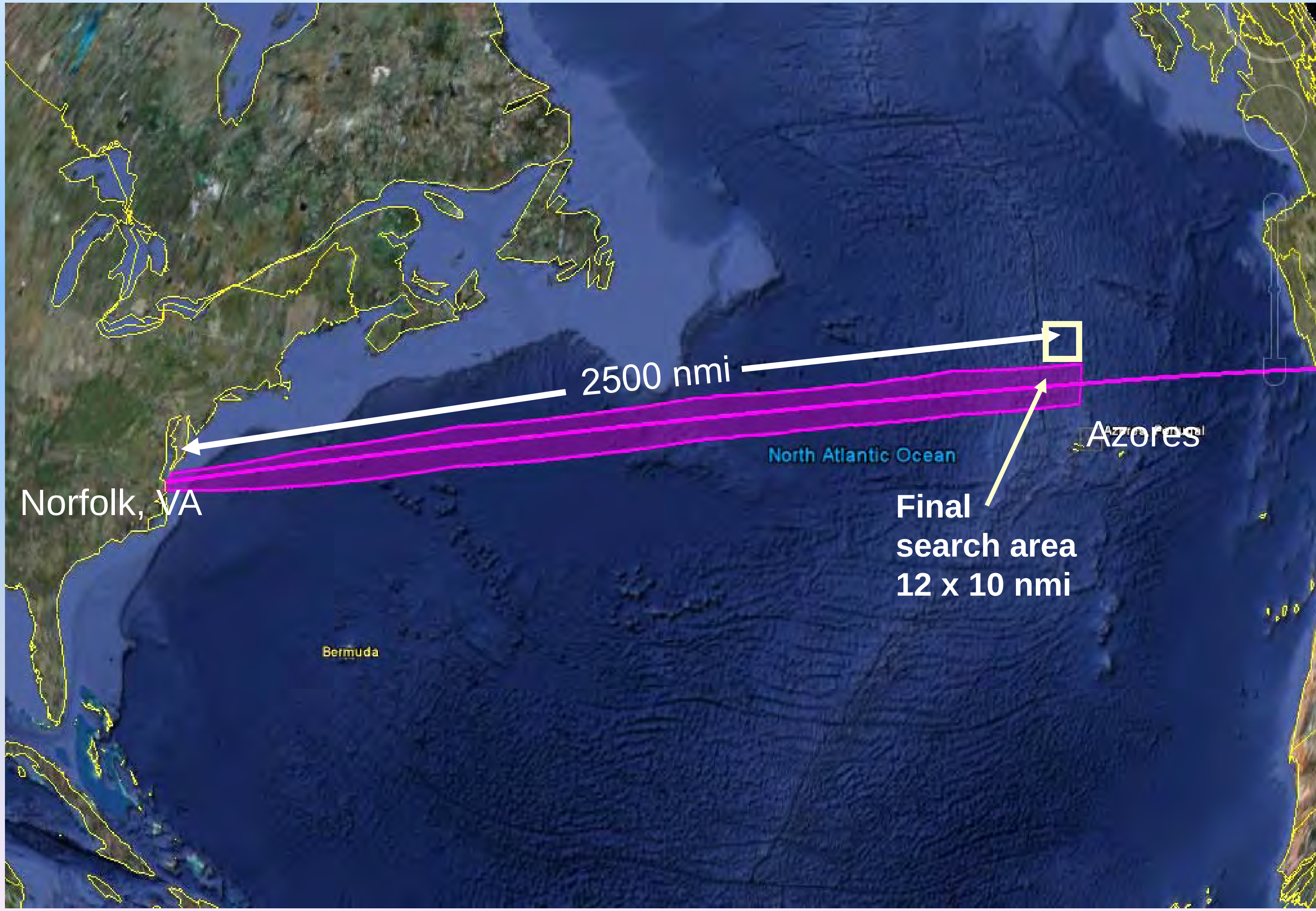
Background

- **Books:**
  - *Theory of Optimal Search,* 2004
    INFORMS Lanchester Prize
  - *Optimal Search for Moving Targets,* 2016
- **Searches**:
  - US Submarine *Scorpion,* 1968
  - *SS Central America,* 1988
  - Fosset 2008
  - AF 447, 2011
- **Search Planning Aids:**
  - SAROPS: US Coast Guard 2007
    Search And Rescue Optimal Planning System –
    "Man Overboard" in *NY Times Magazine*, 1/5/2014
  - OSPRE: Land searches for missing people

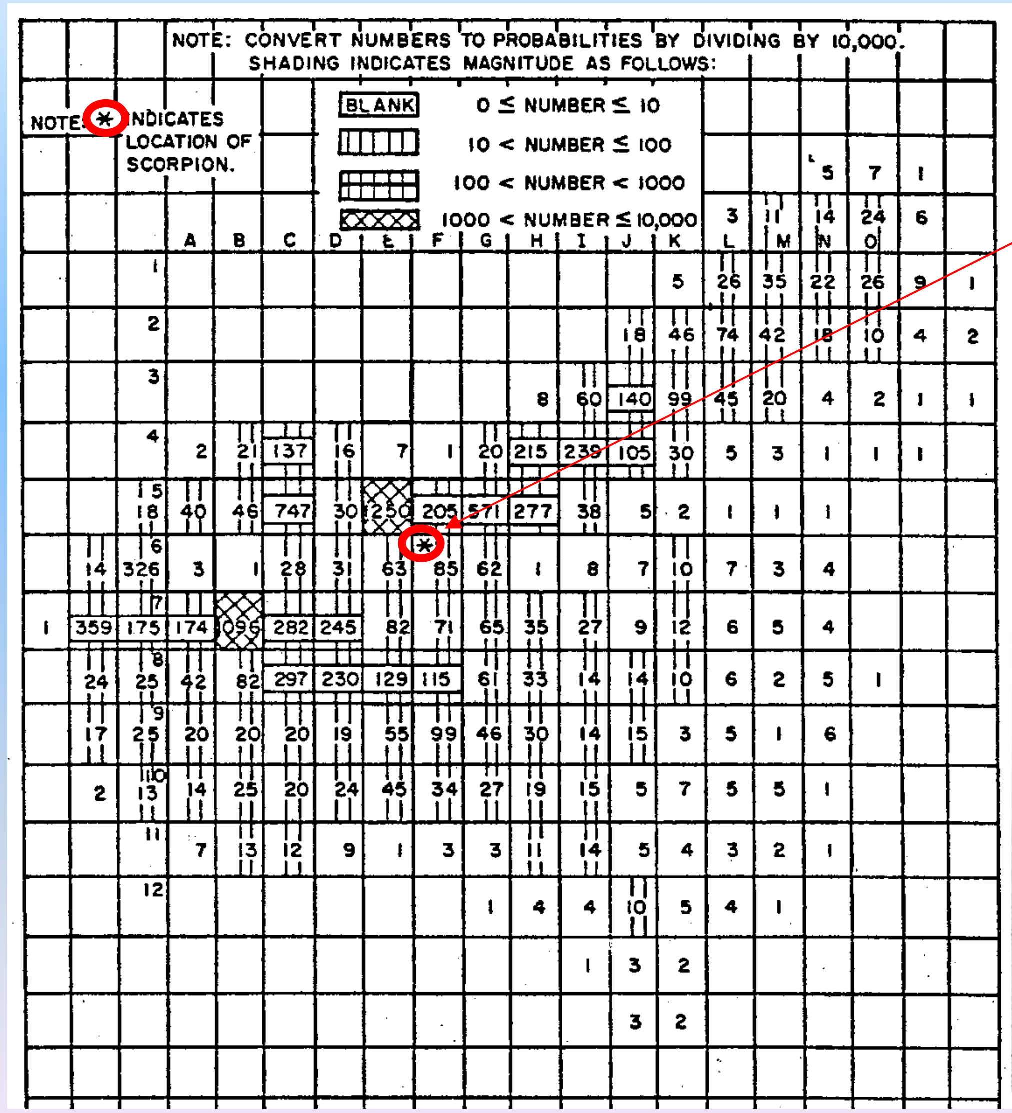




USS Scorpion
Initial Uncertainty Area



Probability Map for Wreck Location



Scorpion located within 260 yards of highest probability cell

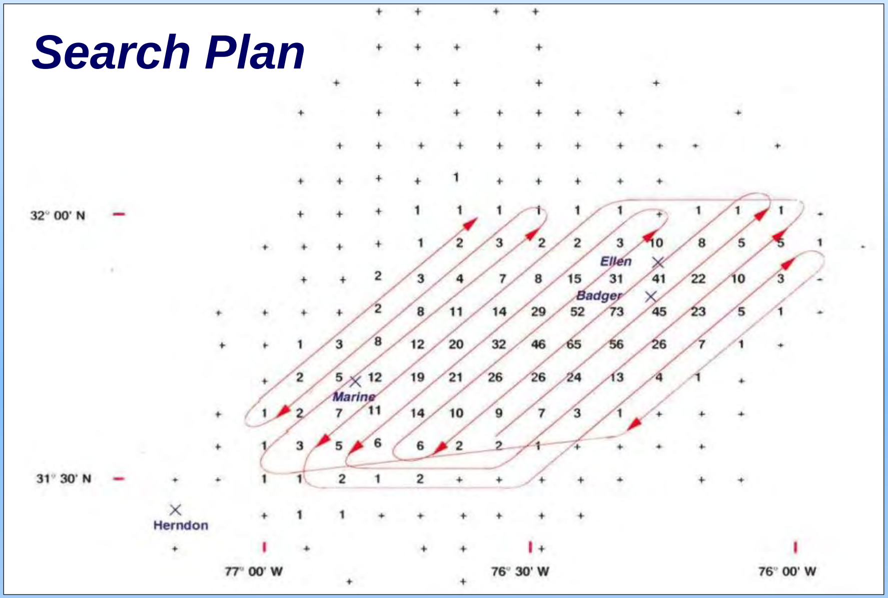
Operations analysis during the underwater search for *Scorpion,* by H R Richardson and L D Stone, *Naval Research Logistics*, 1971, 18:141-157

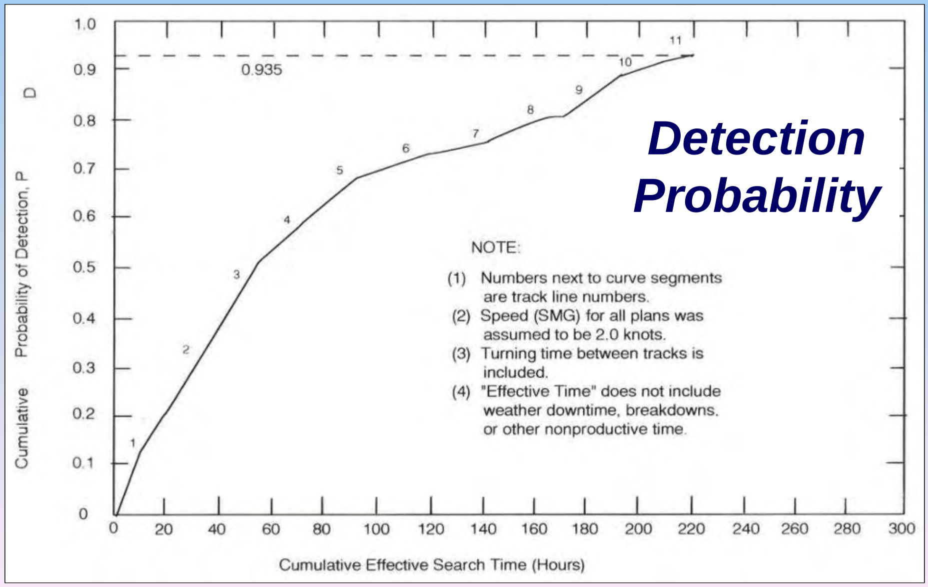
Search for the *S.S. Central America*

- Background
  - The S.S. Central America sank in 1857 off the South Carolina coast while sailing to New York with a cargo of gold and passengers.
  - In 1985, Dr. Stone was hired by a private company to develop a probability map for the location of the wreck.
  - The wreck was found and 1 ton of gold bars and coins were recovered.
- Steps
  - Develop scenarios to predict wreck location
  - Quantify uncertainties using probabilities
  - Simulate to obtain probability map for each scenario
  - Combine to produce composite search map

Search for the *SS Central America*: mathematical treasure hunting by LD Stone, *Interfaces*, 1992, 22:32-54

Discovery of the *S.S. Central America*





Bayesian Search Planning Process

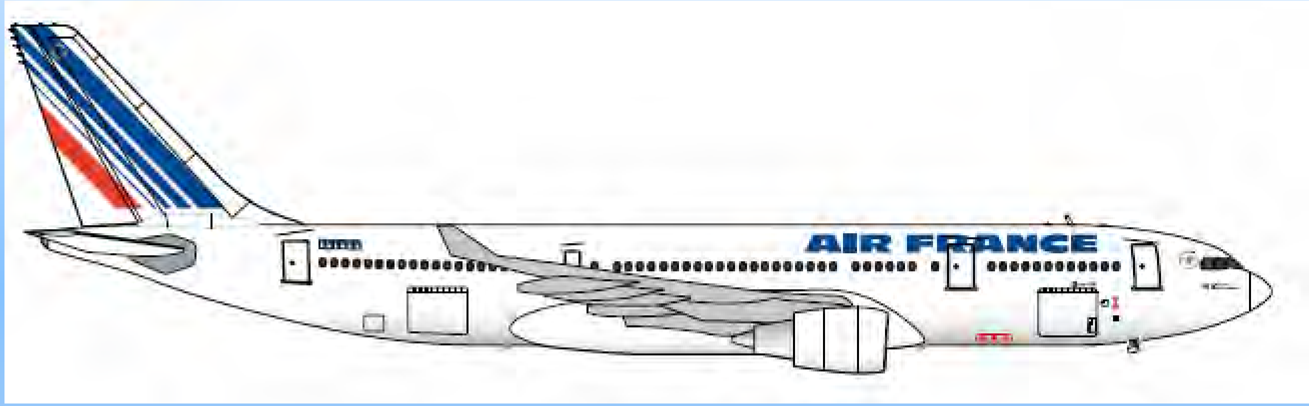
- Construct Prior Distribution
  - Organize information into consistent subsets = scenarios
  - Quantify uncertainties in terms of probabilities
    - Estimate probabilities that scenarios are correct
    - Often requires use of subjective probabilities
  - Compute a probability distribution for each scenario
  - Combine into weighted prior : weights = scenario probabilities
    - Compute posterior given unsuccessful search (if appropriate)
  - Resulting distribution is (usually) partially subjective
    - Represents the decision maker's best understanding of problem
    - Basis for search planning decisions
    - Complicated searches such as AF 447 are typically one-of-a-kind events – can't recreate crash 1000 times to get empirical probability distribution

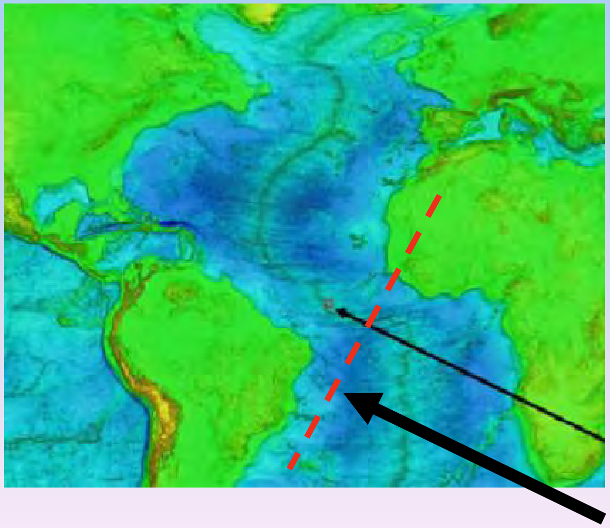
Bayesian Search Planning Process

- Allocate search effort to maximize detection probability
- If search fails, compute posterior given failure
- Use posterior to plan next increment of search

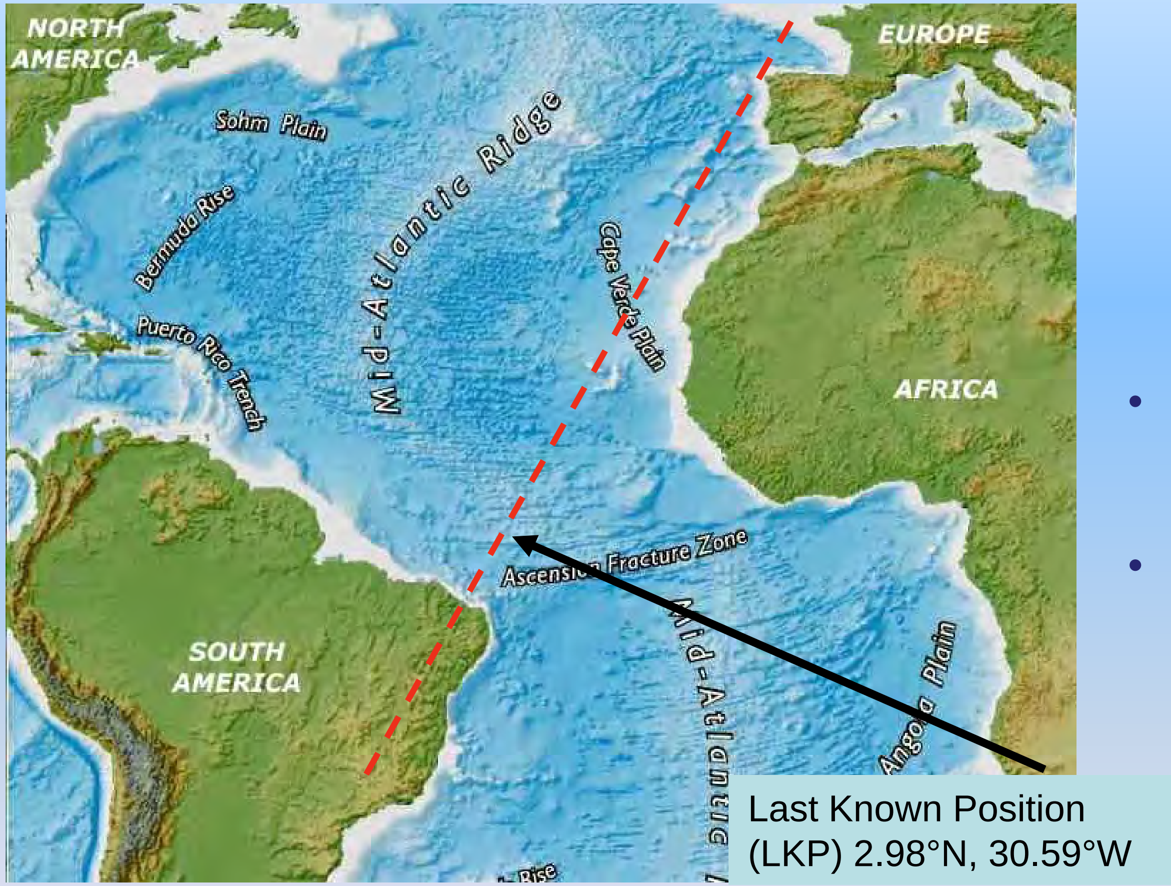
- **Bayesian search planning is an example of Bayesian decision theory**
  - Capable of combining subjective and objective information in principled fashion to allow you to make decisions with the information you have
  - In most search problems you can't wait for more data to resolve uncertainties before acting

Air France Flight 447: Open Ocean Search for Wreckage



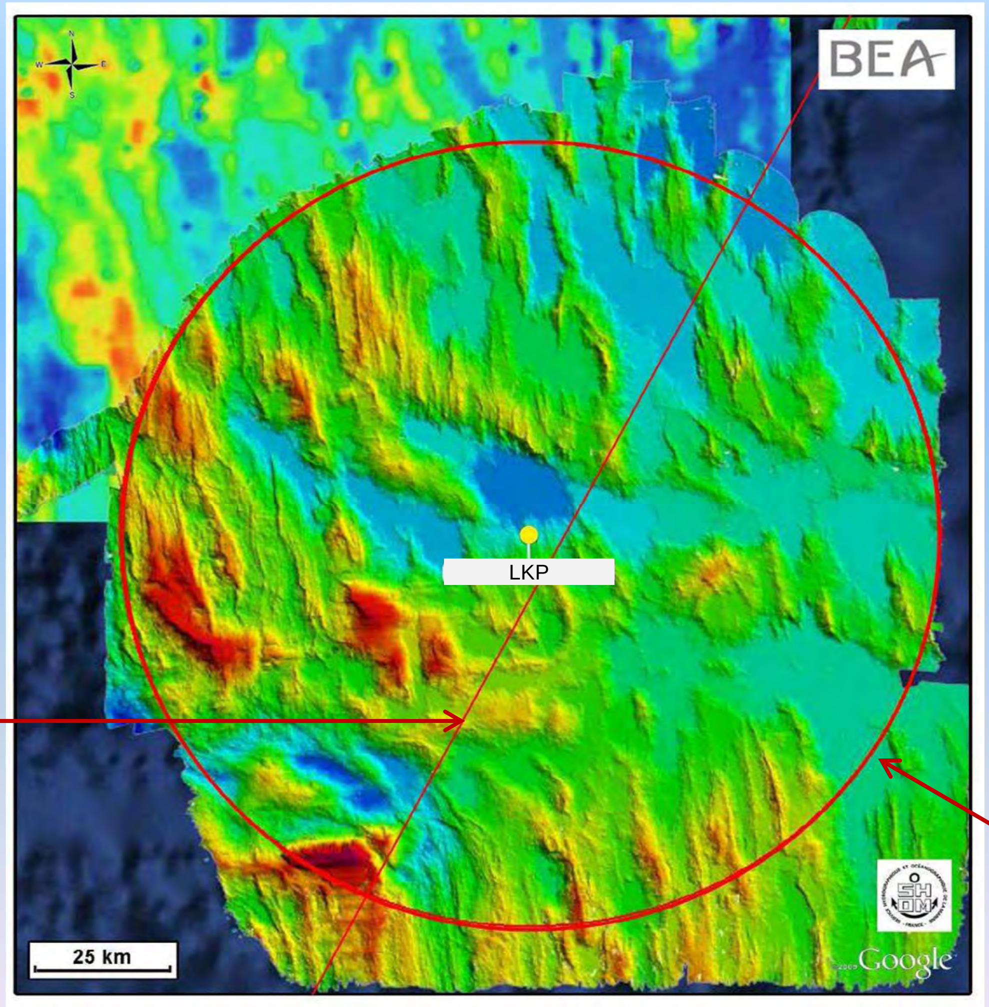


Air France Flight 447 Disappears



- In the early morning hours of June 1, 2009, Air France Flight AF 447 from Rio to Paris with 228 passengers and crew aboard, disappeared during stormy weather over the South Atlantic
- The French Bureau of Enquiries and Analyses (BEA) took charge of the search.
- On April 3, 2011, an autonomous underwater vehicle (AUV) found the wreckage on the ocean bottom at a depth of 13,060 ft

Last Known Position and 40 NM Circle



Intended Flight Path

The BEA determined that the plane could have flown no farther than 40 NM from Last Known Position (LKP)

40 NM Circle:

Unsuccessful Searches

- Air and surface search for floating debris and signs of survivors begins 1 June 2009
  - On 6 June the first bodies and floating debris are found 38 NM north of LKP
- Three unsuccessful underwater search phases in 2009 and 2010
  - Phase 1: Passive sonar search for the Underwater Locator Beacons (ULBs)
  - Phases 2 & 3: Side-looking sonar search for underwater wreckage

Tail Section

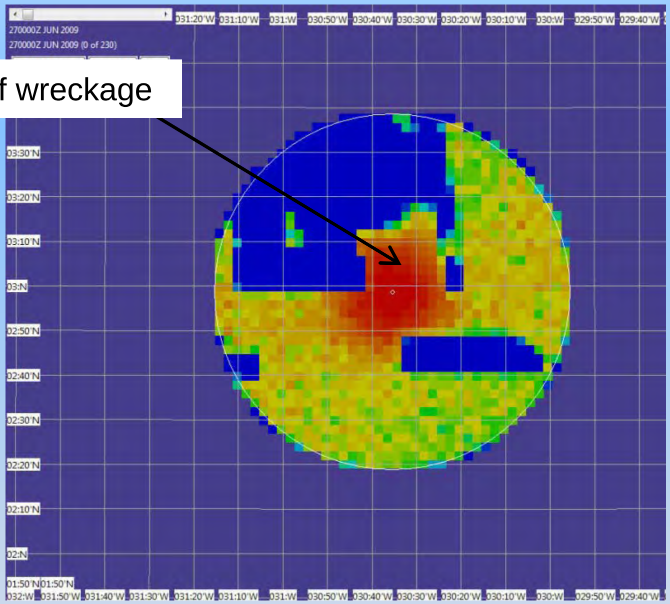
Galley

Probability Map for 2011 Search

- July 2010 BEA tasks Metron to produce probability maps (PDFs) for wreckage location using all available info including unsuccessful searches
- January 2011 Metron delivers PDFs to the BEA[1] showing an area near center of 40 NM circle to be high probability.
- In late March search began.

BEA announced on 8 April 2011[2]

"This [Metron] study, published on the BEA website 20 January 2011, indicated a strong possibility for the discovery of the wreckage near the center of the Circle. It was in this area that it was in fact discovered after one week of exploration…"



PDF after Phase 3 assuming pingers failed

1. Stone, Keller, Kratzke, Strumpfer, *Search Analysis for the Location of the AF447 Underwater Wreckage* Metron Report to BEA, 20 January 2011
2. Troadec,Jean-Paul. *Undersea search operations to find the wreckage of the A 330, flight AF 447: the culmination of extensive searches.* Note from BEA Director, 8 April 2011, on the BEA website at http://www.bea.aero/en/enquetes/flight.af.447/note.from.bea.director.end.phase4.pdf

Analysis Process

Posterior after 2010 active sonar search

Posterior after 2009 active sonar search

Posterior after passive sonar search

Posterior after unsuccessful surface search

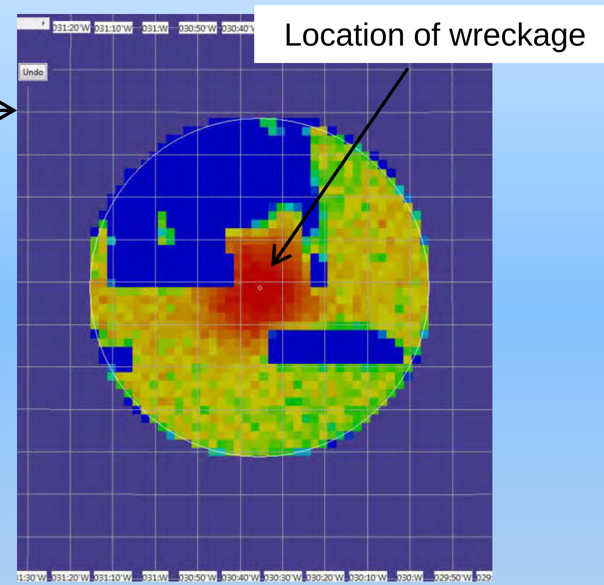
Prior

35%
Uniform over 40
NM Circle

35%
Circular normal based on
LOC accidents

30%
Reverse drift scenario

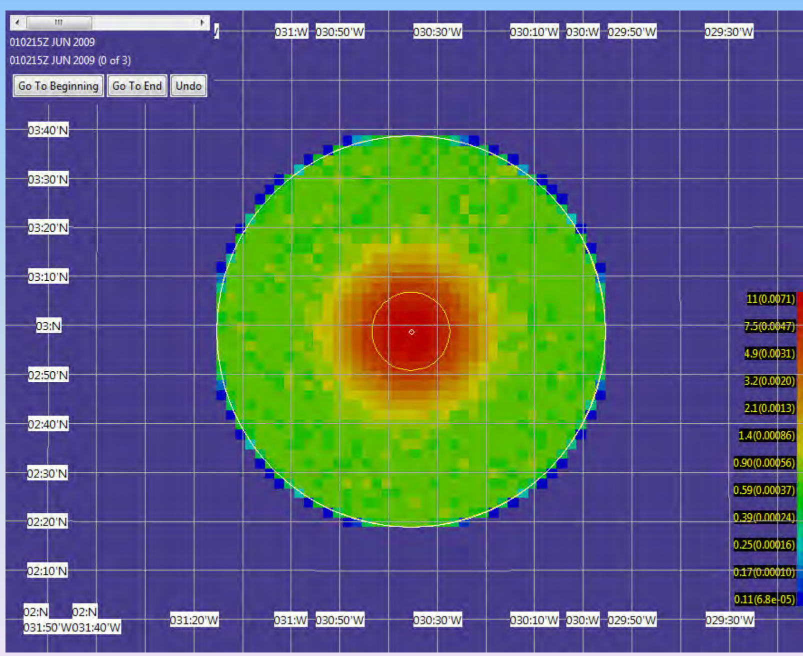
Flight dynamics scenarios



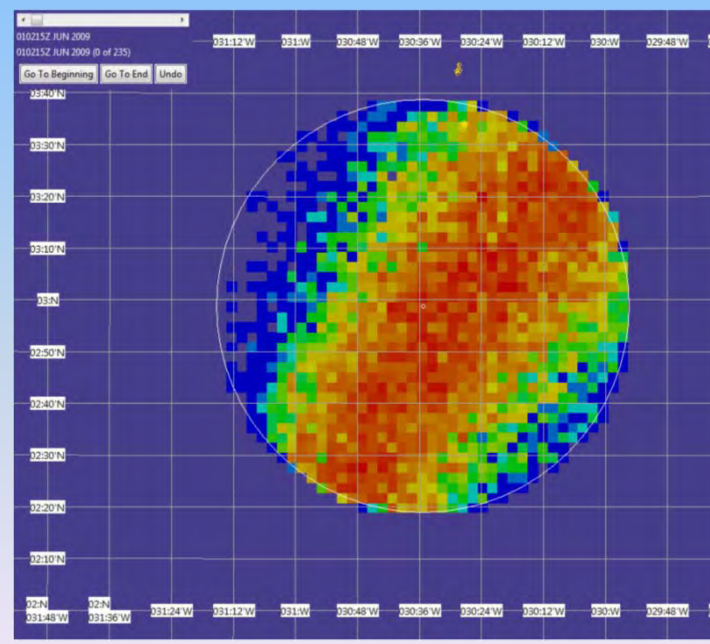
Prob distribution for 2011 search

Flight Dynamics (FD) and Reverse Drift PDFs

- Prior PDF for impact prior to surface search - mixture
  - Flight dynamics (70%)
    1. (50%) Uniform over 40 NM circle about LKP
    2. (50%) Distribution based on nine commercial accidents involving emergency crashes – represented by circular normal with std dev = 8 NM
  - Reverse Drift (30%) prior truncated at 40 NM circle

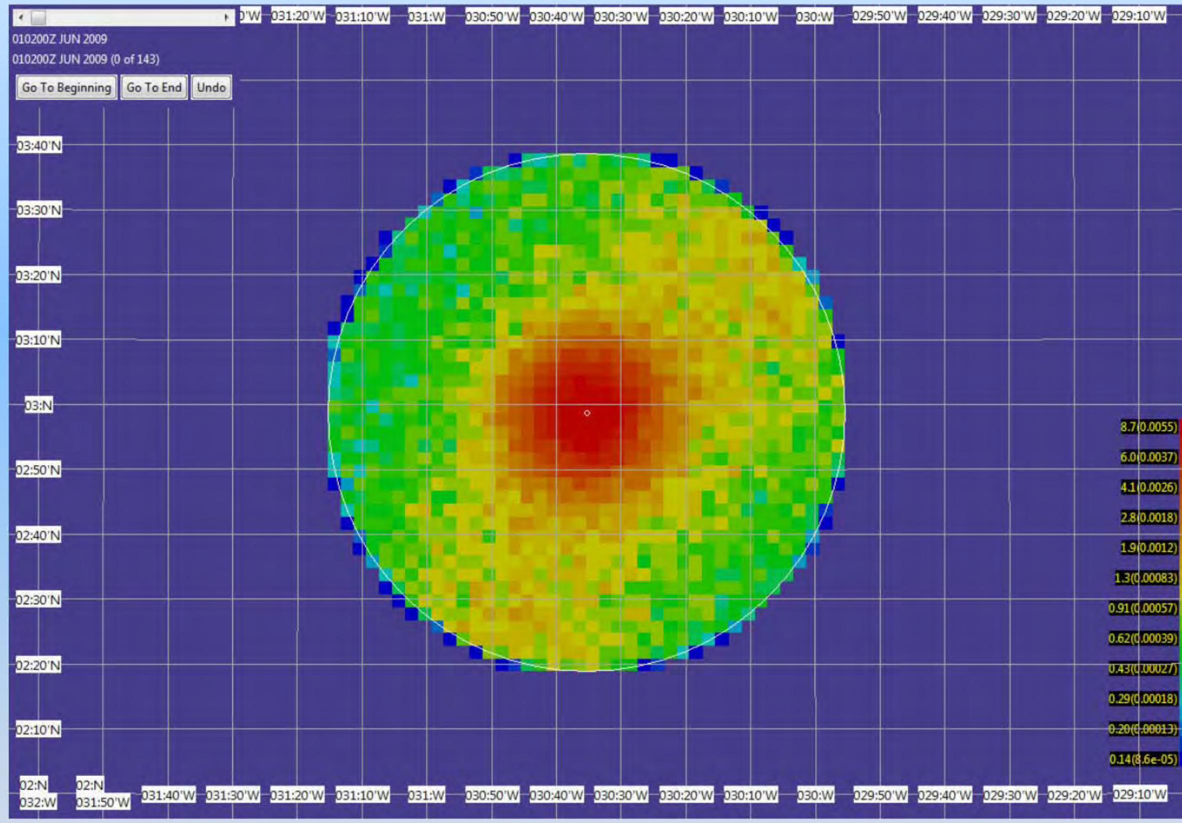


Flight Dynamics PDF



Reverse Drift PDF

Prior



70% Flight Dynamics + 30% Reverse Drift

Accounting for Failed Surface Search

- Aircraft and Ships searched the surface from 1 June to mid-day on 6 June before detecting floating debris – Galley
- Search paths for aircraft and ships put into SAROPS along with estimate of detection capability
- Posterior given failed surface search computed by drifting N = 40,000 particles (paths) forward in time and accounting for unsuccessful search

For $n = 1, \ldots, N,$ compute $\{(w_n, x_n(t)); 0 \le t \le 120\,\text{hrs} = 5\,\text{days}\}$ where

$w_n = 1/40{,}000$

$x_n(0)$ drawn from prior and drifted forward over $[0, T]$ using wind and current models

For $i = 1, \ldots, I$ surface searches compute

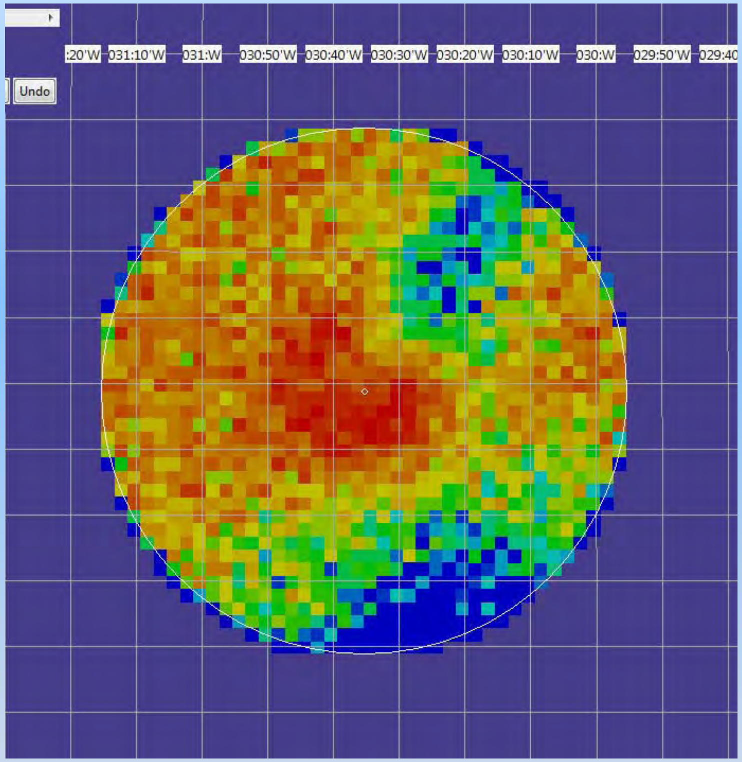
probability $q_i(n)$ that ith search failed to detect galley given it followed path n

$$\tilde{w}_n = w_n \prod_{i=1}^{I} q_i(n) \Big/ C \text{ where } C = \sum_{n=1}^{N} w_n \prod_{i=1}^{I} q_i(n)$$

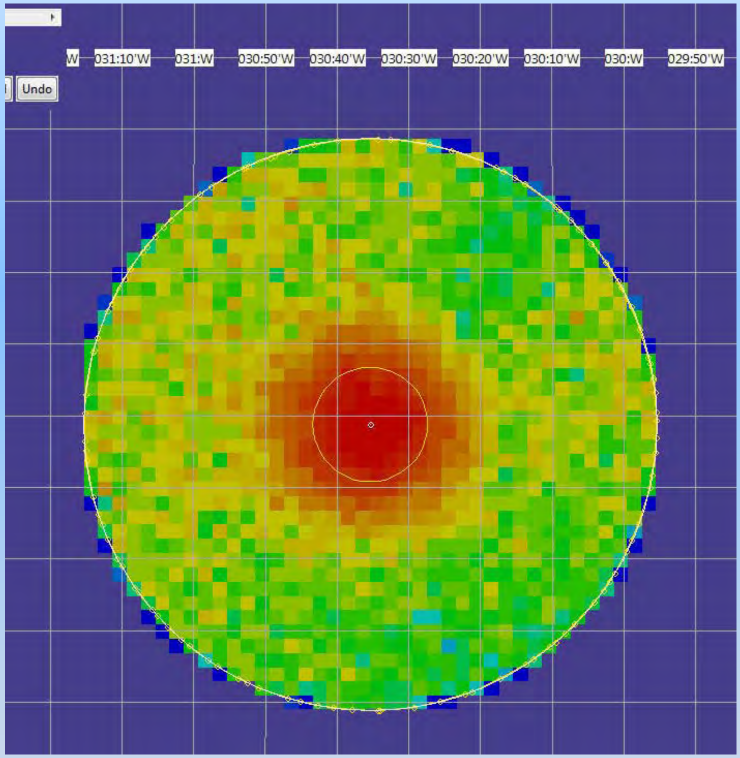
Posterior for crash location given failure of surface search

$$\{(\tilde{w}_n, x_n(0); n = 1, \ldots, N\}$$

Surface Search Likelihood and Posterior

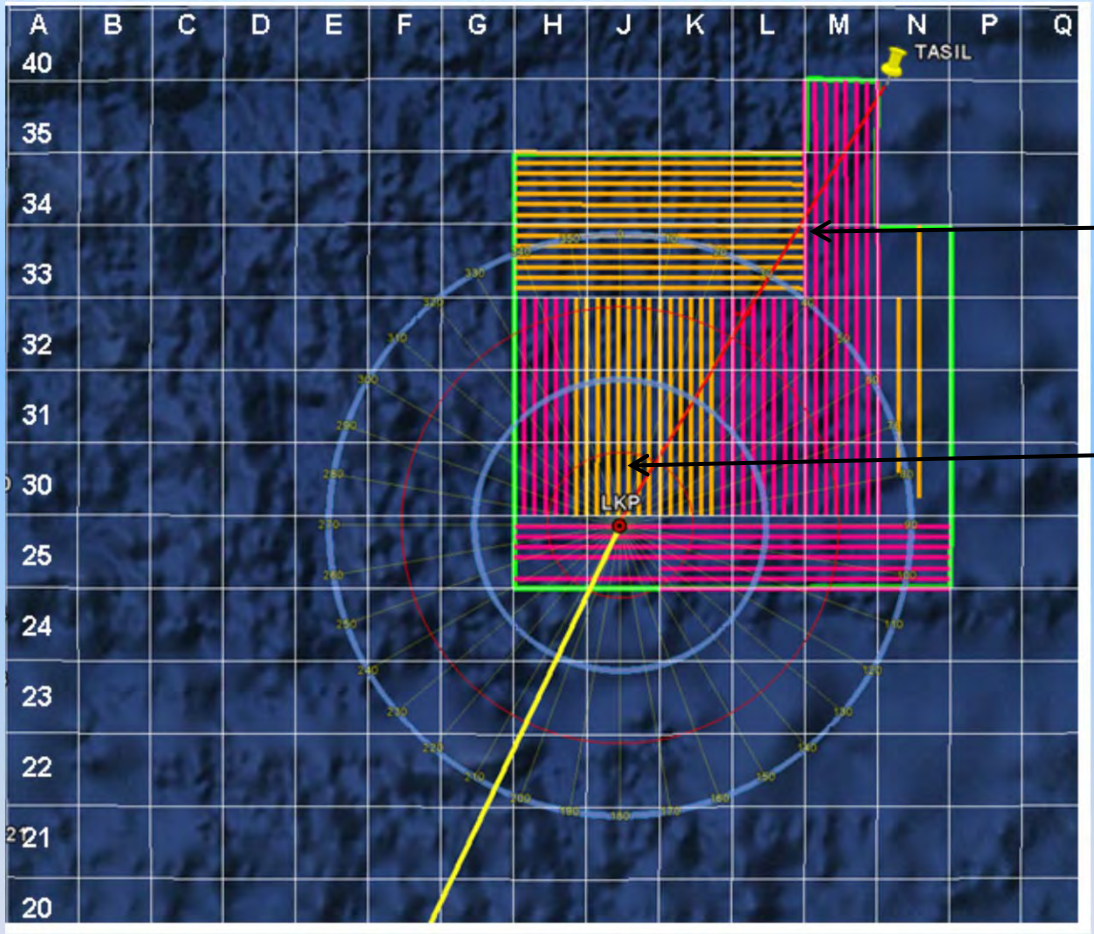


Surface Search Likelihood
- before discounting



Surface Search Posterior

Phase 1: Towed Pinger Locator Search Paths



Fairmount Glacier **(orange) and *Fairmount Expedition*** **(pink) Search Tracks.**
Blue circles are 20 NM and 40 NM circles about the LKP

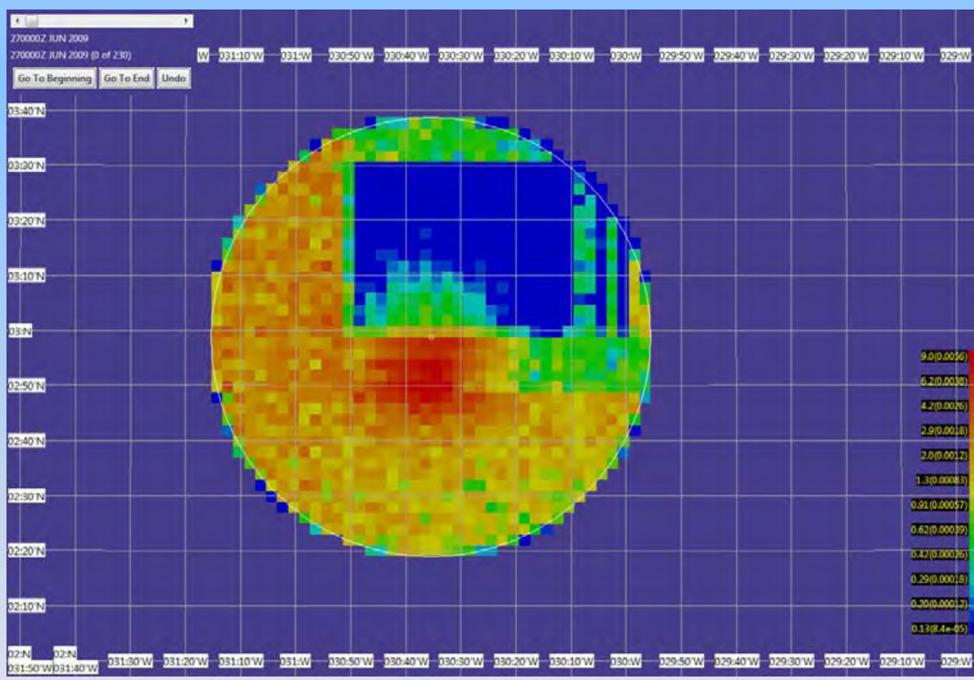
Failure of Towed Pinger Locator (TPL) search led to long and difficult search

Intended flight path

Approximate location of wreckage

Posterior after TPL Search (Phase 1)

- Assumed 0.8 probability of survival of ULB.
  - If survival is independent then 0.92 detection prob within 1730m lateral range
  - If dependent = 0.72.
  - We used 0.77 =(.25)(.92)+(.75)(.72)



Posterior given Failure of TPL Search

Compute posterior given failure of TPL search

Let

$w_n =$ prob of $x_n(0)$ in surface search posterior

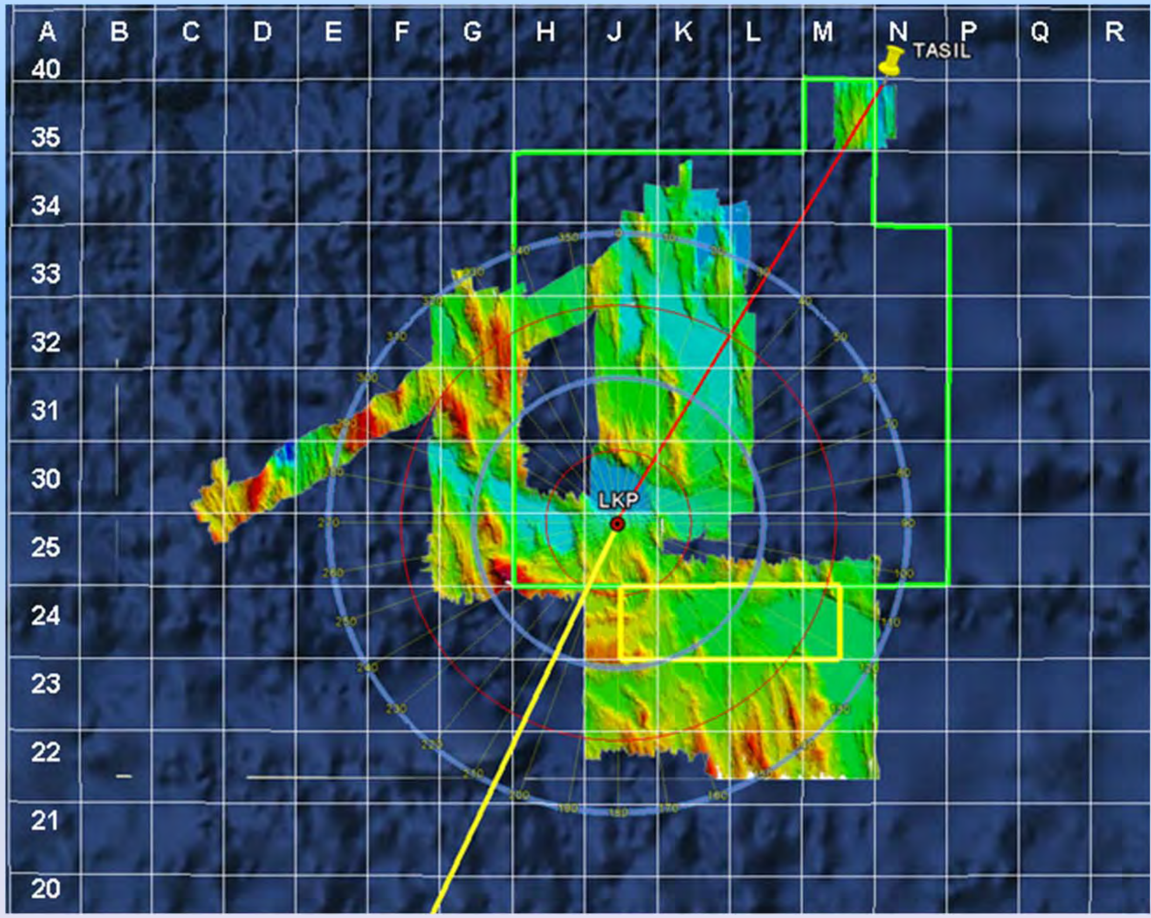
$p_d = 0.77$

$$u_n = \begin{cases} (1 - p_d) w_n & \text{if } x_n(0) \text{ in TPL search region} \\ w_n & \text{otherwise} \end{cases}$$

$C = \sum_{n=1}^{N} u_n$

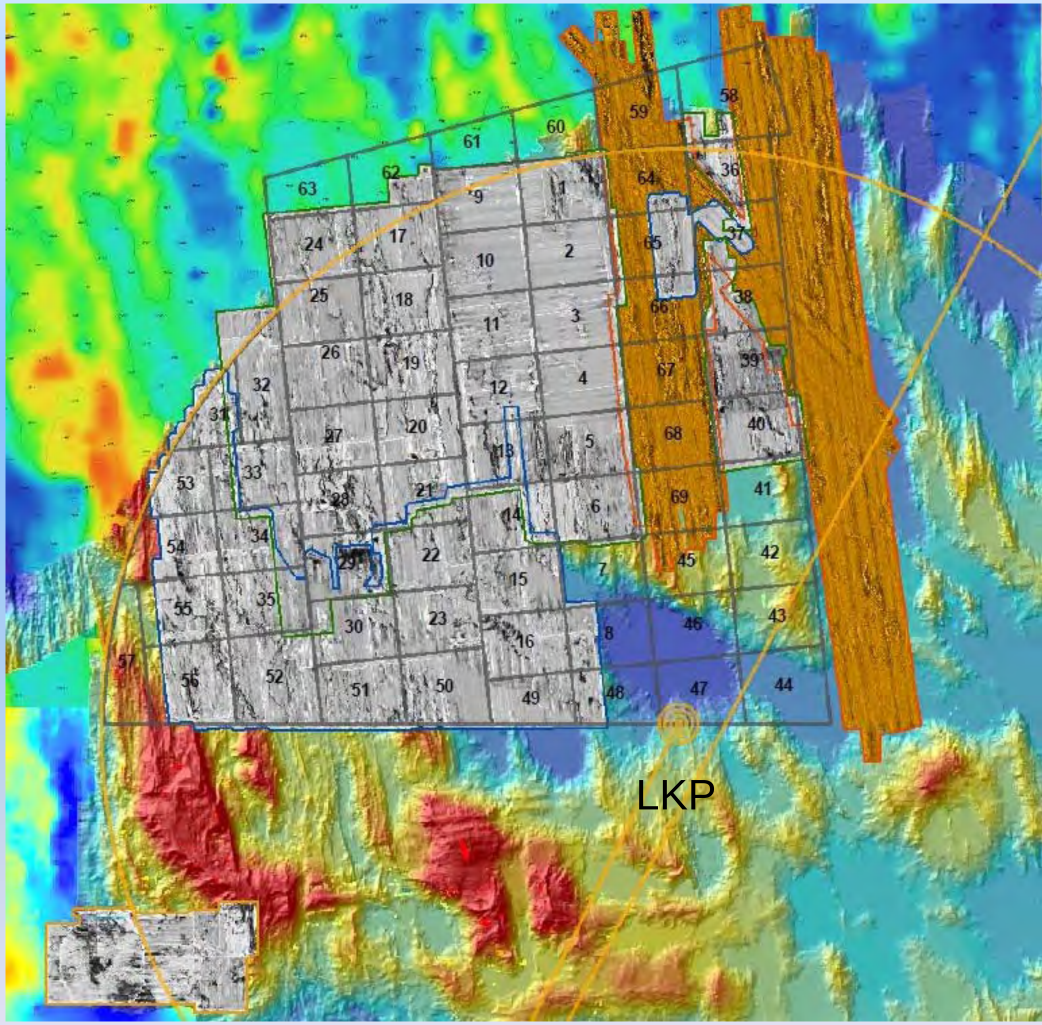
$\tilde{w}_n = u_n / C =$ posterior probability of $x_n(0)$

Phase 2: Side-Looking Sonar

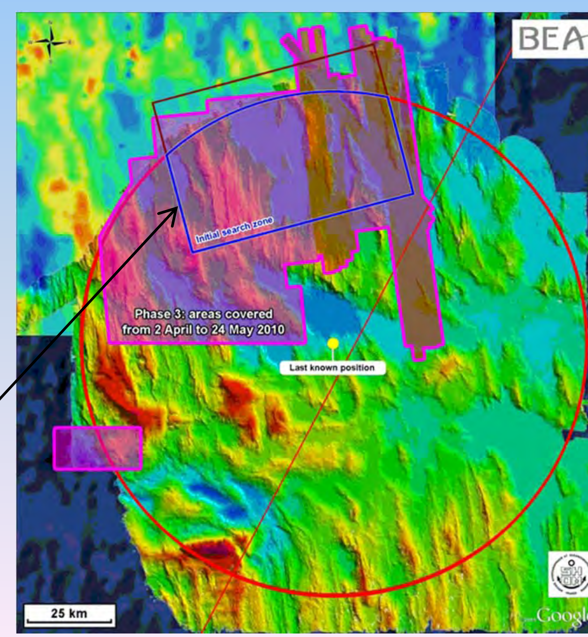


- In August 2009, the BEA decided to search in the cells outlined in yellow with a side-looking sonar towed from the *Pourquoi Pas?*
- Area relatively flat
- Pd estimated at 0.9
- Search performed by IFREMER – French Research Institute for Exploitation of the Sea
- Green outline shows TPL search area

Phase 3: REMUS and ORION Searches



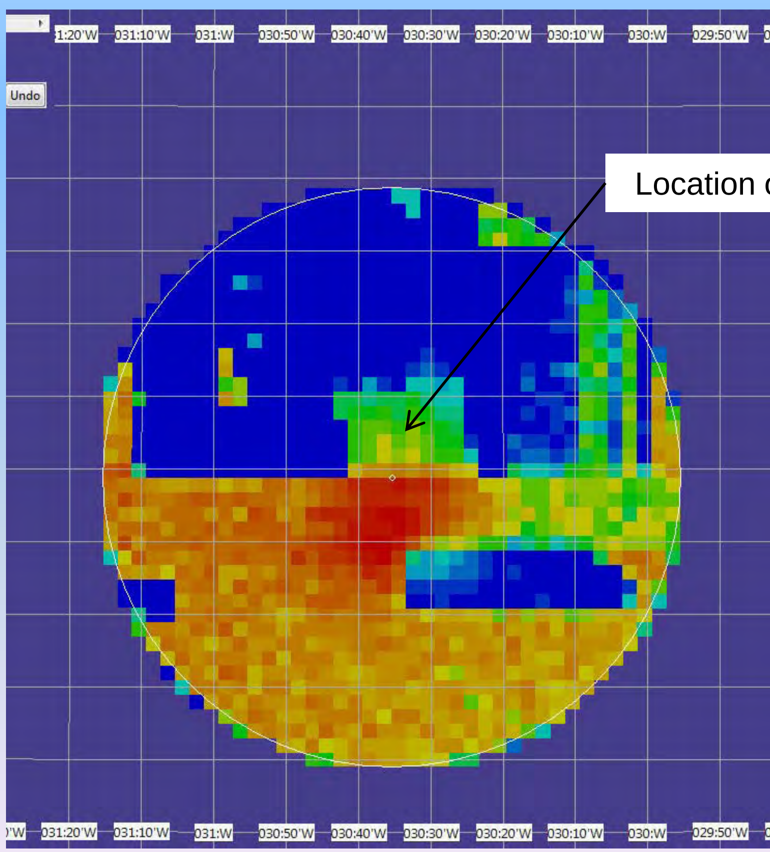
- Woods Hole Oceanographic Institute (WHOI) deployed 3 REMUS 6000 AUVs which searched the grey area.
- US Navy/Phoenix International performed search in orange area using ORION towed side-looking sonar
- Both searches rated highly effective, Pd > 0.9



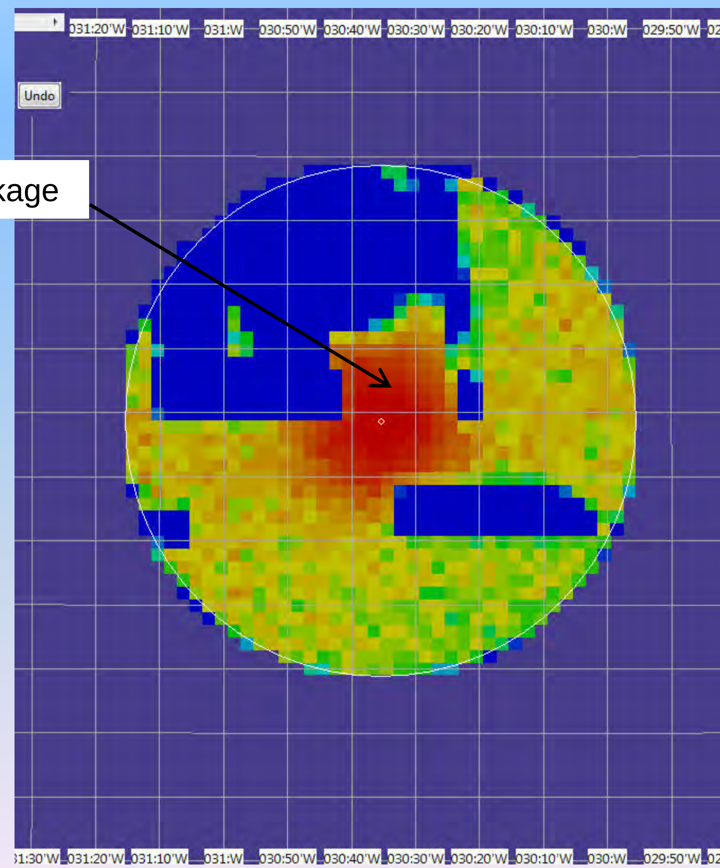
Search area based on reverse drift analysis by an international group of oceanographers

Posterior after Phases 2 and 3

- Using 0.9 detection probability within areas covered by side-looking sonar in phases 2 and 3, we obtained the posterior PDFs –
  - one assuming that ULBs functioned the other that they didn't

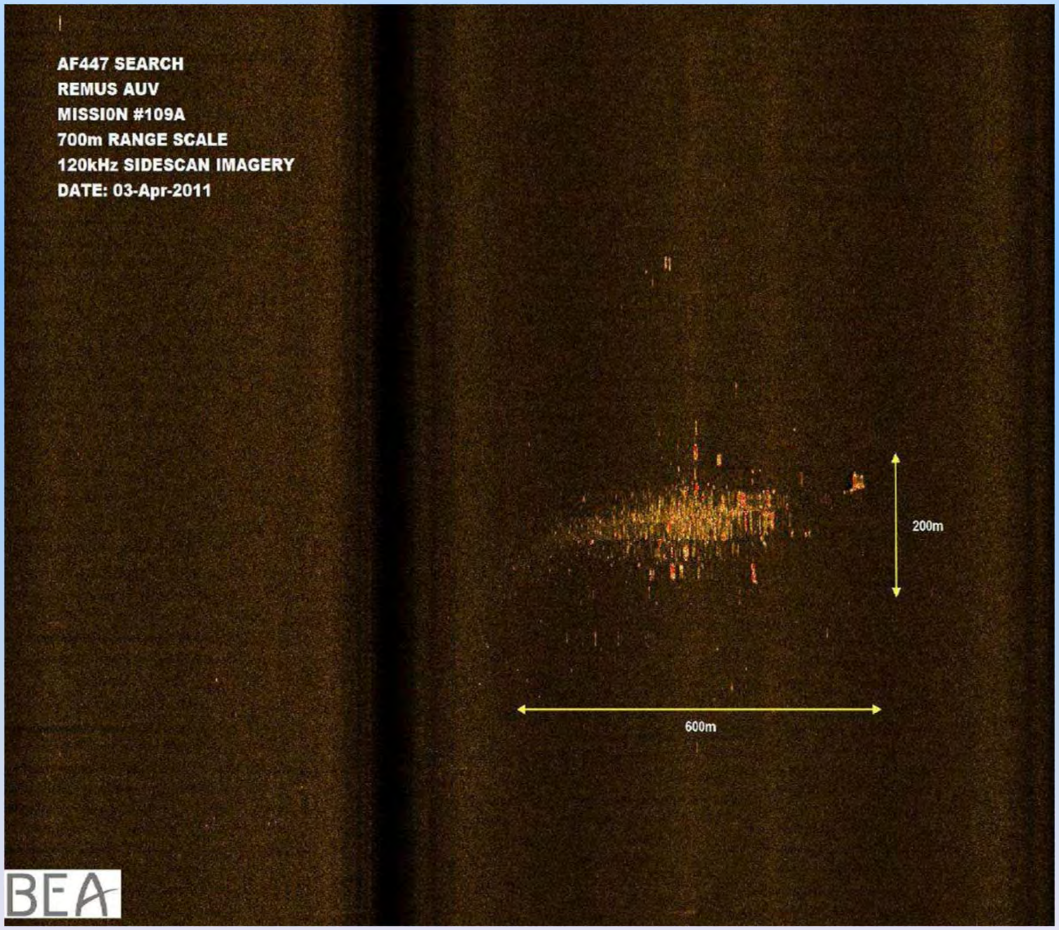


Posterior Assuming ULBs Worked



Posterior Assuming ULBs Failed

Wreck Found on 03 April 2011



Side-Scan Sonar Image of Wreckage

Engine

Landing Gear

Underwater Locator Beacons Recovered

Result and Conclusions

- Result
  - Search began in high probability area of posterior (assuming ULBs failed) and found wreck with one week of effort
- Major Conclusion
  - The success of this effort illustrates of the value of a methodical, Bayesian approach to search planning
- Minor Conclusions
  - The failure of the TPL search to detect the ULBs led to a long and complicated search
    - Despite record of reliability it appears that ULBs failed
      - In test of recovered ULB, the beacon failed to actuate when connected to a newly charged battery
      - Likely both ULBs were damaged in crash and did not function
  - Drift Group recommendation delayed success by a year
    - Used ad-hoc methods to determine search rectangle

Critique of Metron AF447 Work

- Instead of two probability maps, we should have included pinger status (working/not working) in state space and computed probability map on (position, pinger-status)
- A debris likelihood function should be used instead of a reverse drift distribution to incorporate debris info
  - This will be addressed in the MH 370 discussion to follow
- Technical discussion of search planning approach in
  - Search for the wreckage of Air France Flight AF 447, by LD Stone, CM Keller, TM Kratzke, and JP Strumpfer, *Statistical Science* 2014, 29:69-80

Bayesian Search Methods Applied to MH 370[1]

1. Bayesian Methods in the Search for MH370
Davey, Gordon, Holland, Rutten, and Williams, Springer 2016

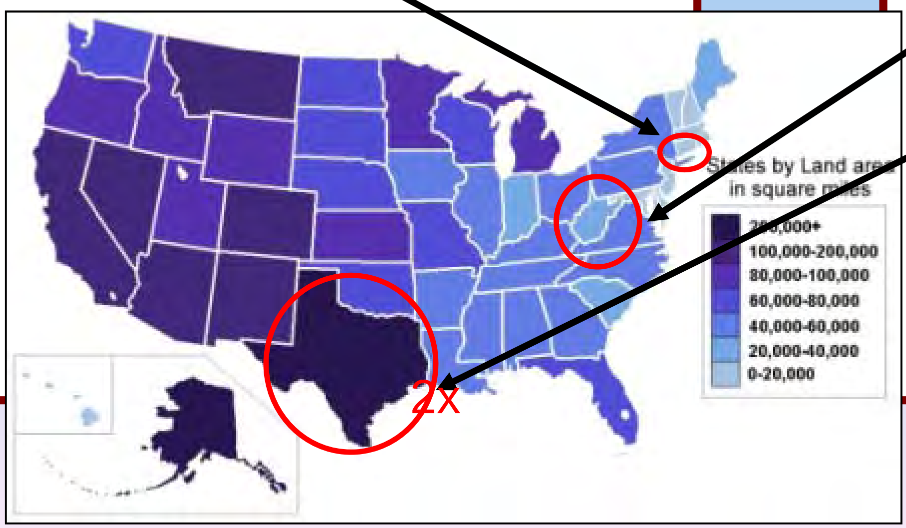
Comparison of AF447 and MH370

AF 447

- Last known position < 2 min and 40 sec before loss
- First debris located within 6 days of loss and 40 nm of LKP
- Area of Uncertainty:
  - 17,240 km^2

MH370

- Last known position 6 hours before loss
- First debris located 506 days after loss 2200 nms from search area
- Area of Uncertainty:
  - 60,000 km^2 – Priority Area
  - 240,000 km^2 – Medium Area
  - 1,120,000 km^2 – Wide Area



Known Flight Path of MH 370

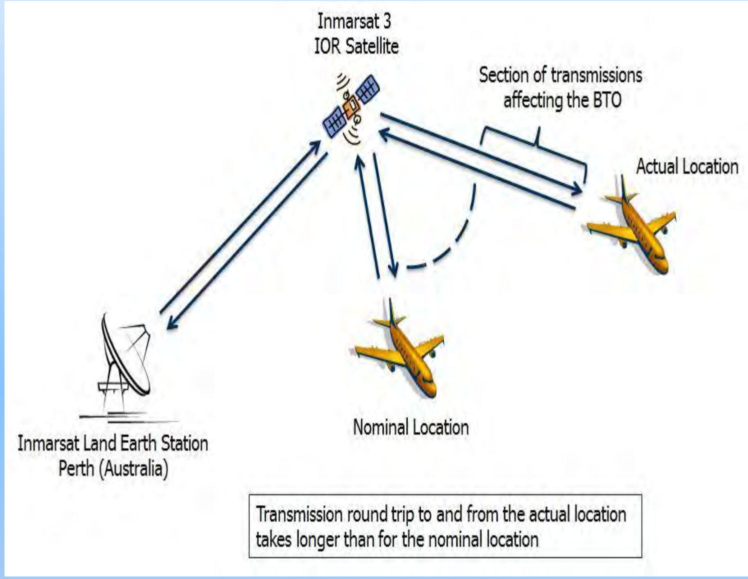
Satellite Logon and Interrogation Requests

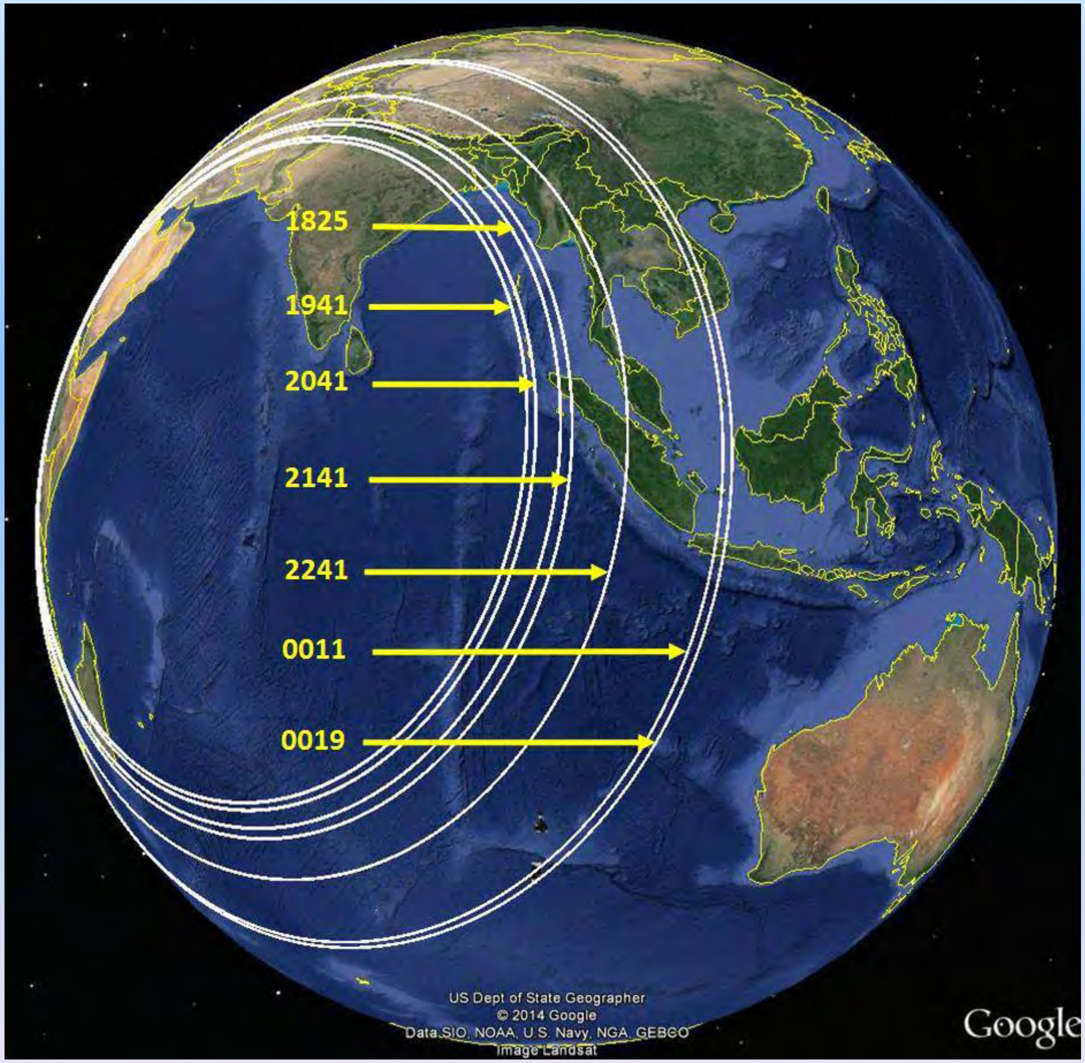
	UTC hhmm.ss
- Last radar fix	1822
- 1st handshake initiated by the aircraft	1825.27
- Unanswered ground to air telephone call	1839.52
- 2nd handshake initiated by the ground station	1941.00
- 3rd handshake initiated by the ground station	2041.02
- 4th handshake initiated by the ground station	2141.24
- 5th handshake initiated by the ground station	2241.19
- Unanswered ground to air telephone call	2313.58
- 6th handshake initiated by the ground station	0010.58
- 7th handshake initiated by the aircraft	0019.29
- Aircraft did not respond to log-on interrogation from the satellite earth ground station	0115.56

Information from handshake: Burst Timing Offset (BTO) - time delay
Burst Frequency Offset (BFO) – Doppler

Info from ground to air phone call: BFO only

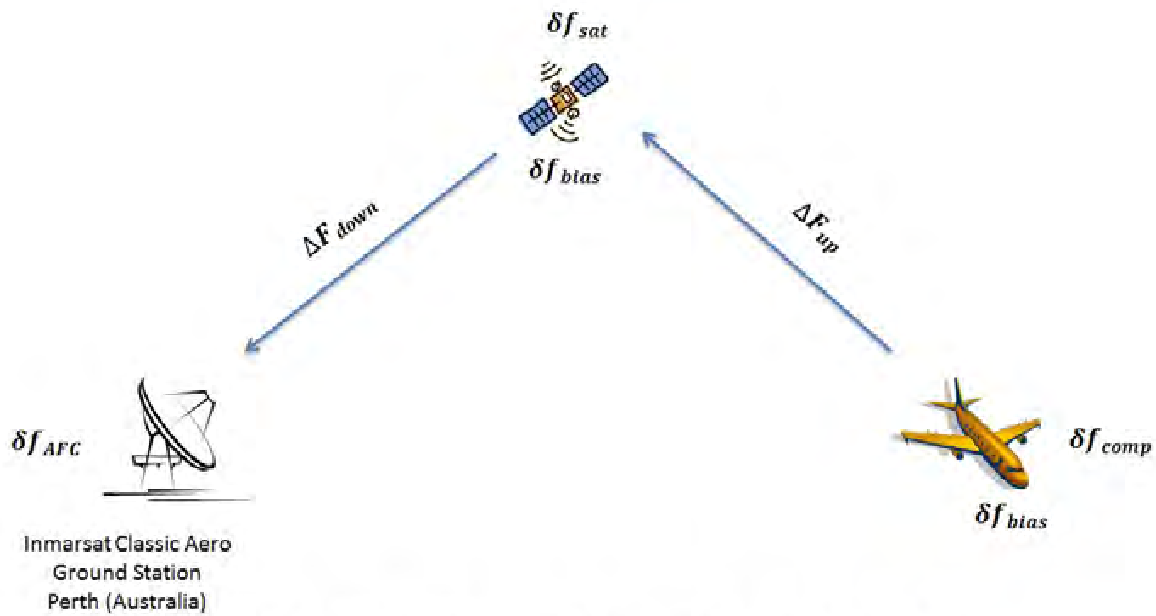
BTO Provides Range from Satellite





- Range rings for BTOs from Satellite
- Ground Earth Station initiates logon request roughly every hour if nothing heard from aircraft.

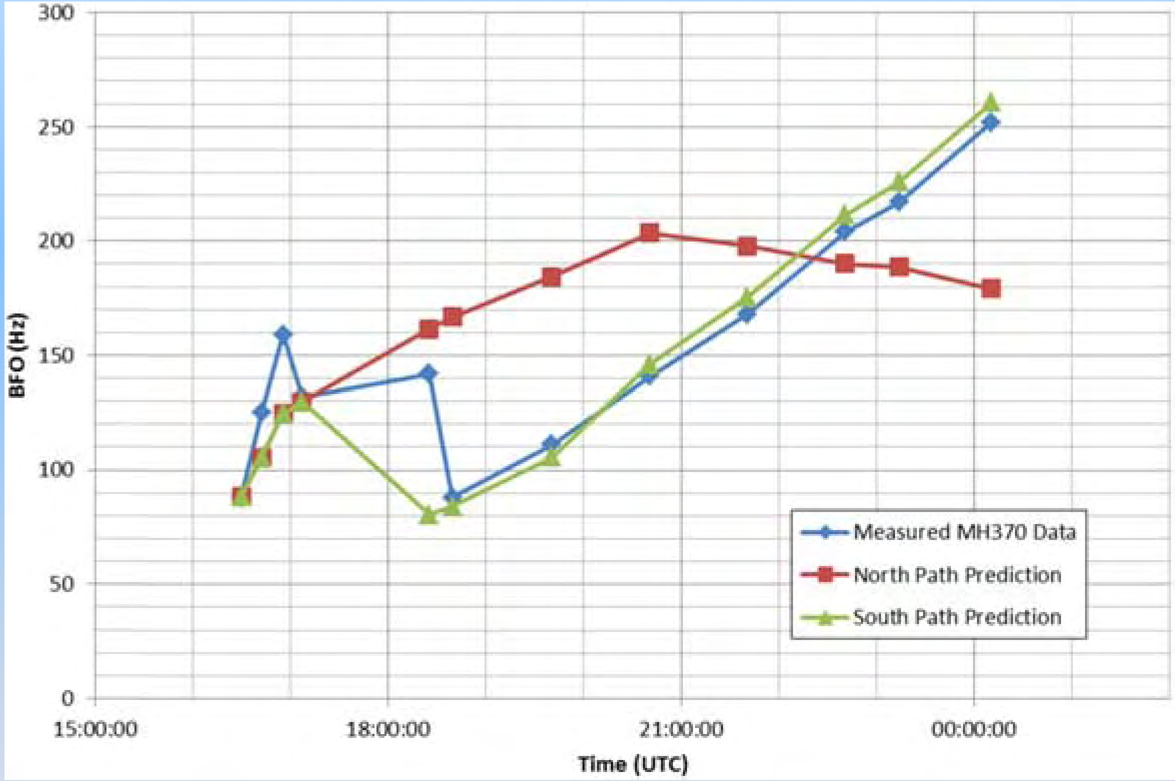
BFO provides Doppler



$$BFO = \Delta F_{up} + \Delta F_{down} + \delta f_{comp} + \delta f_{sat} + \delta f_{AFC} + \delta f_{bias}$$

ΔF_{up}	*Doppler on the signal passing from the aircraft to the satellite*
ΔF_{down}	*Doppler on the signal passing from the satellite to the ground station*
δf_{comp}	*Frequency compensation applied by the aircraft, assuming level flight and a fixed satellite location*
δf_{sat}	*Variation in satellite translation frequency*
δf_{AFC}	*Frequency compensation applied by the ground station receive chain*
δf_{bias}	*Fixed offset due to errors in the aircraft and satellite oscillators*

Comparison of North and South Paths



How Did DST Group Proceed to Develop Prior?

- **First:** developed a prior distribution on flight paths to crossing of 7th arc
  - Flight paths started at the 18:01 radar measurement and provided position and (ground) velocity to the 00:19 arc crossing.
- **Second:** derived measurement (error) models for the BTO and BFO measurements – developed theoretically and verified empirically
  - Construct likelihood functions. A likelihood function computes the probability of receiving a measurement given a position and (ground) velocity of the aircraft at the time of the measurement
- **Third:** computed the prior distribution on location of crash. Specifically
  - Compute the likelihood of each flight path given the measurements.
  - Multiply the likelihood times the prior probability of the path
  - Normalize to compute posterior probability of each path
    - Gives posterior probability on location when crossing 7th arc
  - Distribution on heading and distance from the crossing used to produce probability distribution on crash location

Constructing Flight Paths

- **Simulated a large number of possible paths:**
  - Start paths at 18:01 radar fix generally heading Northwest. Paths turn south to match18:25 range ring crossing
  - Then follow procedures developed by Boeing: For each leg
    - Altitude − uniform [25,000, 43,000] feet
    - Speeds − uniform Mach [0.73, 0.84]
    - Various autopilot modes (constant true track, constant true heading, constant magnetic heading, and great circle)
    - Speed and heading modeled by an Ornstein-Uhlenbeck process about mean
    - Modeled the affect of winds on ground speed
  - Crash location within 20 nm of position at time 00:19 arc crossing
    - Distribution weighted to have most locations less then 10 nm.
- **Result:** A set of N paths giving 3-d position and velocity

$$\left\{\left(x_n(t), v_n(t)\right); n = 1, \ldots, N\right\} \text{ for } 18:01 \leq t \leq 00:19 \text{ incl final crash position}$$

  - Let p_n = probability of path n where $\sum_{n=1}^{N} p_n = 1$
  - Branching procedure used to prevent particle starvation

Likelihood Functions

- Let g_1 and g_2 be the density functions for measurement errors of the BTO and BFO. (Normal with mean 0 and stand dev = 29μs, 4.3 Hz)
- Suppose we have BTO and BFO measurements Δt_i and Δf_i at time t_i
- For each path n, we calculate the predicted BTO $\delta t(t_i, n)$ and BFO $\delta f(t_i, n)$ at time t_i given the position and velocity $\left(x_n(t_i), v_n(t_i)\right)$ of the aircraft on path n at time t_i
- The likelihood l of these measurements given the aircraft follows the n th path is

$$
\begin{aligned}
l\left(\left(\Delta t_i, \Delta f_i\right) \mid (x_n(t_i), v_n(t_i)\right) &= \Pr\left\{BTO = \Delta t_i \;\&\; BFO = \Delta f_i \mid \left(x_n(t_i), v_n(t_i)\right)\right\} \\
&= g_1\left(\Delta t_i - \delta t(t_i, n)\right) g_2\left(\Delta f_i - \delta f(t_i, n)\right)
\end{aligned}
$$

Computing the Posterior

- The likelihood of obtaining the 9 measurements from the logon requests and interrogations listed above given the aircraft follows the n th path is

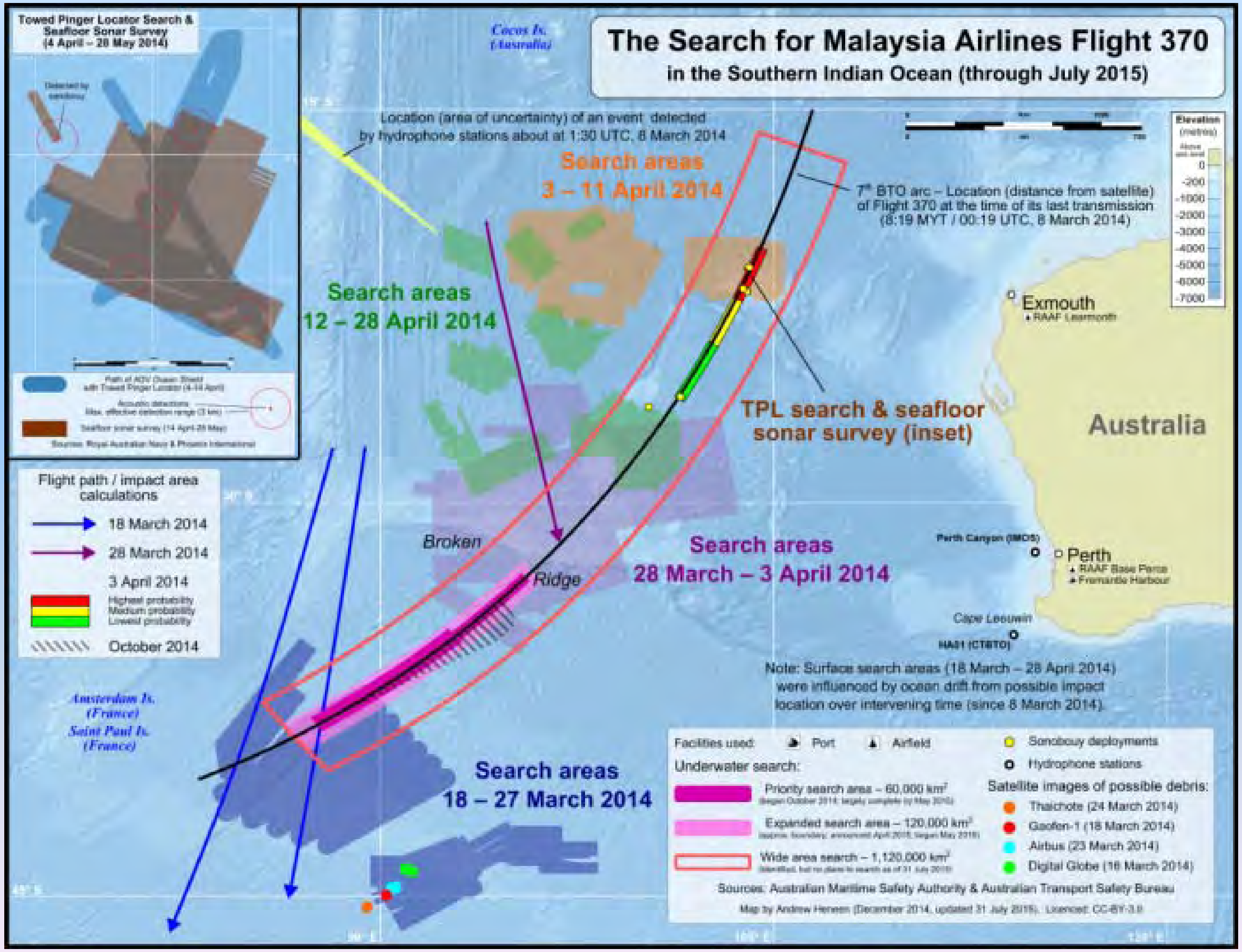
$$L(n) = \prod_{i=1}^{9} g_1\left(\Delta t_i - \delta t(t_i, n)\right) g_2\left(\Delta f_i - \delta f(t_i, n)\right)$$

- The posterior probability on path n is

$$\tilde{p}_n = L(n)\, p_n \Big/ \sum_{m=1}^{N} L(m)\, p_m$$

- Which yields the posterior probability on the 00:19 arc crossing point and crash location.

Search Areas



MH370 Aircraft Debris

- Flaperon from MH370 recovered on Reunion Island 508 days after loss – 2200 nm from the search zone
- How do you include this information in the probability map for the crash location?
- Answer: Compute likelihood function and apply Bayes rule

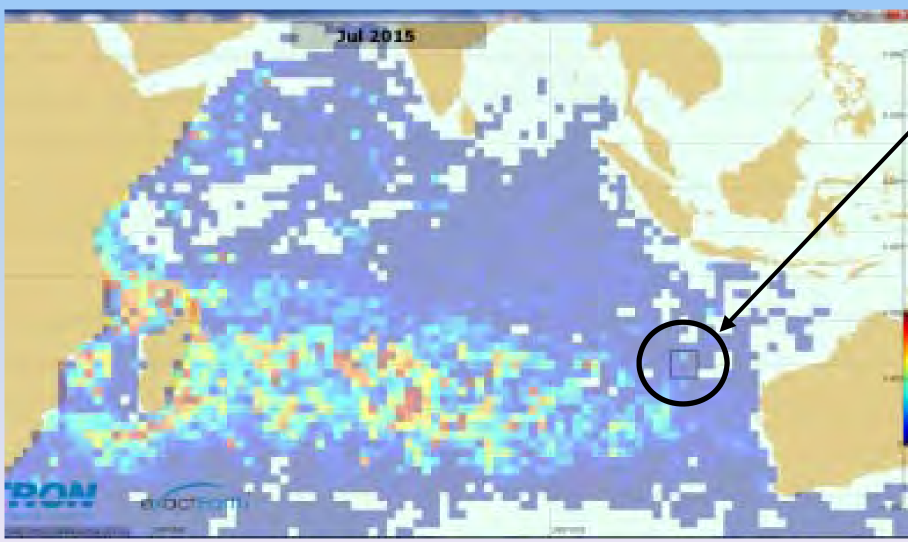
Debris Likelihood Function

Let: $(y,T) =$ event: debris found at y by time T

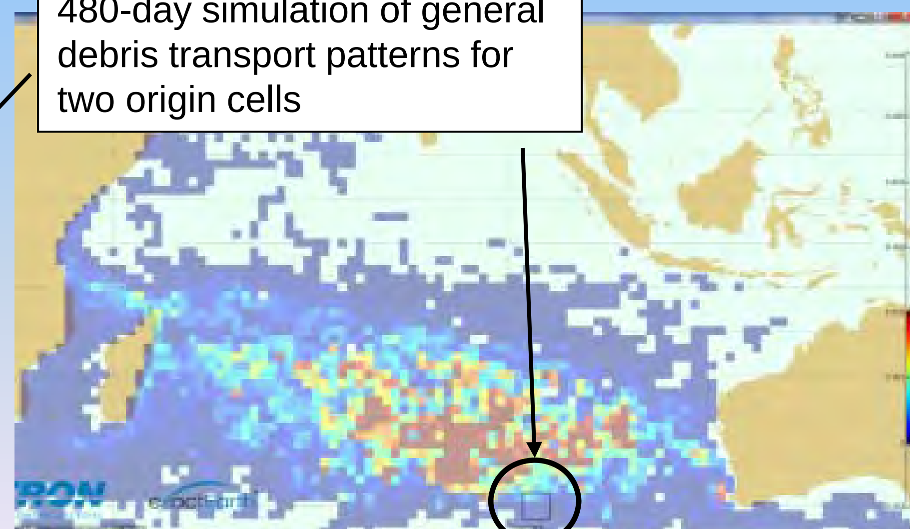
$j = 1,\ldots,J$ be the cells of the prior

$\boldsymbol{L}\big((y,T)\mid j\big) = \Pr\{\text{debris arrives at } y \text{ by time } T \mid \text{origin in cell } j\}$

Use Drift Model Simulation to Calculate Likelihood

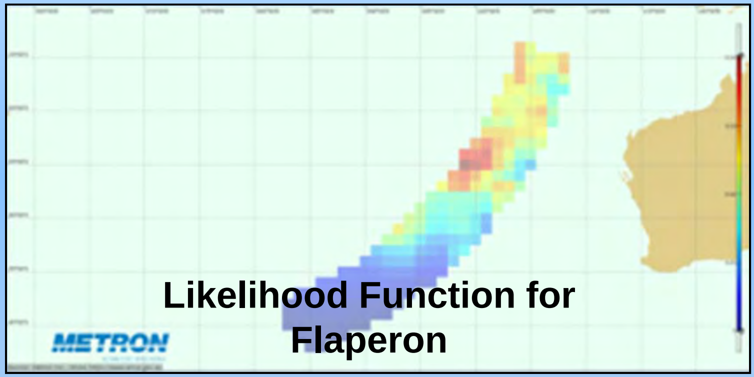


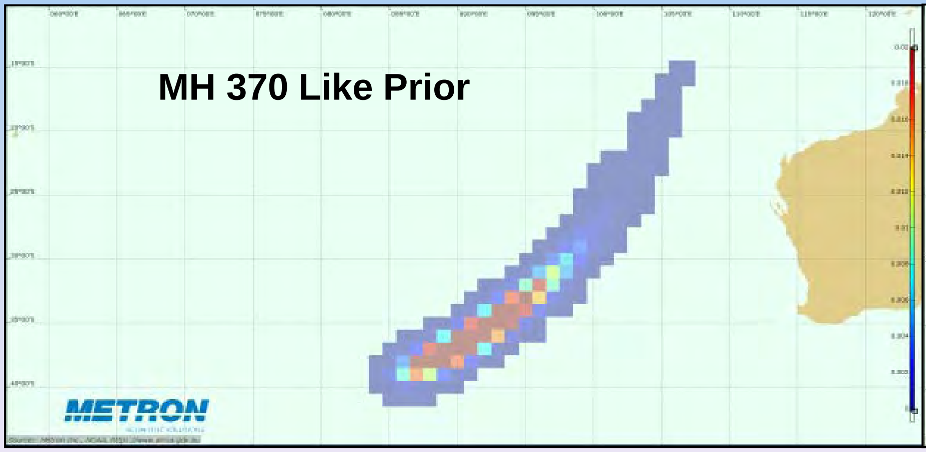
480-day simulation of general debris transport patterns for two origin cells

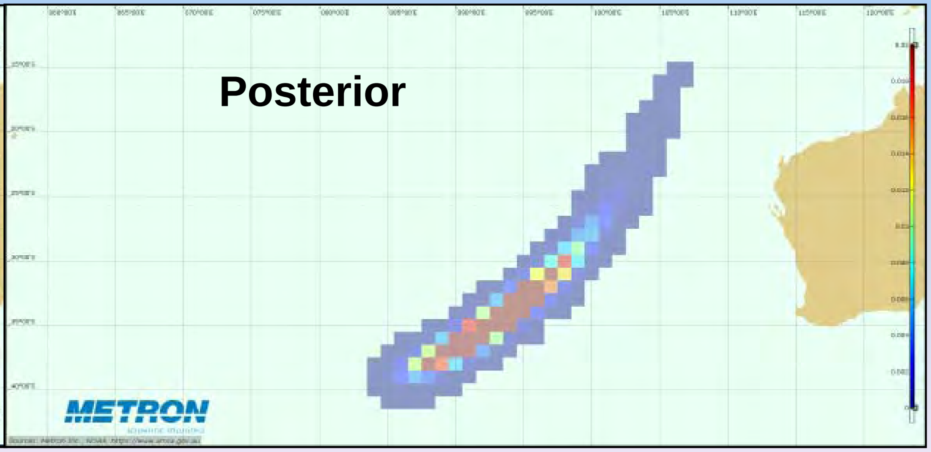


Posterior Distribution Given Flaperon Find

$$\text{Posterior } \tilde{p}(j) = \frac{L\big((y,T) \mid j\big)\, p(j)}{\sum_{j'=1}^{J} L\big((y,T) \mid j'\big)\, p(j')} \quad \text{for } j = 1, \ldots, J$$

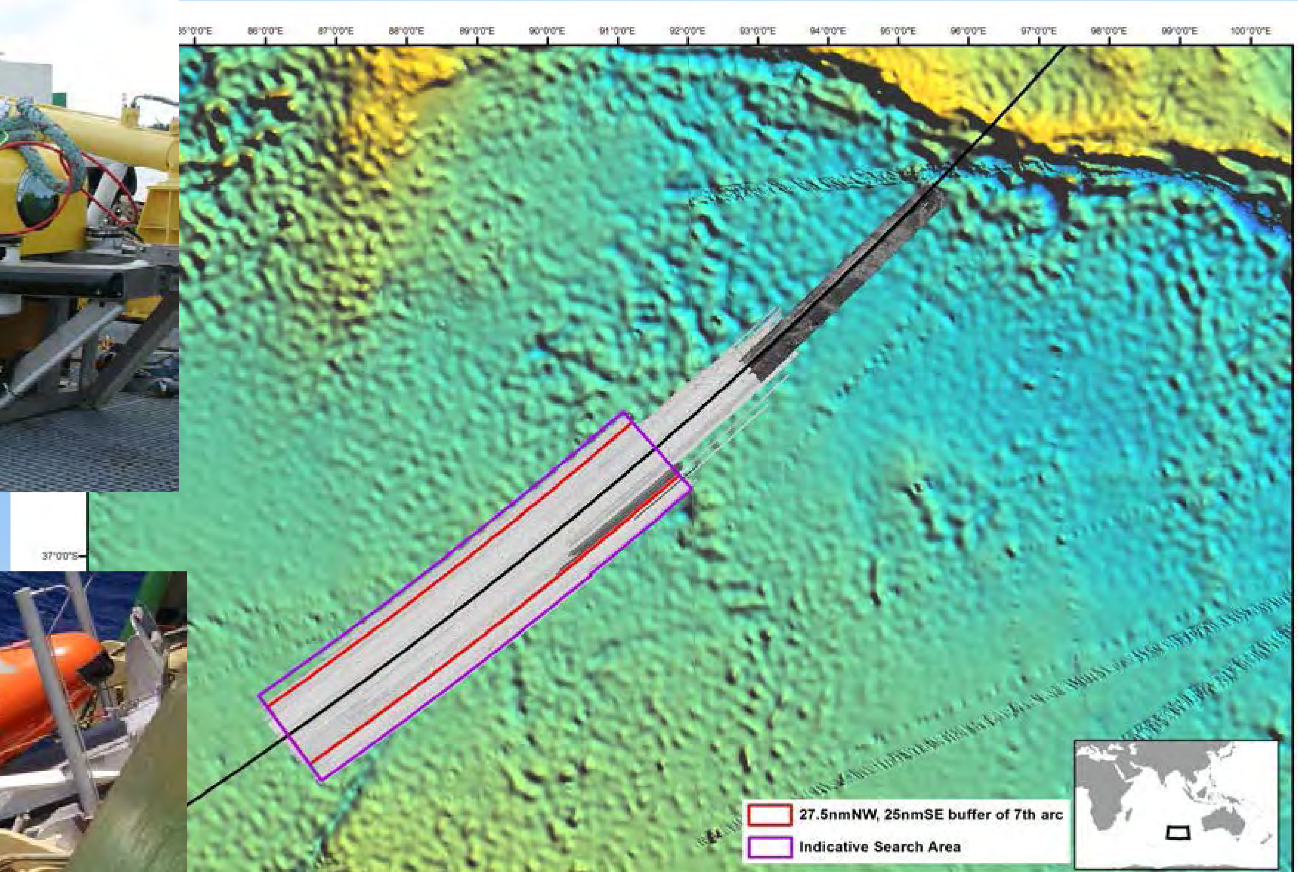






Search Sensors and Areas

Towed Side Looking Sonar

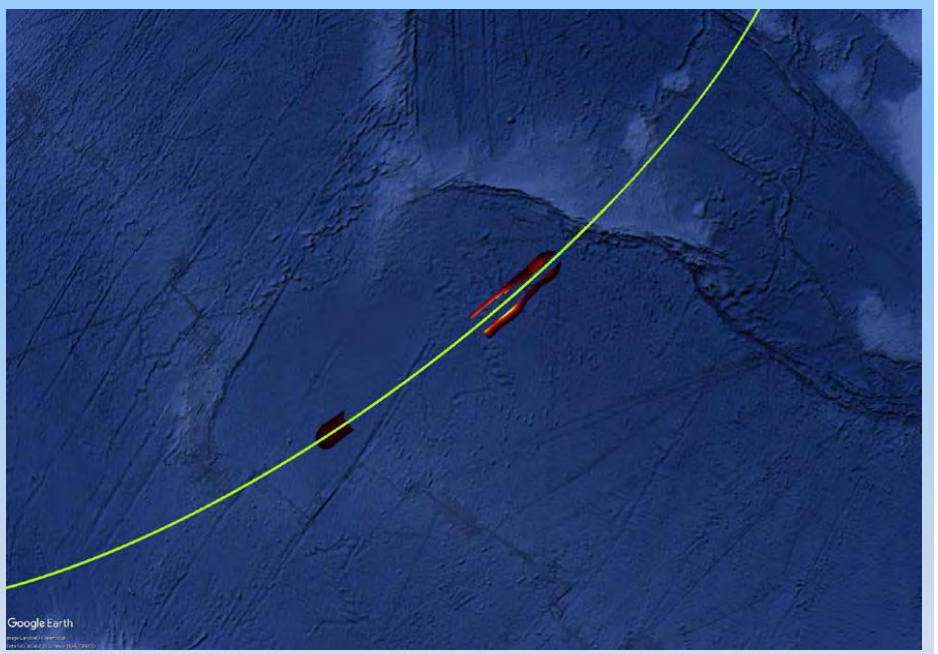


Gray Shows Area Searched as of November 2016

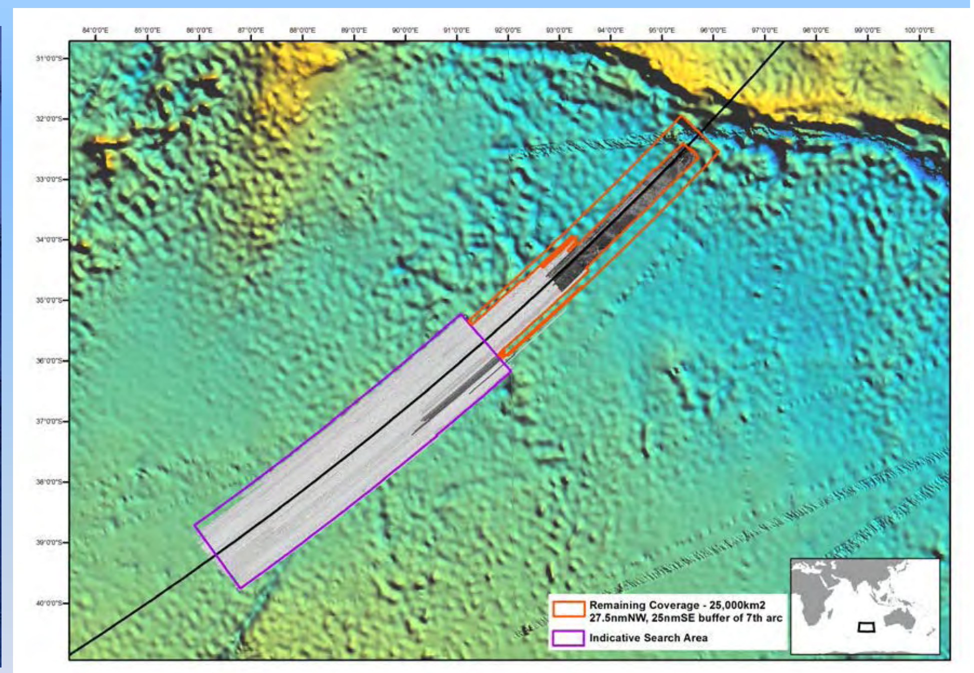
Autonomous Underwater Vehicle

Recommended Area for Further Search

- November 2016: Given the failure of the sonar search and using information from the MH370 debris, the analysis team computed the posterior and recommend a 25,000 km^2 area for continued search[1,2] (recommendation not taken) **Search officially called off on January 17, 2017**



Posterior



Recommended Search Area: Orange Outline

1. MH370 - First Principles Review, ATSB Transport Safety Report, Aviation External Investigation: AE 2014-054, 20 December 2016.
2. MH370 – Search and debris examination update, ATSB Transport Safety Report, Aviation External Investigation: AE-2014-054, 2 November 2016

Summary

- **Benefits of Bayesian search planning**
  - Provides a principled method of incorporating all information (objective and subjective) about the location of a search object to produce a probability distribution (PDF) on object location
  - Produces a PDF that provides the basis for efficient allocation of search effort
  - Allows incorporation of feedback from the search
    - Unsuccessful search and additional information can be incorporated via Bayes Theorem to produce posterior PDF which becomes the basis for planning next increment of effort
  - Provides analytical estimates of the effort required to achieve a given level of probability of success and measures the effectiveness of search to date.
- **Bayesian methodology has been applied successfully** to a number of difficult searches involving lost aircraft and other objects
- **The US Coast Guard routinely uses this methodology in its SAROPS** system for planning searches for people and boats lost at sea.
  - You don’t have to be an Bayesian search expert to use it.

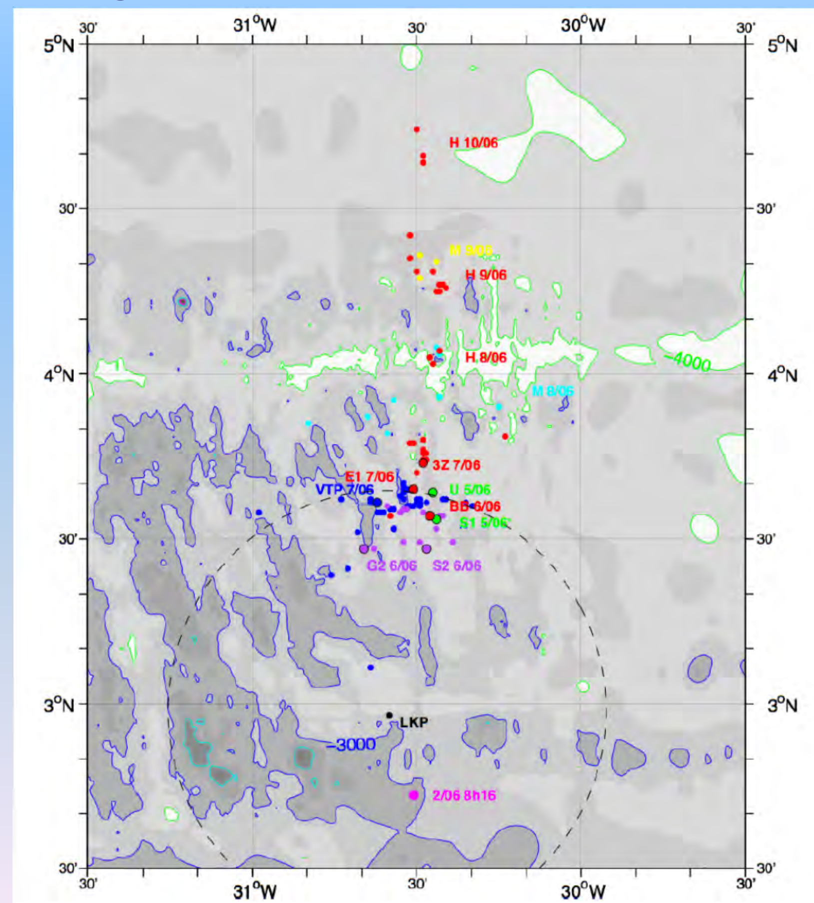
References

- *Bayesian Methods in the Search for the MH370* by Davey, Gordon, Holland, Rutten, and Williams, Springer 2016
- MH370 - First Principles Review, ATSB Transport Safety Report, Aviation External Investigation: AE 2014-054, 20 December 2016.
- MH370 – Search and debris examination update, ATSB Transport Safety Report, Aviation External Investigation: AE-2014-054, 2 November 2016
- The search for MH370 by C Ashton, AS Bruce, C Colledge, and M Dickinson, *The Journal of Navigation*, The Royal Institute of Navigation, 2014.
- Search for the wreckage of Air France Flight AF 447, by LD Stone, CM Keller, TM Kratzke, and JP Strumpfer, *Statistical Science* 2014, 29:69-80
- Search for the *SS Central America*: mathematical treasure hunting by LD Stone, *Interfaces*, 1992, 22:32-54
- Operations analysis during the underwater search for *Scorpion,* by H R Richardson and L D Stone, *Naval Research Logistics*, 1971, 18:141-157
- Search and rescue optimal planning system by T M Kratzke, L D Stone, and J R Frost, *Proceedings of 13th International Conference on Information Fusion*, Edinburgh, Scotland, 26 -29 July, 2010.

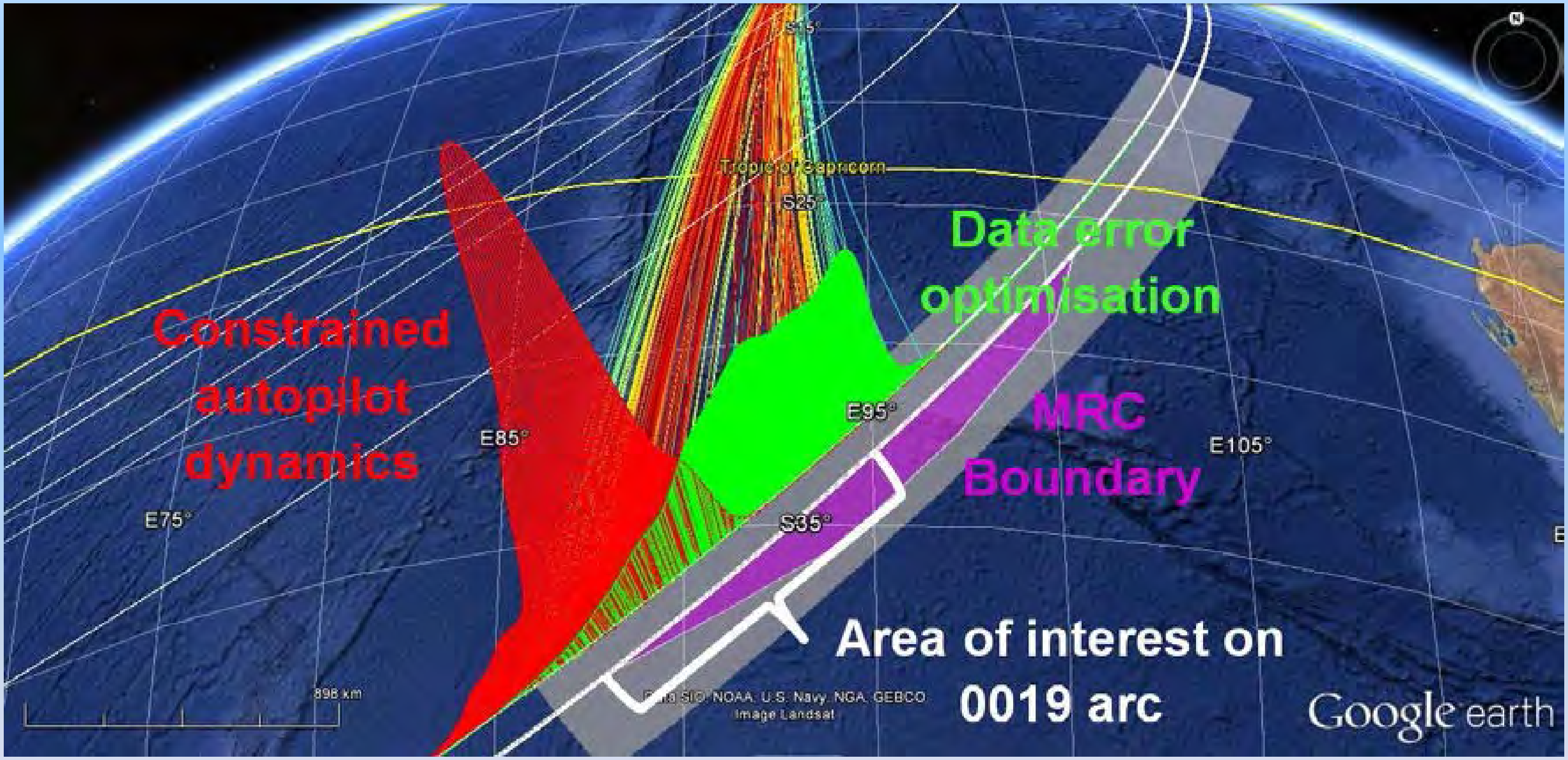
BACKUP SLIDES

Search for Surface Debris

- Air and surface search for floating debris and signs of survivors begins 1 June 2009
  - On 6 June the first bodies and floating debris are found 38 NM north of LKP

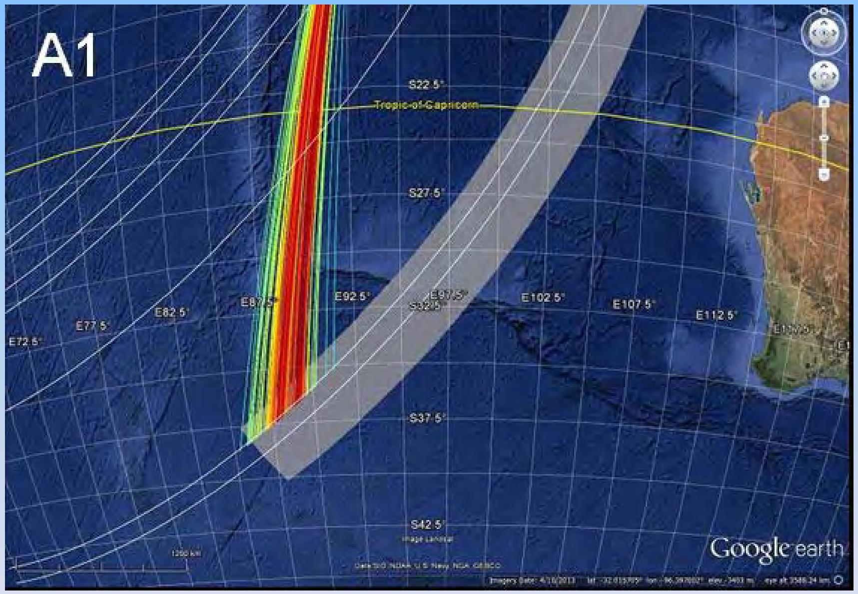


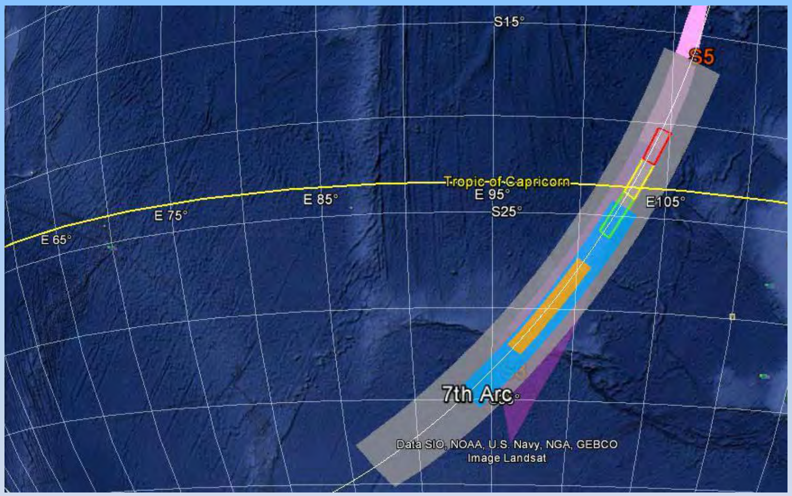
Combined Constrained Autopilot and Data Error Minimization Analysis



Results

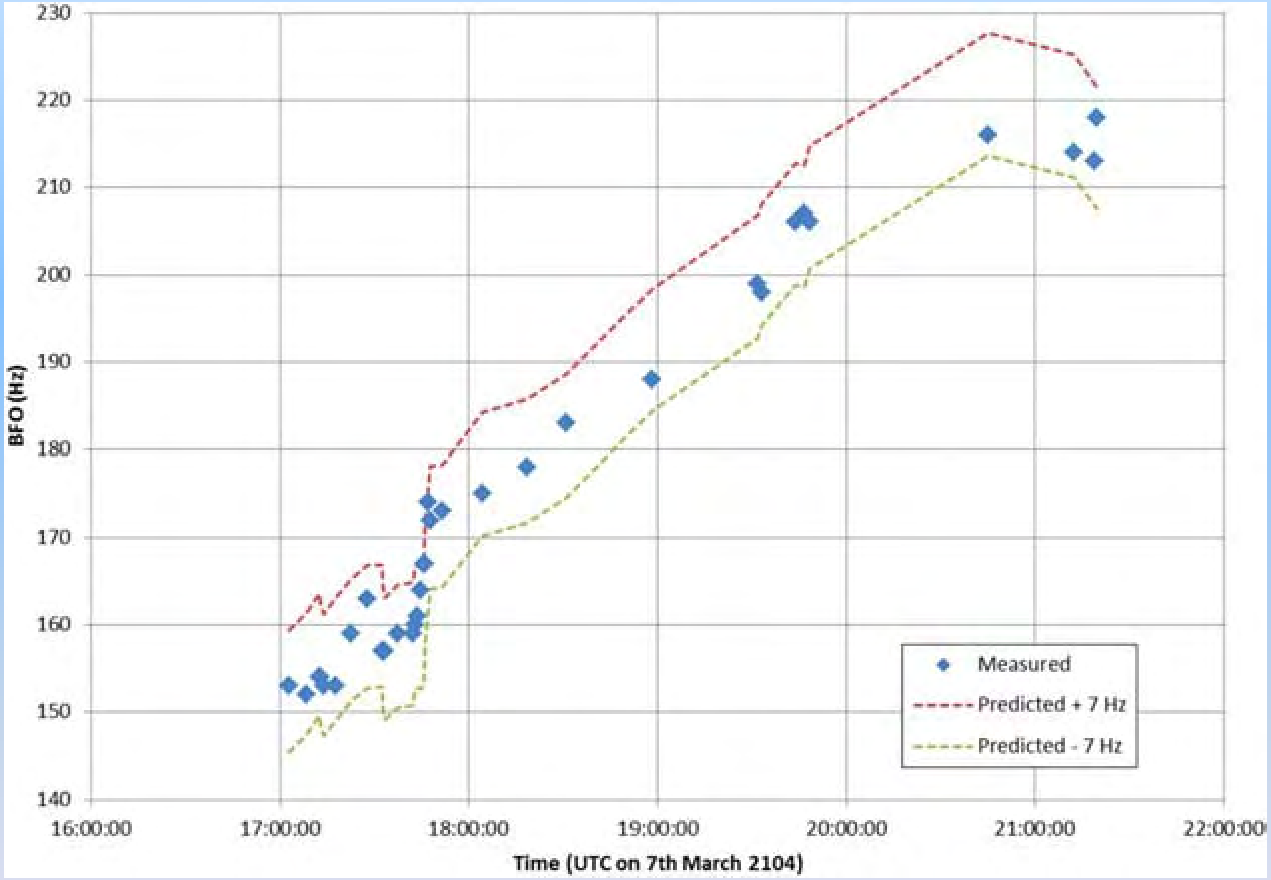
- The result would look something like the following figure from the 8 Oct 2014 ATSB report,
  - but with the paths extended to crash points to produce a probability distribution on crash location
  - Since we don't have aircraft paths, hard to say how this would compare with search areas given by ATSB





Search areas:
Grey = wide
Blue = medium
Orange = priority

BFO Error Analysis



Measured vs Predicted BFO for flight MH 21
(Kuala Lumpur to Amsterdam) on March 7, 2014