Decision Making
in Robots and Autonomous Agents

Control: How should a robot stay "in place"?

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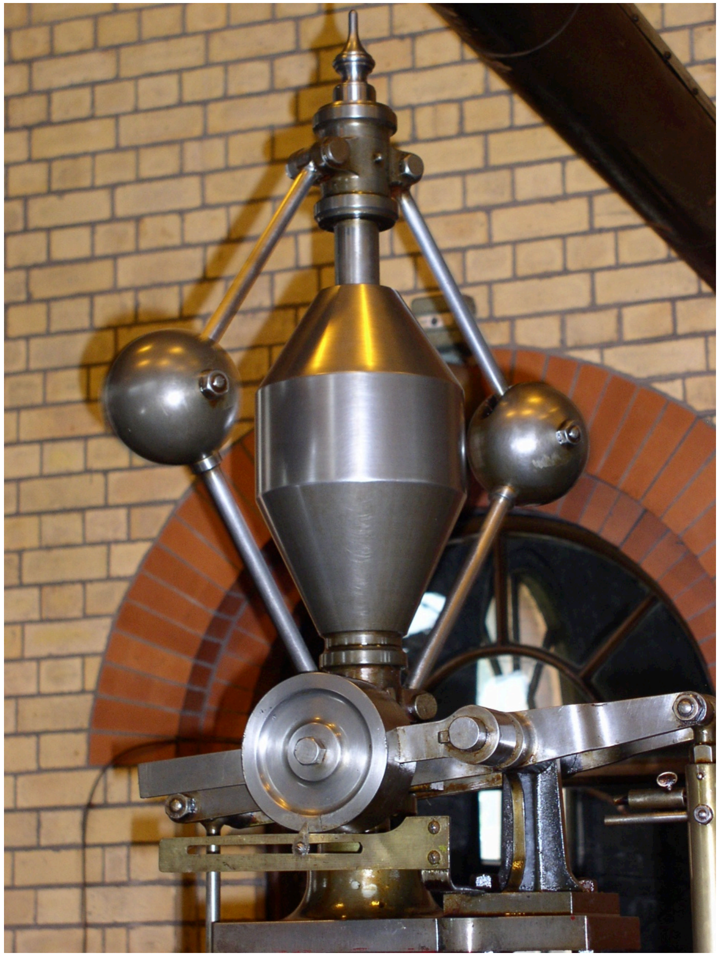
19 January, 2018

Objectives of this Lecture

- Give a selective recap of key ideas from control theory, as a very first approach to the "synthesis of robot motion"
  - If you have studied control before, you should recognize the concepts although the narrative may still be new
  - If you have not studied control before, this should give you useful background that will help contextualize other concepts to come later

- After a first half surveying a few key concepts, we will spend the second half of the lecture thinking concretely about the design of a controller for one particular model system: inverted pendulum

Centrifugal Governor (James Watt, 1788)



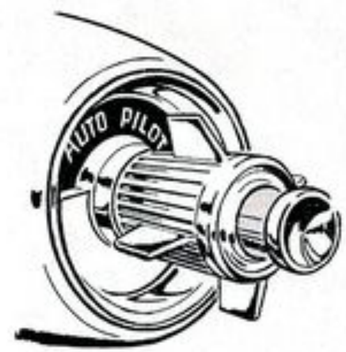
Not Only of Historical Interest…

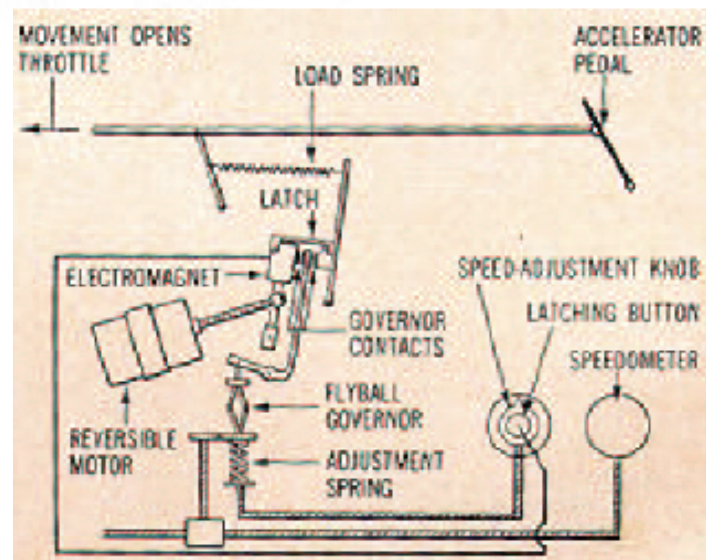
. . . with AUTO PILOT
for *CHRYSLER and IMPERIAL*

Just set the convenient instrument panel dial to your desired speed. Then drive in your usual manner. When you reach the pre-set speed you feel a gentle nudge of the accelerator on your foot telling you you've reached your desired speed.

For completely automatic control, pull the control knob when you feel the nudge of the pedal and remove your foot from the accelerator. Then, drive relaxed with your eyes on the road.

A touch of your brake pedal instantly returns the control to manual. To return to automatic control, just accelerate until you feel the nudge and remove your foot from the accelerator.





How does a Governor Work? Proportional Control

- A feedback system that controls the speed of an engine by regulating the amount of fuel (or working fluid) admitted
- Goal is to maintain a near-constant speed, irrespective of the load or fuel-supply conditions.

A sequence of operations:

1) Power is supplied to the governor from the engine's output shaft. The governor is connected to a throttle valve that regulates the flow of working fluid (steam) supplying the prime mover.

How does a Governor Work? Proportional Control

2) As the speed of the prime mover increases, the central spindle of the governor rotates at a faster rate and the kinetic energy of the balls increases.

3) This allows the two masses on lever arms to move outwards and upwards against gravity.

4) If the motion goes far enough, this motion causes the lever arms to pull down on a thrust bearing, which moves a beam linkage, which reduces the aperture of a throttle valve.

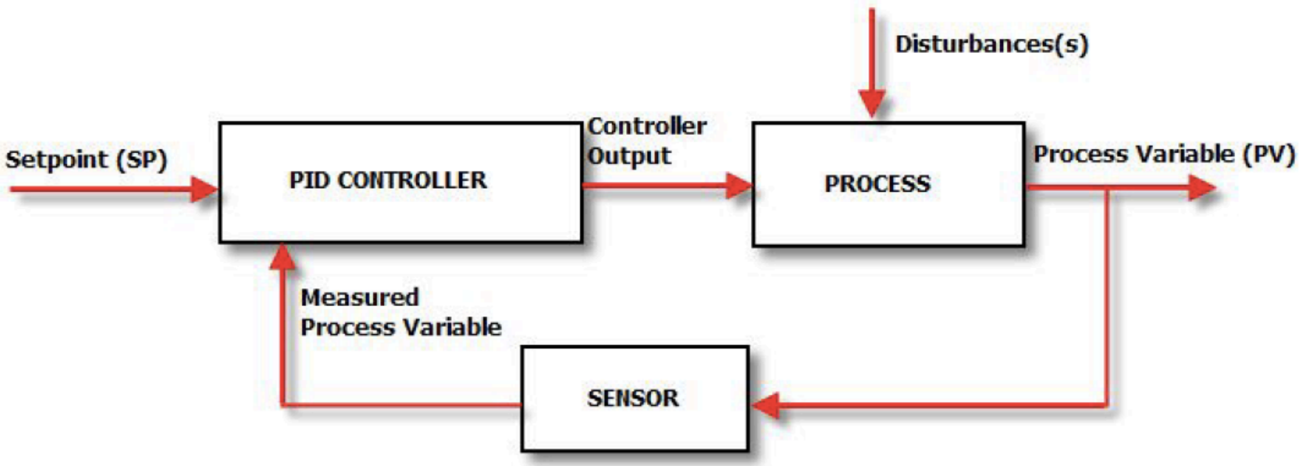
5) The rate of working-fluid entering the cylinder is thus reduced and the speed of the prime mover is controlled, preventing over-speeding.

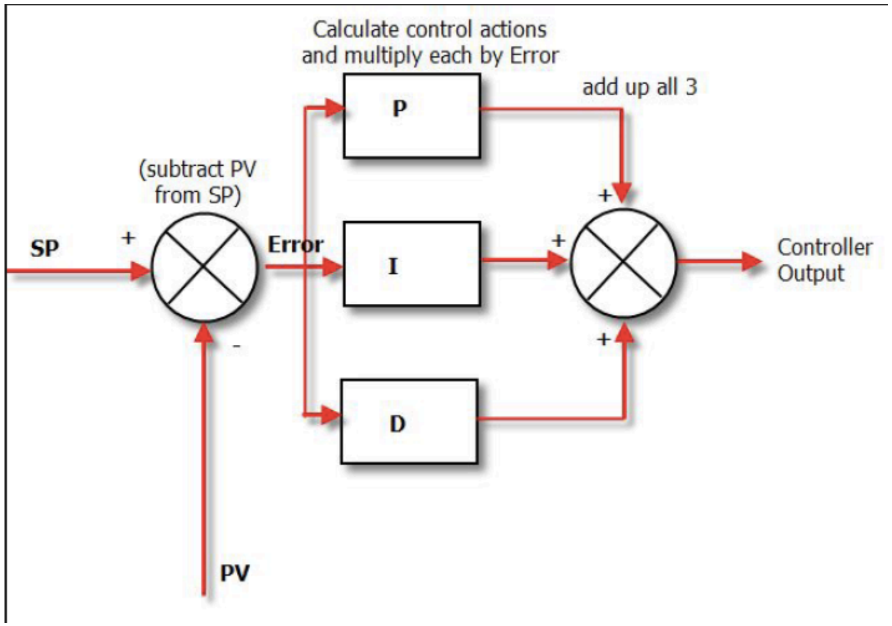
Proportional Control

- We want to hold system "in place" – in this case, at a certain rate of flow
- When flow exceeds desired value, the mechanism applies a correction which is proportional to the excess

- This idea of regulation is quite valuable in all engineered systems
- However, the quantity being regulated is not always flow
- How to write down the principle mathematically?
  - We also need to say how to describe the system

PID Controllers





Proportional-Integral-Derivative Control

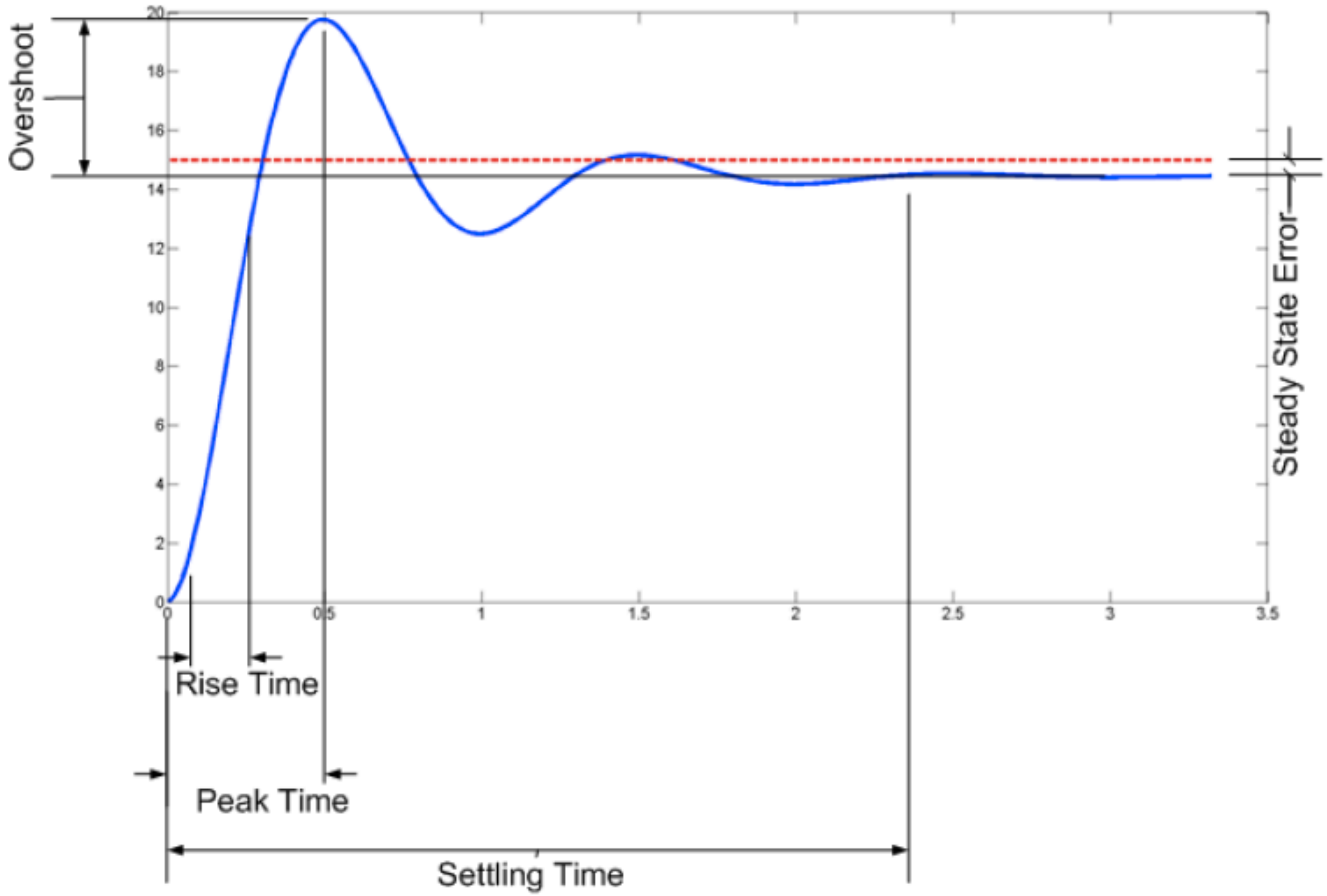
- The control signal, $u(t)$, is given in terms of the error $e(t)$ as,

$$u(t) = K_p e(t) + K_i \int_{t_0}^{t} e(\tau) d\tau + K_d \dot{e}(t)$$

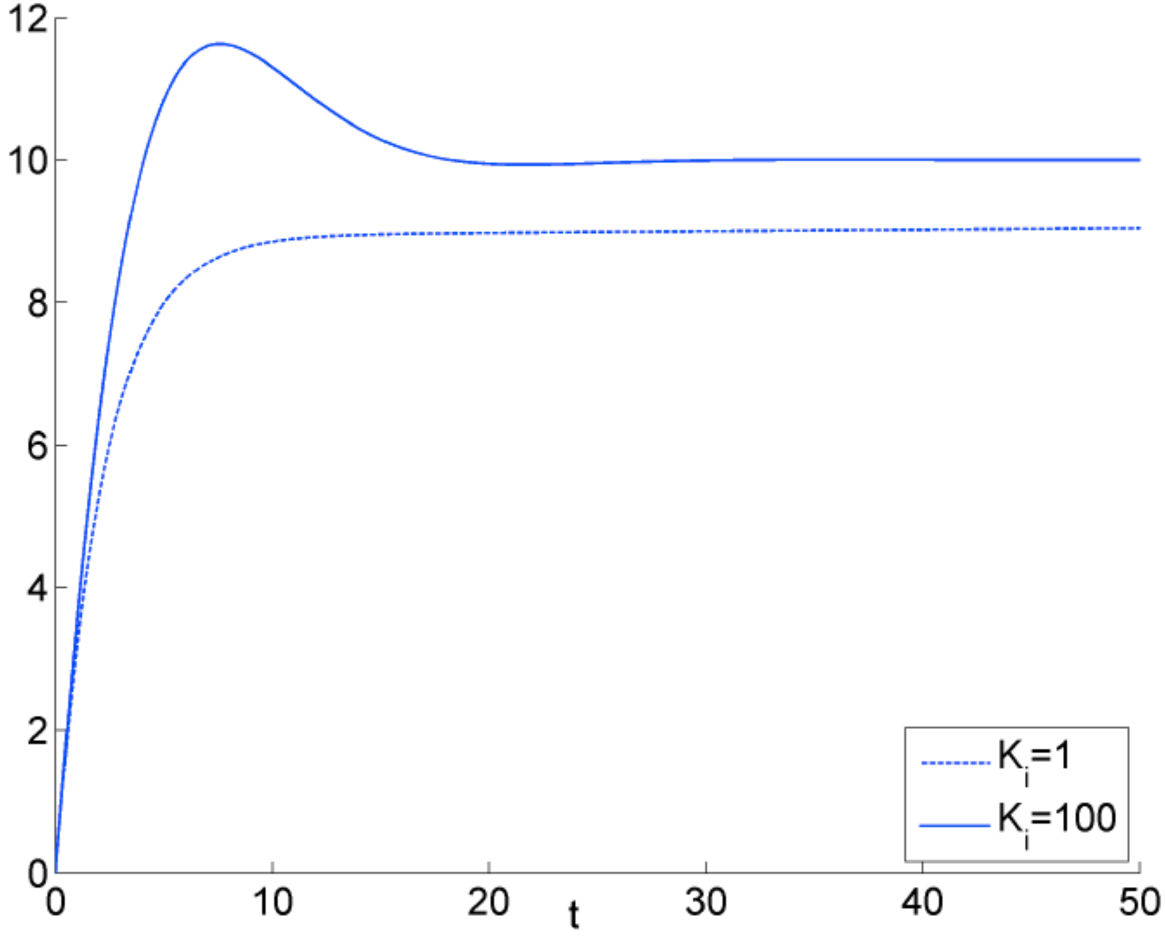
- This simple algorithm is most useful when processes are known to be stable and not very oscillatory
  - Parameters may not be well known, however

- Why is each term needed?
- How could we set the scale factors (the Ks)?

Typical Step Response of 2nd Order System with Proportional Control



Step Response with Different Levels of Integral Gain (Setpoint = 10)



Effects of Different Components

Control Action	Rise Time	Overshoot	Settling Time	Steady State Error
Increasing K_p	reduces	increases	small change	reduces
Increasing K_i	reduces	increases	increases	eliminates
Increasing K_d	small change	reduces	reduces	small change

Many Design Heuristics, e.g., Ziegler-Nichols Rules (1942)

- Trial and error procedure, entirely empirical
- Gradually reduce proportional gain alone until the system begins to oscillate (with loop gain, K_u, and period, T_u)
- Then, set the gains to be:

$$K_p = \frac{1}{2} K_u$$

$$K_i = \frac{2}{T_u} K_p$$

$$K_d = \frac{T_u}{8} K_p$$

- How to think about design and dynamics, more generally?

Linear Time Invariant (LTI) Systems

- Consider the simple spring-mass-damper system:
- The force applied by the spring is $F_s = -kz(t)$
- Correspondingly, for the damper: $F_d = \gamma \dot{z}(t)$
- The combined equation of motion of the mass becomes:

$$x\ddot{z}(t) = -\gamma \dot{z}(t) - kz(t)$$

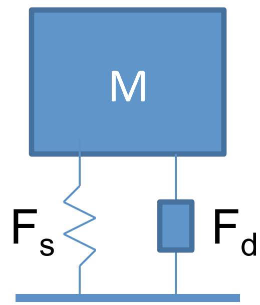
- One could also express this in state space form:

$$x(t) = [x_1(t), x_2(t)]' = [z(t), \dot{z}(t)]'$$

$$\dot{x}(t) = \begin{pmatrix} \dot{z}(t) \\ \ddot{z}(t) \end{pmatrix} = \begin{pmatrix} x_2(t) \\ -\frac{1}{m}(\gamma x_2(t)) + kx_1(t) \end{pmatrix}$$

Linear ODE ←

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & \frac{\gamma}{m} \end{pmatrix} x(t) = Ax(t)$$



Solution of a Linear ODE

$\dot{x} = kx,\ x \in \mathbb{R}$

For initial condition $\phi(0) = x_0$, the solution is $\phi(t) = e^{kt}x_0$

i.e., time evolution of state is given by operator $g^t = e^{kt}$, with velocity $\mathbf{v} = kt$

This type of "exponential term" is a feature of all linear dynamical systems

The multivariate case $x(t) = e^{A(t-t_0)}x_0$

$$\boxed{e^{A(t-t_0)}} = \sum_{i=0}^{\infty} \frac{A^i (t-t_0)^i}{i!} = I_{n\times n} + A(t-t_0) + \frac{A^2(t-t_0)^2}{2!} + \ldots$$

This is state transition matrix $\phi(t)$:
In linear algebra, there are numerous ways to compute this…

Example

Determine the matrix exponential, and hence the state transition matrix, and the homogeneous response to the initial conditions $x_1(0) = 2$, $x_2(0) = 3$ of the system with state equations:

$$\begin{aligned} \dot{x}_1 &= -2x_1 + u \\ \dot{x}_2 &= x_1 - x_2. \end{aligned}$$

The system matrix is

$$\mathbf{A} = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}.$$

Example, contd.

$$
\begin{aligned}
\boldsymbol{\Phi}(t) &= e^{\mathbf{A}t} \\
&= \left(\mathbf{I} + \mathbf{A}t + \frac{\mathbf{A}^2 t^2}{2!} + \frac{\mathbf{A}^3 t^3}{3!} + \ldots + \frac{\mathbf{A}^k t^k}{k!} + \ldots\right) \\
&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} t + \begin{bmatrix} 4 & 0 \\ -3 & 1 \end{bmatrix} \frac{t^2}{2!} \\
&\qquad + \begin{bmatrix} -8 & 0 \\ 7 & -1 \end{bmatrix} \frac{t^3}{3!} + \ldots \\
&= \begin{bmatrix} 1 - 2t + \frac{4t^2}{2!} - \frac{8t^3}{3!} + \ldots & 0 \\ 0 + t - \frac{3t^2}{2!} + \frac{7t^3}{3!} + \ldots & 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \ldots \end{bmatrix}.
\end{aligned}
$$

$$
\boldsymbol{\Phi}(t) = \begin{bmatrix} e^{-2t} & 0 \\ e^{-t} - e^{-2t} & e^{-t} \end{bmatrix}
$$

Example, contd.

$$\mathbf{x}_h(t) = \boldsymbol{\Phi}(t)\mathbf{x}(0)$$

$$\begin{aligned} x_1(t) &= x_1(0)e^{-2t} \\ x_2(t) &= x_1(0)\left(e^{-t} - e^{-2t}\right) + x_2(0)e^{-t}. \end{aligned}$$

$$\begin{aligned} x_1(t) &= 2e^{-2t} \\ x_2(t) &= 2\left(e^{-t} - e^{-2t}\right) + 3e^{-t} \\ &= 5e^{-t} - 2e^{-2t}. \end{aligned}$$

Basic Notion: Stability

- Simple question:

  Given the system, $\dot{x}(t) = Ax(t)$

  where in phase space, $(x, \dot{x})$, will it come to rest?

Any guesses?
Think about solution in previous slide…

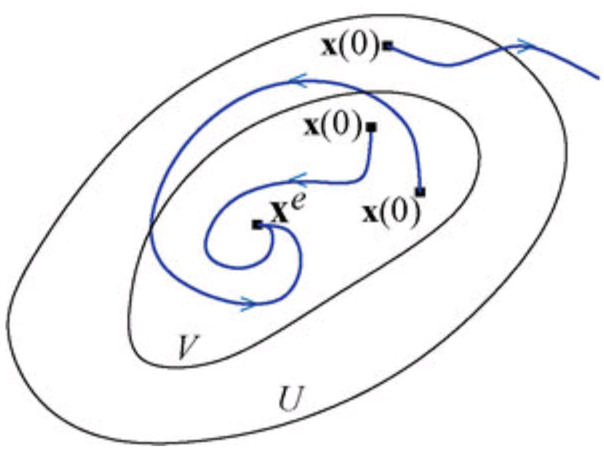
Do you know what this is? (Whiteboard)

- This point is called the equilibrium point
  - If initialized there, dynamics will not take it away
  - If perturbed, system will eventually return and stay there

Stability

An equilibrium position $x = 0$ is *stable* (in Lyapunov's sense) if given $\epsilon > 0$, $\exists \delta > 0$ (not dependent on t), s.t. $\forall x_0, |x_0| < \delta$ the solution satisfies $|\phi(t)| < \epsilon$, $\forall t > 0$

Asymptotic stability: Lyapunov stabile and $\lim_{t \to +\infty} \phi(t) = 0$



Stability for an LTI System, $\dot{x}(t) = Ax(t)$

Unforced (homogeneous) response: $x_i(t) = \sum_{j=1}^{n} m_{ij} e^{\lambda_j t}$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ m_{21} & m_{22} & \dots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \dots & m_{nn} \end{bmatrix} \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \\ \vdots \\ e^{\lambda_n t} \end{bmatrix}$$

$$\mathbf{x_h}(t) = \mathbf{M} \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \\ \vdots \\ e^{\lambda_n t} \end{bmatrix}$$

Stability for an LTI System

If you differentiate the homogeneous response, $\dfrac{dx_i}{dt} = \sum_{j=1}^{n} \lambda_j m_{ij} e^{\lambda_j t}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} \lambda_1 m_{11} & \lambda_2 m_{12} & \dots & \lambda_n m_{1n} \\ \lambda_1 m_{21} & \lambda_2 m_{22} & \dots & \lambda_2 m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1 m_{n1} & \lambda_2 m_{n2} & \dots & \lambda_n m_{nn} \end{bmatrix} \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \\ \vdots \\ e^{\lambda_n t} \end{bmatrix}.$$

The system being considered is $\dot{x}(t) = Ax(t)$, so:

$$\begin{bmatrix} \lambda_1 m_{11} & \lambda_2 m_{12} & \dots & \lambda_n m_{1n} \\ \lambda_1 m_{21} & \lambda_2 m_{22} & \dots & \lambda_2 m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1 m_{n1} & \lambda_2 m_{n2} & \dots & \lambda_n m_{nn} \end{bmatrix} \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \\ \vdots \\ e^{\lambda_n t} \end{bmatrix} = \mathbf{A} \begin{bmatrix} m_{11} & m_{12} & \dots & m_{1n} \\ m_{21} & m_{22} & \dots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \dots & m_{nn} \end{bmatrix} \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \\ \vdots \\ e^{\lambda_n t} \end{bmatrix}$$

LTI Stability, in algebraic equations

- The above equation leads to an eigenvalue problem:

$$\lambda_i \mathbf{m}_i = \mathbf{A}\mathbf{m}_i \qquad\qquad i = 1, 2, \ldots, n.$$

$$[\lambda_i \mathbf{I} - \mathbf{A}]\,\mathbf{m}_i = 0$$

- For this to have nontrivial solutions:

Characteristic eqn.

$$\Delta(\lambda_i) = \det[\lambda_i \mathbf{I} - \mathbf{A}] = 0.$$

$$\lambda^n + a_{n-1}\lambda^{n-1} + a_{n-2}\lambda^{n-2} + \ldots + a_1\lambda + a_0 = 0$$

$$(\lambda - \lambda_1)(\lambda - \lambda_2)\ldots(\lambda - \lambda_n) = 0.$$

Stability: LTI System, $\dot{x}(t) = Ax(t)$

Theorem. *Let λ_i, $i \in \{1, 2, \ldots, n\}$ denote the eigenvalues of A. Let $re(\lambda_i)$ denote the real part of λ_i. Then the following holds:*

1. $x_e = 0$ *is stable if and only if* $re(\lambda_i) \leq 0$, $\forall i$
2. $x_e = 0$ *is asymptotically stable if and only if* $re(\lambda_i) < 0$, $\forall i$
3. $x_e = 0$ *is unstable if and only if* $re(\lambda_i) > 0$, *for some* i

For the spring-mass-damper example, the eigenvalues are:

$$\frac{\gamma \pm \sqrt{\gamma^2 - 4km}}{2m}$$

With positive damping, we get asymptotic stability

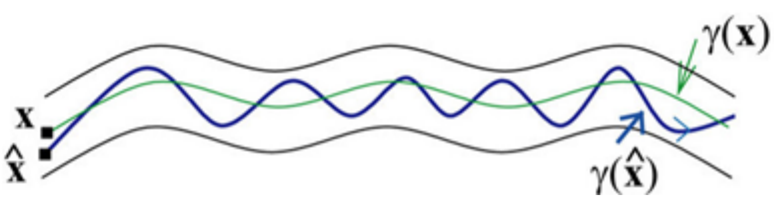
Exercise (ponder at home)

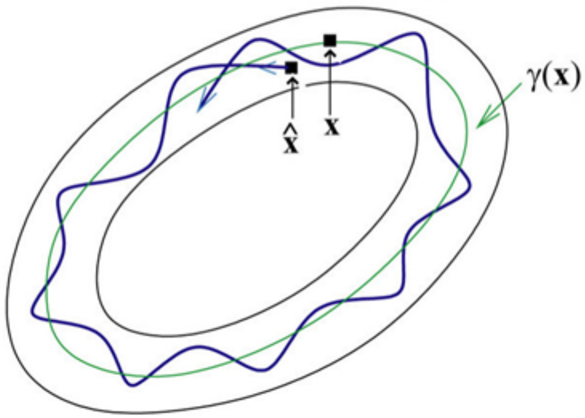
Can you visualize (i.e., draw the curve vs. time) state variables for the case of asymptotic stability, instability and the borderline in between?

Other Related Notions: Orbital Stability

- Stability doesn't only refer to being at rest at a point
  - could be defined in terms of staying in a subset, e.g., path

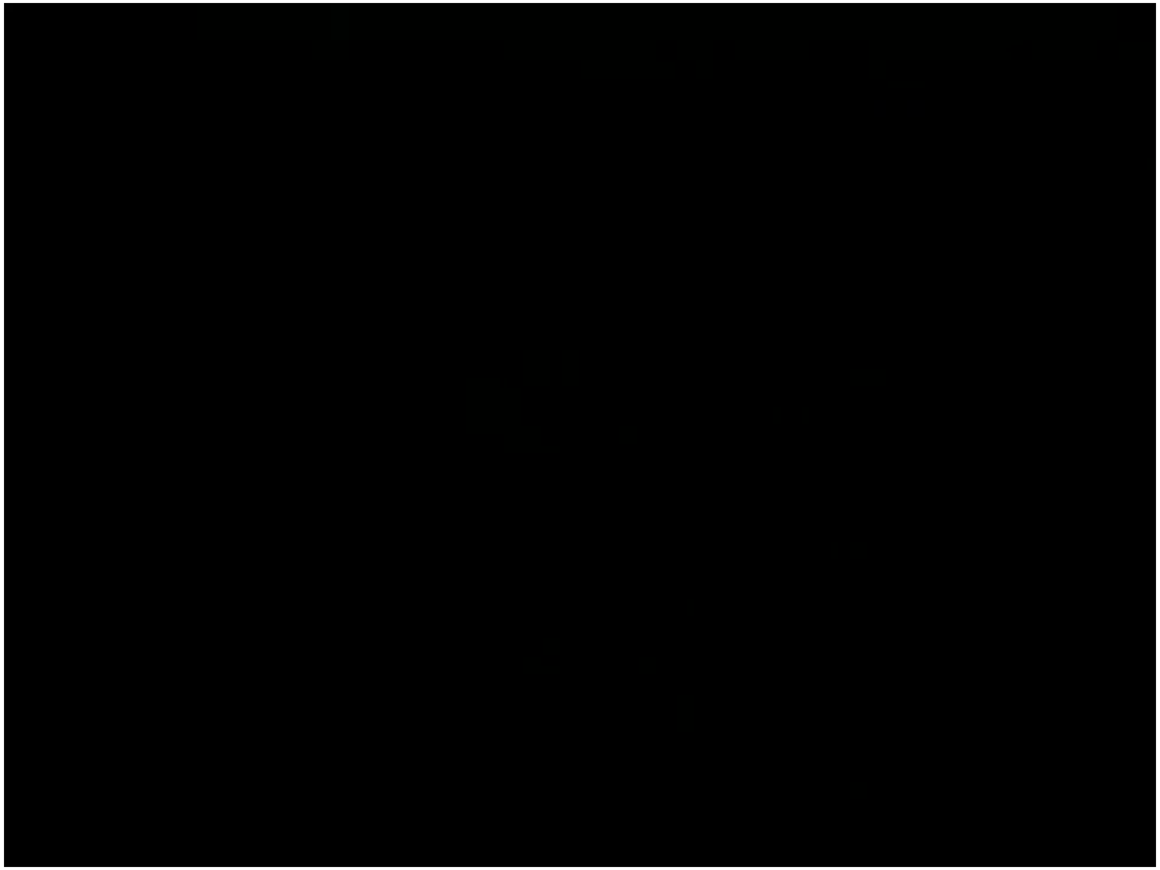
Definition. *An orbit $\gamma(x)$ is orbitally stable if for any $\epsilon > 0$, there is a neighbourhood V of x so that for all $\hat{x}$ in V, γx and $\gamma \hat{x}$ are ϵ-close.*
Loosely speaking, $|\gamma(x) - \gamma(\hat{x})| < \epsilon$ at all times.





Is Stability Really an Issue?

Some Aircrafts are *Designed* to be Statically Unstable! Why?



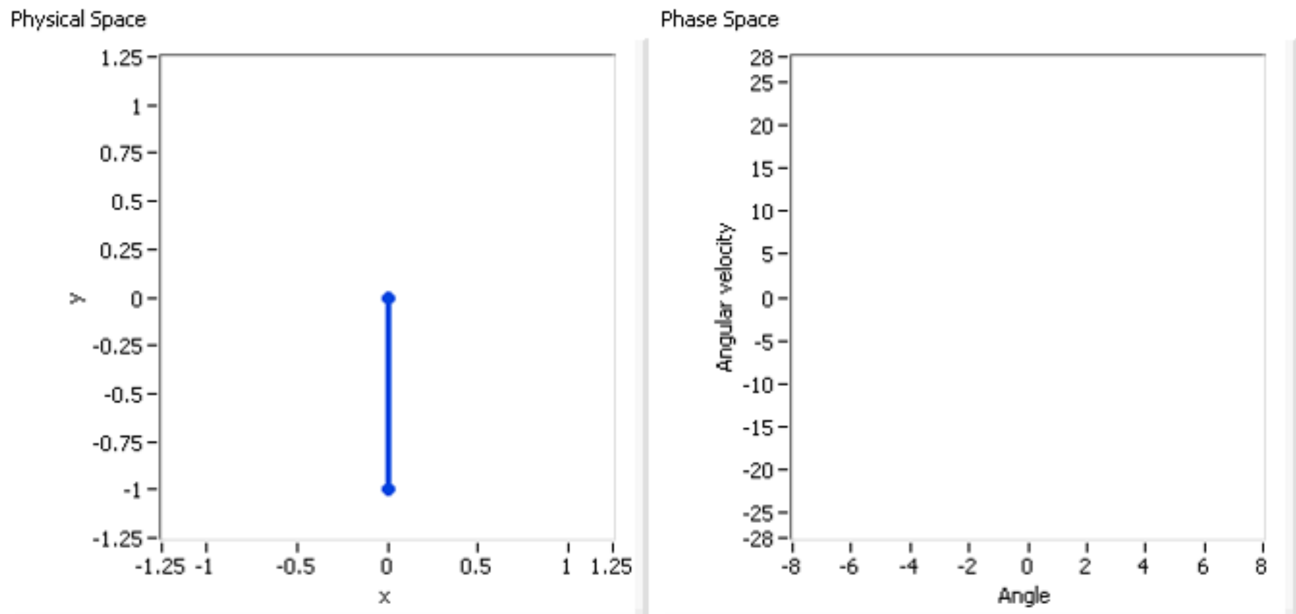
Jump to 2:07 for exciting bit!

[https://www.youtube.com/watch?v=2CUyoi634wc]

A Simple Complete Example: Inverted Pendulum

B.J. Kuipers, S. Ramamoorthy, Qualitative modeling and heterogeneous control of global system behavior. In C. J. Tomlin & M. R. Greenstreet (Eds.), Hybrid Systems: Computation and Control, LNCS 2289: 294-307, 2002.

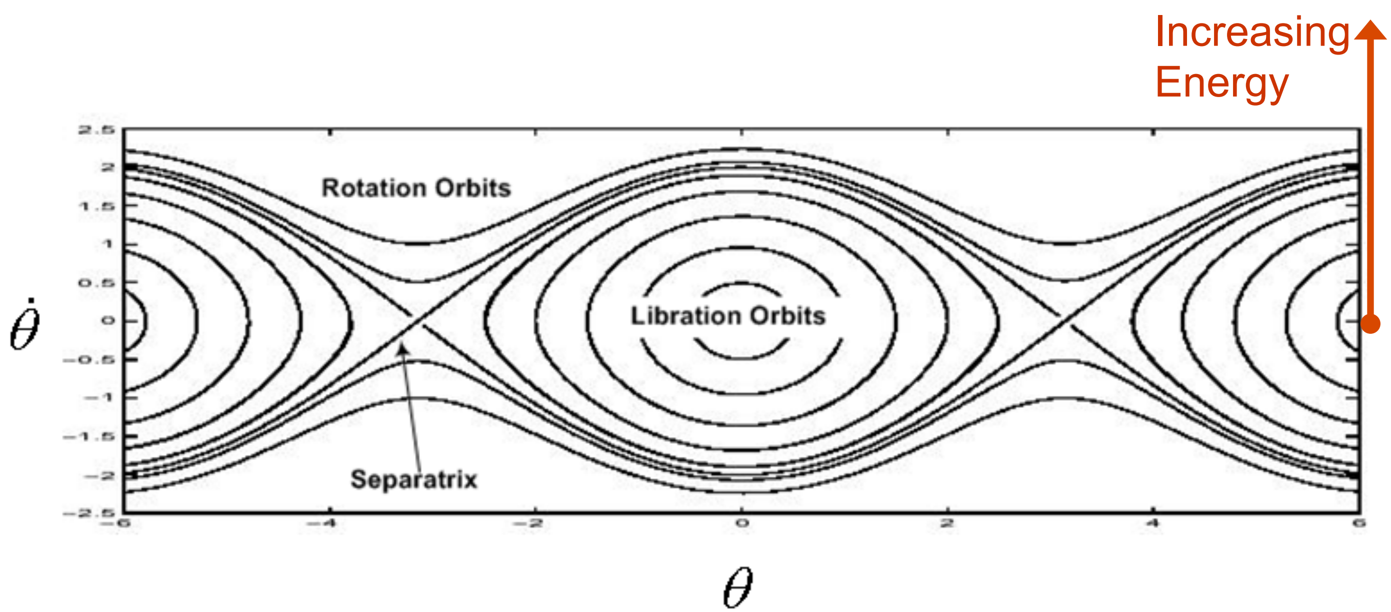
Pendulum Phase Space



- Phase space is organized into families (open sets) of trajectories
- Trajectories may be parameterized by a single variable: energy

Design Strategy: Use Natural Dynamics

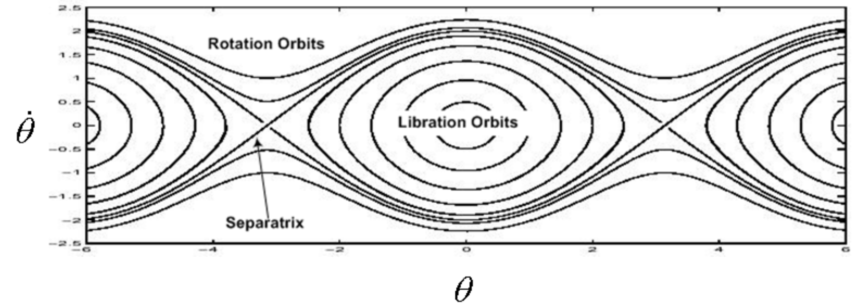
- Passively "ride" orbits ⇔ Energy Efficiency
- Parameterized families of trajectories ⇔ Flexibility
- Topology, structural stability ⇔ Robustness



Using Natural Dynamics for Motion Planning

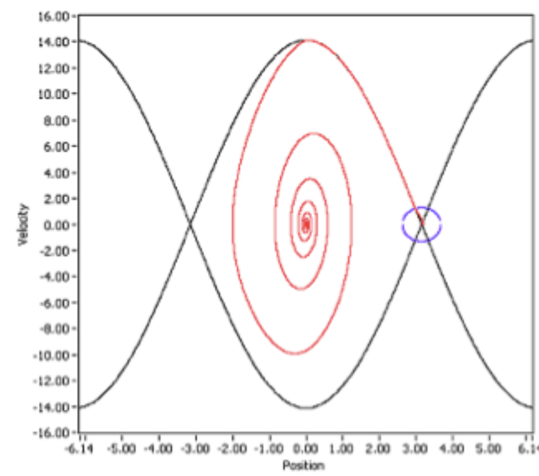
Generate trajectories, on-line,

- From the whole phase space
- To inverted position



Solution:

- Change *E* to move towards separatrix
- Two trajectory classes:
  pump (libration)/ spin (rotation)
- Ride the separatrix, once there



Let us now walk through this construction in some detail!

Remarks: Use of *Qualitative* Models

- A qualitative differential equation (QDE) expresses partial knowledge of a dynamical system.
  – One QDE describes a set of ODEs,
  – non-linear as well as linear systems.

- A QDE can express partial knowledge of a plant or a controller design.

- QSIM can predict all possible behaviors of all ODEs described by the given QDE.

Qualitative Design of a Heterogeneous Controller

- Design local models with the desired behavior.
- Identify qualitative constraints to guarantee the right transitions.
- Provide weak conditions sufficient to guarantee desired behavior.
  - Remaining degrees of freedom are available for optimization by any other criterion.
- Demonstrate with a global pendulum controller.
  - Local models: Pump, Balance, Spin.

Some Remarks about QSIM notation

- Each variable is a *reasonable* function.
  - Continuously differentiable, etc.
  - Range described by landmark values and intervals.
- Constraints link variables.
  - ADD, MULT, MINUS, D/DT
  - Monotonic functions: $y=f(x)$ for f in M_0^+
  - $[x]_0=\text{sign}(x)$
- Semi-quantitative bounds and envelopes.
- QSIM predicts all possible behaviors.
- Temporal logic model-checking can prove theorems about ODEs from QSIM prediction.

The Monotonic Damped Spring

- The spring is defined by Hooke's Law:

$$F = ma = m\ddot{x} = -k_1 x$$

- Include damping friction

$$m\ddot{x} = -k_1 x - k_2 \dot{x}$$

- Rearrange and redefine constants

$$\ddot{x} + b\dot{x} + cx = 0$$

- Generalize to QDE with monotonic functions

$$\ddot{x} + f(\dot{x}) + g(x) = 0$$

Lemma 1:
The Monotonic Damped Spring

Let a system be described by

$$\ddot{x} + f(\dot{x}) + g(x) = 0$$

where

$$f \in M_0^+ \text{ and } [g(x)]_0 = [x]_0$$

Then it is asymptotically stable at (0,0), with a Lyapunov function:

$$V(x,\dot{x}) = \tfrac{1}{2}\dot{x}^2 + \int_0^x g(x)\,dx$$

- Proof in the paper.

Lemma 2:
The Spring with Anti-Damping

Suppose a system is described by

$$\ddot{x} - f(\dot{x}) + g(x) = 0$$

where

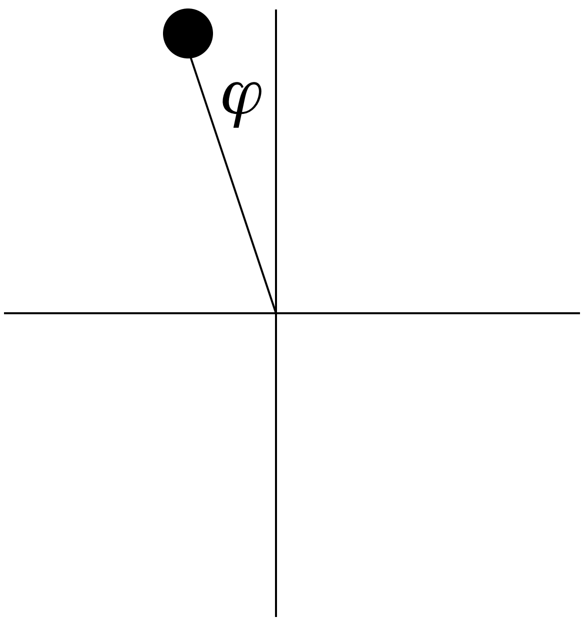
$$f \in M_0^+ \text{ and } [g(x)]_0 = [x]_0$$

Then the system has an unstable fixed-point at (0,0), and no limit cycle (i.e., stable periodic orbit).

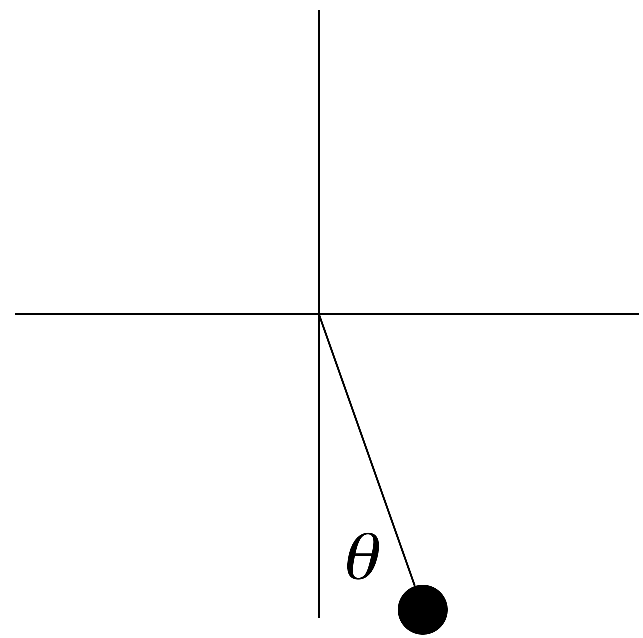
- Proof in the paper.

Pendulum Models: Equations of Motion

- Near the top.
- Near the bottom.



$$\ddot{\phi} + f(\dot{\phi}) - k\sin\phi = 0$$



$$\ddot{\theta} + f(\dot{\theta}) + k\sin\theta = 0$$

Balance the Pendulum

- Design the control input *u* to make the pendulum into a damped spring.

$$\ddot{\phi} + f(\dot{\phi}) - k\sin\phi + u(\phi,\dot{\phi}) = 0$$

- Define the **Balance** controller:

$$u(\phi,\dot{\phi}) = g(\phi)$$

such that

$$[g(\varphi) - k\sin\varphi]_0 = [\varphi]_0$$

- Lemma 1 shows that it converges to (0,0).

$$\ddot{\phi} + f(\dot{\phi}) + g(\phi) - k\sin\phi = 0$$

The Balance Region

- If the control action has upper bound $u_{\max}$ then gravity defines the limiting angle:

$$u_{\max} = k \sin \varphi_{\max}$$

- Energy defines maximum velocity at top:

$$\tfrac{1}{2}\dot{\phi}_{\max}^2 = \int_0^{\phi_{\max}} g(\phi) - k \sin \phi \, d\phi$$

- Define the **Balance** region:

$$\frac{\phi^2}{\phi_{\max}^2} + \frac{\dot{\phi}^2}{\dot{\phi}_{\max}^2} \leq 1$$

Pump the Hanging Pendulum

- Define the control action *u* to make the pendulum into a spring with negative damping.

- Define the **Pump** controller

$$u(\theta,\dot{\theta}) = -h(\dot{\theta})$$

such that

$$h - f \in M_0^+$$

gives

$$\ddot{\theta} - (h - f)(\dot{\theta}) + k\sin\theta = 0$$

- Lemma 2 proves it pumps without a limit cycle.

Slow the Spinning Pendulum

If the pendulum is spinning rapidly, define the **Spin** control law to augment natural friction:

$$u(\theta,\dot{\theta}) = f_2(\dot{\theta})$$

such that $f_2 \in M_0^+$

The Pump-Spin Boundary

- Prevent a limit-cycle behavior that cycles between **Pump** and **Spin** regions, overshooting **Balance**.
- Define the Pump-Spin boundary to be the *separatrix* of the undamped pendulum.
- **Pump** and **Spin** create what is known as a *sliding mode* controller
  - Special type of switching based control strategy
- The separatrix leads straight to the heart of **Balance.**

The Separatrix as Boundary

- A separatrix is a trajectory that begins and ends at the unstable saddle point of the undamped, uncontrolled pendulum:

$$\ddot{\theta} + k\sin\theta = 0$$

- Points on the separatrix have the same energy as the balanced pendulum:

$$KE + PE = \tfrac{1}{2}\dot{\theta}^2 + \int_0^\theta k\sin\theta\, d\theta = 2k$$

- Simplify to define the separatrix:

$$s(\theta,\dot{\theta}) = \tfrac{1}{2}\dot{\theta}^2 - k(1+\cos\theta) = 0$$

The Sliding Mode Controller

- Differentiate to see how *s* changes with time:

$$\dot{s} = -\dot{\theta} f(\dot{\theta}) - \dot{\theta}\, u(\theta, \dot{\theta})$$

- In the **Pump** region:

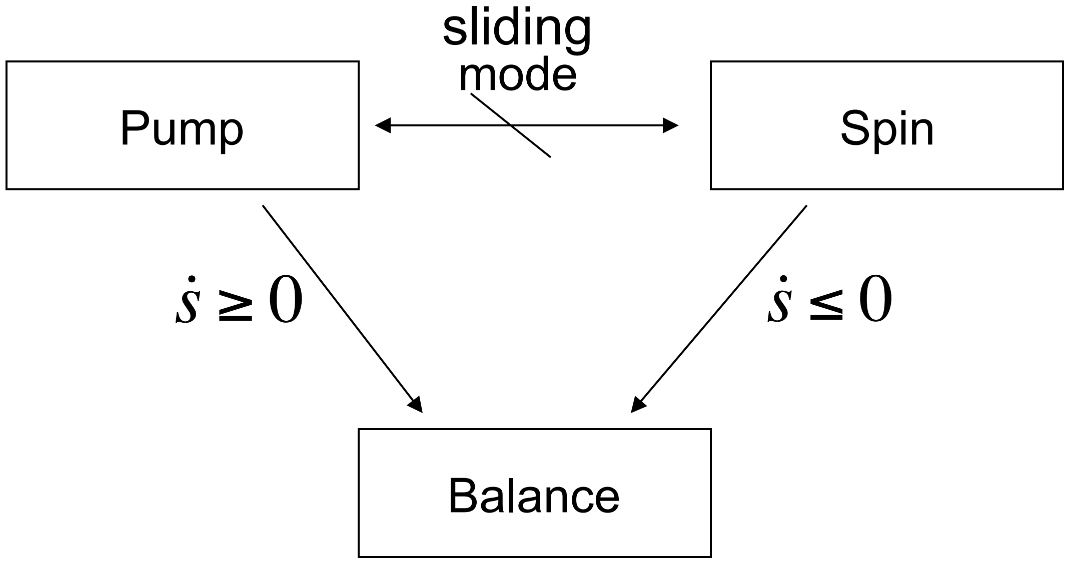
$$s < 0 \qquad \text{and} \qquad \dot{s} = \dot{\theta}(h - f)(\dot{\theta}) \geq 0$$

- In the **Spin** region:

$$s > 0 \qquad \text{and} \qquad \dot{s} = -\dot{\theta}(f + f_2)(\dot{\theta}) \leq 0$$

- Therefore, both regions approach $s = 0$

The Global Pendulum Controller



The Global Controller

The control law:

if **Balance**

$$u(\phi,\dot{\phi}) = g(\phi)$$

else if **Pump**

$$u(\theta,\dot{\theta}) = -h(\dot{\theta})$$

else **Spin**

$$u(\theta,\dot{\theta}) = f_2(\dot{\theta})$$

Constraints:

$$[g(\varphi) - k\sin\varphi]_0 = [\varphi]_0$$

$$h - f \in M_0^+$$

$$f_2 \in M_0^+$$

Pendulum Controller Example

System:

$$\ddot{\theta} + c\dot{\theta} + k\sin\theta + u(\theta,\dot{\theta}) = 0$$

$$c = 0.01, k = 10, u_{\max} = 4$$

Balance:

$$c_{11} = 0.4, c_{12} = 0.3$$

$$u = (c_{11} + k)(\theta - \pi) + c_{12}\dot{\theta}$$

$$\phi_{\max} = 0.4, \dot{\phi}_{\max} = 0.3$$

Spin:

$$u = c_2\dot{\theta}$$

$$c_2 = 0.5$$

Pump:

$$u = -(c + c_3)\dot{\theta}$$

$$c_3 = 0.5$$

Pendulum Example, cont.

The switching strategy:

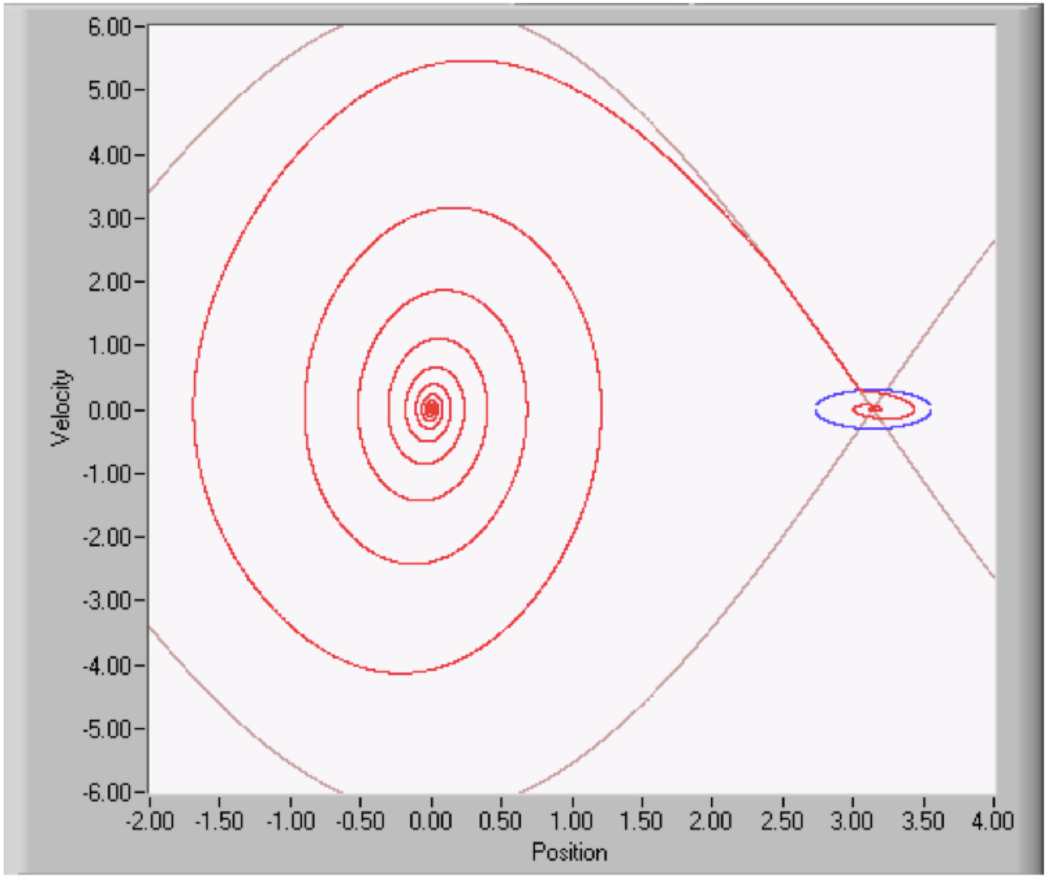
If $\alpha \leq 1$ then ***Balance***

else if $s < 0$ then ***Pump***

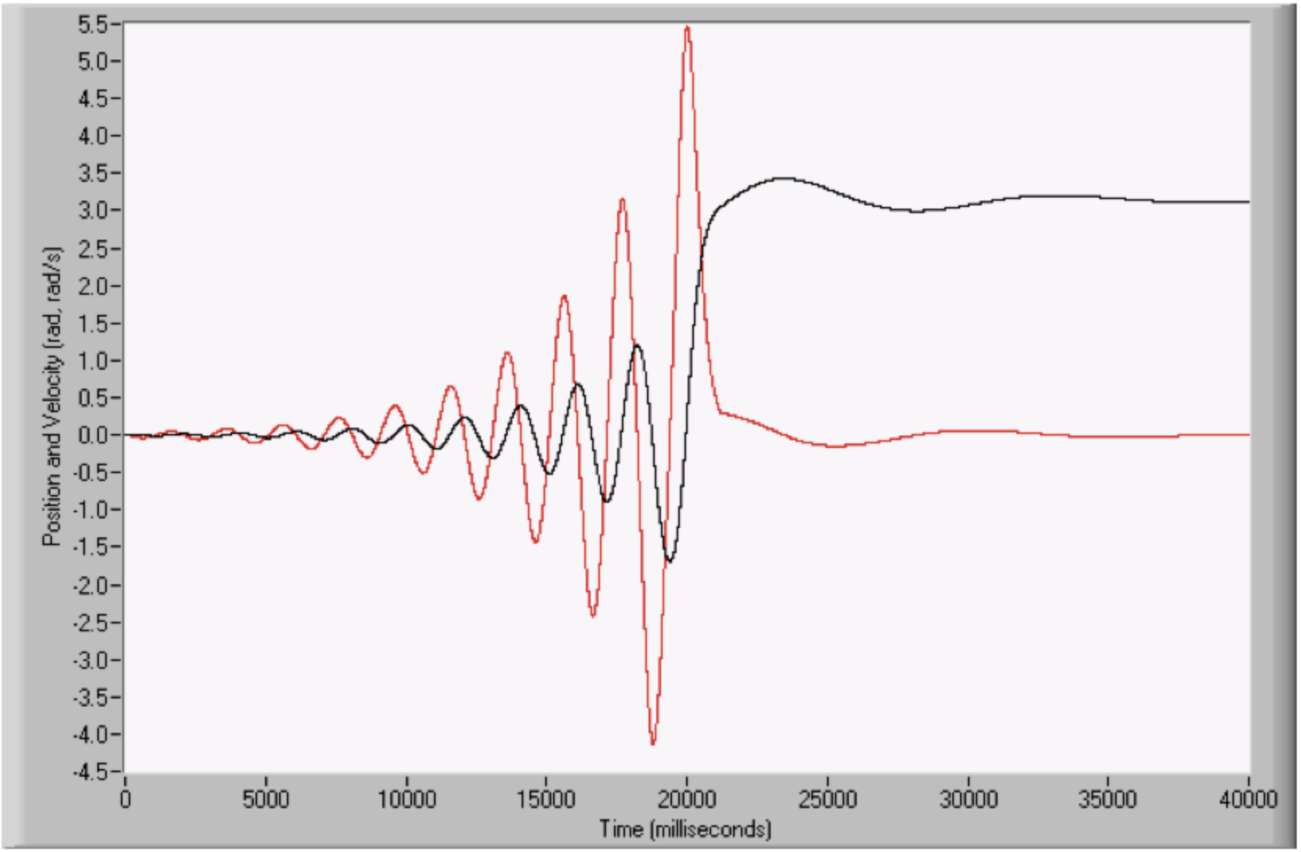
else ***Spin***

$$\alpha = \frac{\phi^2}{\phi_{\max}^2} + \frac{\dot{\phi}^2}{\dot{\phi}_{\max}^2} \qquad s = \frac{1}{2}\dot{\theta}^2 - k(1 + \cos\theta)$$

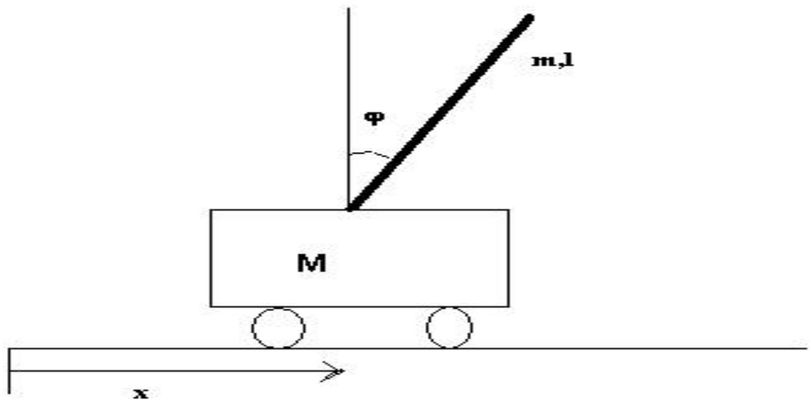
The Controlled Pendulum



The Controlled Pendulum



Now quite the full system, yet: The Cart-Pole System



$\ddot{x}$: Control Action

$\phi = \theta + \pi$

Cart Pole System:

$$\ddot{\phi} + f(\dot{\phi}) - k\sin\phi - \ddot{x}\cos\phi = 0$$

$$\ddot{\theta} + f(\dot{\theta}) + k\sin\theta + \ddot{x}\cos\theta = 0$$

Example:
Heterogeneous Cart-Pole Controller

The Cart-Pole System

$$\ddot{\theta} + f(\dot{\theta}) + k\sin\theta + \ddot{x}\cos\theta = 0$$

Compare to Pivot-Torque Pendulum System:

$$\ddot{\theta} + f(\dot{\theta}) + k\sin\theta + u(\theta,\dot{\theta}) = 0$$

Heterogeneous Cart-Pole Controller:

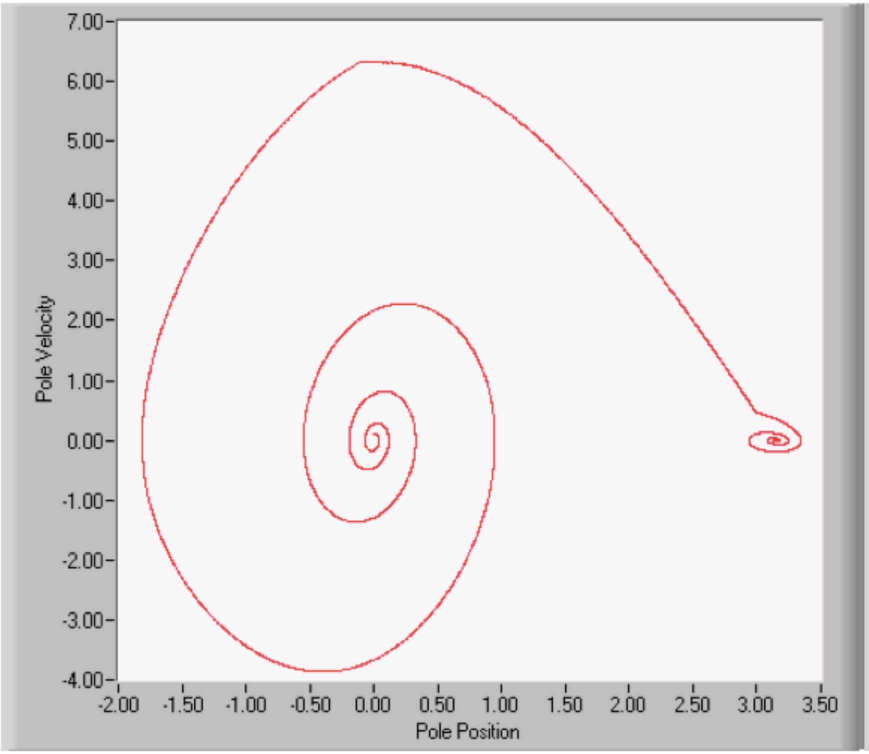
$$\ddot{x} = sat\left\{-\frac{f_1(\dot{x})}{\cos\theta} - \frac{g_1(x)}{\cos\theta} + \frac{u(\theta,\dot{\theta})}{\cos\theta}\right\}$$

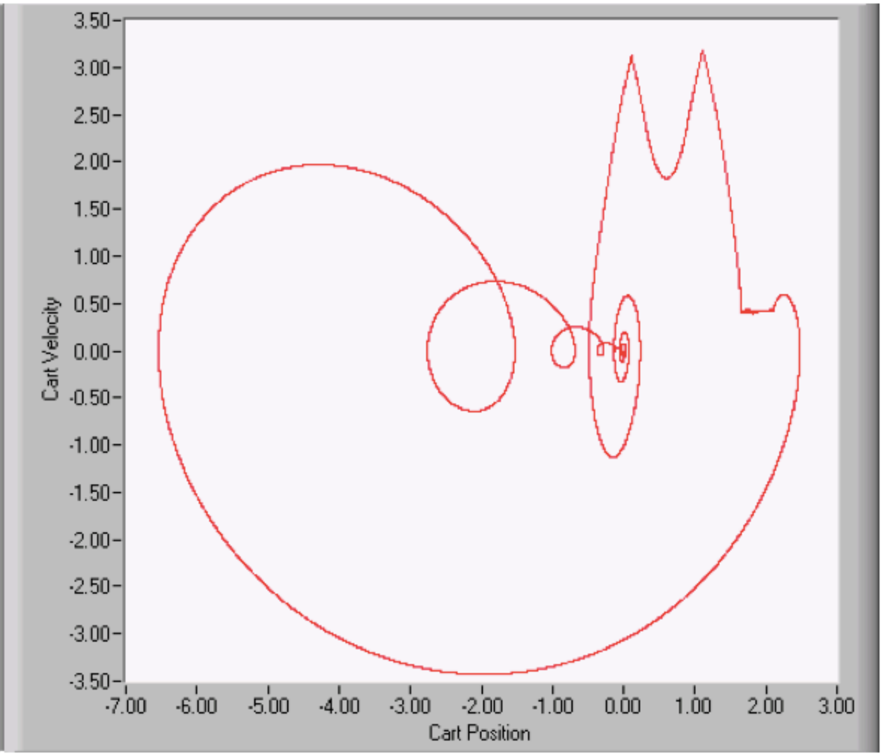
Heterogeneous Cart-Pole Controller

- Pivot torque controller stabilizes the pole (heterogeneous, 3 regions)
- Negative feedback stabilizes the cart, Lemma 1
- Combination of the two should preserve sliding mode for the heterogeneous pole controller
- We can derive the desired constraints:

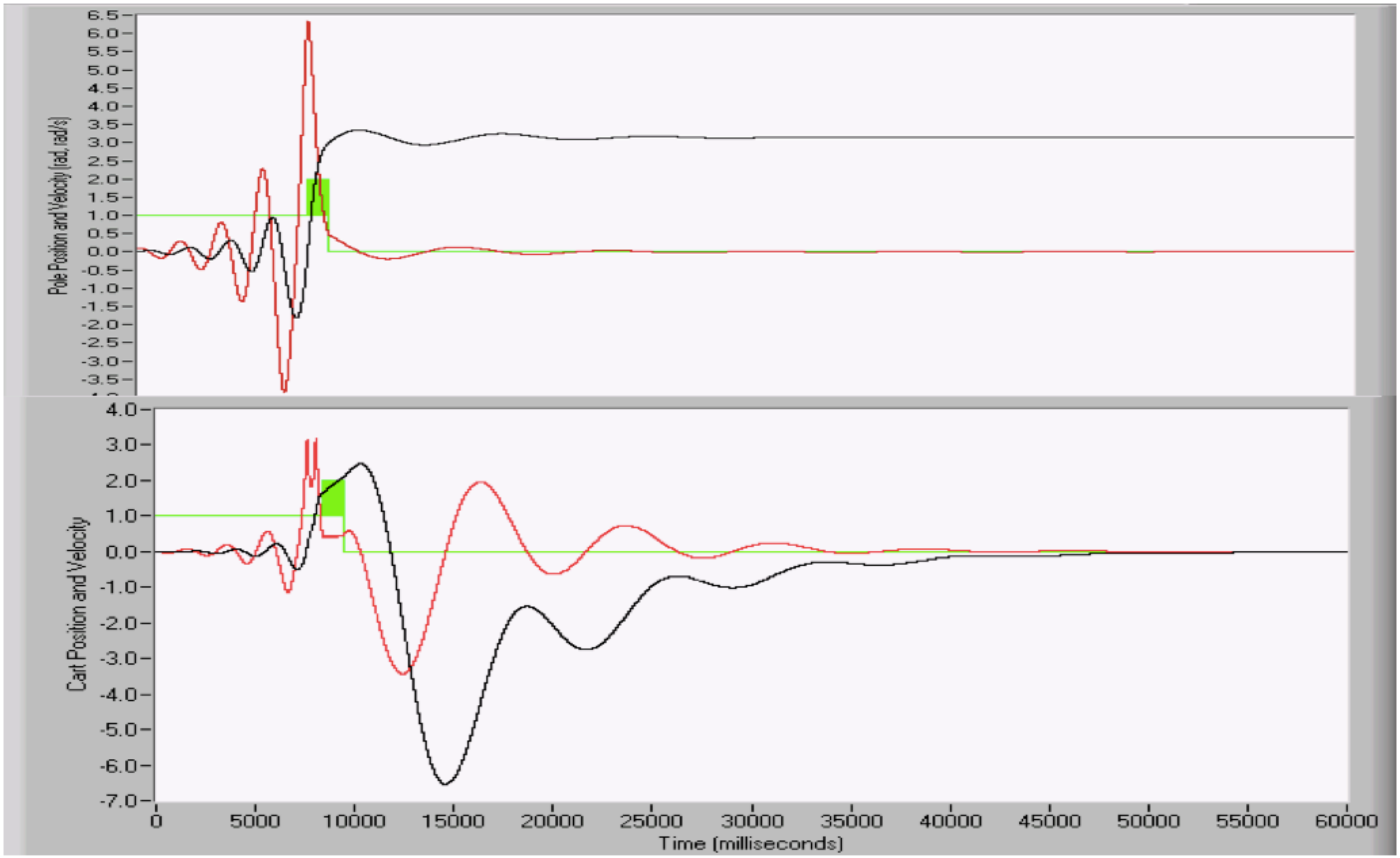
$$[(h-f)(\dot{\theta})+f_1(\dot{x})+g_1(x)]_0=[\dot{\theta}]_0$$
$$[(f+f_d)(\dot{\theta})-f_1(\dot{x})-g_1(x)]_0=[\dot{\theta}]_0$$

The Controlled Cart-Pole System





The Controlled Cart-Pole System



Take Home Messages from Example

- Real control problems in robotics require more complex specifications than is common in traditional control theory
- So-called hybrid systems (switching between different dynamical system models/regimes) is common
- Reasoning qualitatively with dynamical systems models provides a useful approach to specifying the controller for a large class of non-linear systems.
  - identifies weak sufficient conditions required for controller operation.
  - any instance of QDE will achieve the behavior. So the designer can optimize the control for any desired criteria.
- Organizes continuous phase portrait within a transition graph

Some Acknowledgements

- Slide 3: By Globbet - Own work by uploader, with permission from the Mill Meece Pumping Station Preservation Trust, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=7421535

- Slide 4: http://www.cds.caltech.edu/~murray/cdspanel/