

*Decision Making in Robots  
and Autonomous Agents*

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**Learning in Repeated Interactions**

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# Learning in Repeated Interactions

- ▶ How can agent **learn** to interact with other agents?
- ▶ What kind of behaviour do we want to learn?
- ▶ Learn individually or together?
- ▶ Many different methods...
- ▶ In this lecture: **reinforcement learning**

# Recap

Markov Decision Process:

- ▶ states  $S$ , actions  $A$
- ▶ stochastic transition  $P(s'|s, a)$
- ▶ utility/reward  $u(s, a)$  (*can be random variable*)

Reinforcement Learning:

- ▶ “reinforce” good actions
- ▶ learn optimal action policy  $\pi^*$
- ▶ e.g. value iteration, policy iteration, ...
  - require knowledge of model, e.g.  $P/u$

# Q-Learning

What if transition and reward function unknown?

- ▶ take action  $a^t$  in current state  $s^t$
- ▶ only see immediate reward  $r^{t+1}$  and next state  $s^{t+1}$ 
  - need **model-free** reinforcement learning

**Q-Learning** (Watkins & Dayan, 1992)

- ▶ store table  $Q(s, a)$  for  $s \in S, a \in A$
- ▶ simple update rule:

$$Q(s^t, a^t) \leftarrow (1 - \alpha)Q(s^t, a^t) + \alpha \left[ r^{t+1} + \gamma \max_{a' \in A} Q(s^{t+1}, a') \right]$$

- ▶ learns optimal Q-values under certain conditions

# Q-Learning in Stochastic Games

Can we use Q-learning for interactive setting?

- ▶ general and simple nature appealing
- ▶ just learn to interact “on the fly”
- ▶ **but:** application not straight-forward, many problems...
  - will discuss some problems later

We consider two examples:

- ▶ Joint Action Q-Learning (Claus & Boutillier, 1998)
- ▶ Nash Q-Learning (Hu & Wellman, 2003)

(Other examples exist)

# Joint Action Q-Learning (JAL) (Claus & Boutillier, 1998)

- ▶ Assume two players,  $i$  and  $j$
- ▶ We observe state  $s^t$ , actions  $a_i^t, a_j^t$ , and results  $s^{t+1}, r_i^{t+1}$
- ▶ Store table  $Q(s, a_i, a_j)$  where  $s \in S, a_i \in A_i, a_j \in A_j$
- ▶ Update rule:

$$Q(s^t, a_i^t, a_j^t) \leftarrow (1-\alpha)Q(s^t, a_i^t, a_j^t) + \alpha \left[ r_i^{t+1} + \gamma \max_{a'_i \in A_i} EV(s^{t+1}, a'_i) \right]$$

$$EV(s, a_i) = \sum_{a_j \in A_j} P_j(s, a_j) Q(s, a_i, a_j)$$

- ▶  $P_j(s, a_j)$  is empirical frequency distribution of  $j$ 's past actions in state  $s$  (**fictitious play**, Brown 1951)

# JAL and Nash Equilibrium

- ▶ Assume both players controlled by JAL agent
- ▶ Assume common payoffs (e.g. players receive same rewards)
- ▶ Many other assumptions...

**Theorem 1** *Let  $E_t$  be a random variable denoting the probability of a (deterministic) equilibrium strategy profile being played at time  $t$ . Then for both ILs and JALs, for any  $\delta, \varepsilon > 0$ , there is an  $T(\delta, \varepsilon)$  such that*

$$\Pr(|E_t - 1| < \varepsilon) > 1 - \delta$$

*for all  $t > T(\delta, \varepsilon)$ .*

(Claus & Boutillier, 1998)

## Nash Q-Learning (NashQ) (Hu & Wellman, 2003)

- ▶ Assume two players,  $i$  and  $j$
- ▶ We observe state  $s^t$ , actions  $a_i^t, a_j^t$ , and results  $s^{t+1}, r_i^{t+1}, r_j^{t+1}$
- ▶ Store table  $Q(s, a_i, a_j)$  where  $s \in S, a_i \in A_i, a_j \in A_j$
- ▶ Update rule:

$$Q(s^t, a_i^t, a_j^t) \leftarrow (1 - \alpha)Q(s^t, a_i^t, a_j^t) + \alpha [r_i^{t+1} + \gamma \text{Nash}Q(s^{t+1})]$$

$$\text{Nash}Q(s) = \sum_{a_i \in A_i} \sum_{a_j \in A_j} \pi_i(s, a_i) \pi_j(s, a_j) Q(s, a_i, a_j)$$

- ▶  $(\pi_i, \pi_j)$  is (possibly mixed) **Nash equilibrium profile** for matrix game defined by  $Q(s, \cdot, \cdot)$



# NashQ and Nash Equilibrium

- ▶ Assume both players controlled by NashQ agent
- ▶ Assume several other restrictions ... including:

**Assumption 3** *One of the following conditions holds during learning.*<sup>3</sup>

**Condition A.** *Every stage game  $(Q_t^1(s), \dots, Q_t^n(s))$ , for all  $t$  and  $s$ , has a global optimal point, and agents' payoffs in this equilibrium are used to update their  $Q$ -functions.*

**Condition B.** *Every stage game  $(Q_t^1(s), \dots, Q_t^n(s))$ , for all  $t$  and  $s$ , has a saddle point, and agents' payoffs in this equilibrium are used to update their  $Q$ -functions.*

(Hu & Wellman, 2003)

- ▶ Then the learning converges to a Nash equilibrium

# Assumptions in Learning Methods

Different methods may make different assumptions, e.g.

Things that can be “seen”:

- ▶ JAL:  $s^t a_i^t a_j^t s^{t+1} r_i^{t+1}$
- ▶ NashQ:  $s^t a_i^t a_j^t s^{t+1} r_i^{t+1} r_j^{t+1}$

Implicit behavioural assumptions:

- ▶ JAL:  $j$  plays **fixed distribution** in each state
- ▶ NashQ:  $j$  plays **Nash equilibrium** strategy in each state

Many other types of assumptions about structure of game, behaviour of players, ability to observe, etc.

# Assumptions in Learning Methods

Often, method can still be used even if assumptions violated:

- ▶ Q-learning assumes stationary transition probabilities
  - *Is this true in interactive setting?*
- ▶ what happens if assumptions violated?
- ▶ **know and understand assumptions!**

Bonus question:

What happens if different methods play against each other?

- ▶ e.g. JAL vs NashQ
- ▶ (Albrecht & Ramamoorthy, 2012)

## **Excursion:**

Ad Hoc Coordination in Multiagent Systems

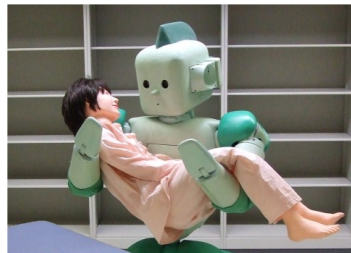
# Ad Hoc Coordination

1. You control single agent in system with other agents
2. You and other agents have **goals** (common or conflicting)
3. You want to be **flexible**: other agents may have large variety of behaviours
4. You want to be **efficient**: not much time for learning, trial and error, etc.
5. You don't a priori know how other agents behave

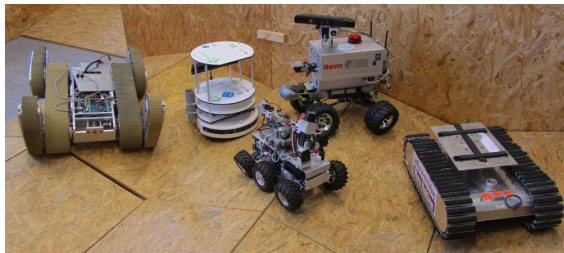
# Ad Hoc Coordination

Applications:

- ▶ Human-robot interaction
- ▶ Robot search and rescue
- ▶ Adaptive user interfaces
- ▶ Financial markets
- ▶ ...



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# Ad Hoc Coordination

Human-robot interaction:

- ▶ Humans can exhibit large variety of behaviours for given task  
→ need **flexibility**!
- ▶ Humans expect machines to learn and react quickly  
→ need **efficiency**!
- ▶ Machine does not know ahead of time how human behaves  
→ **no prior coordination** of behaviours!

# Ad Hoc Coordination

Hard problem:

- ▶ Agents may have large variety of behaviours
- ▶ Behaviours **initially unknown**

General learning algorithms not suitable:

- ▶ Require **long learning periods** (e.g. RL)
- ▶ Often designed for **homogeneous setting**
- ▶ Many **restrictive assumptions** (discussed earlier)



# Idea

Reduce complexity of problem by assuming that:

1. Agents draw their **latent policy** from some set
2. Policy assignment governed by **unknown distribution**

If policy set known:

- ▶ Learn distribution, play best-response

If policy set unknown:

- ▶ “Guess” policy set, find closest policy, play best-response

Idea

## Hypothesise (“guess”) Policy Types



# Stochastic Bayesian Game

- ▶ state space  $S$ , initial state  $s^0 \in S$ , terminal states  $\bar{S} \subset S$
- ▶ players  $N = \{1, \dots, n\}$  and for each  $i \in N$ :
  - ▶ set of actions  $A_i$  (where  $A = \times_i A_i$ )
  - ▶ **type space**  $\Theta_i$  (where  $\Theta = \times_i \Theta_i$ )
  - ▶ payoff function  $u_i : S \times A \times \Theta_i \rightarrow \mathbb{R}$
  - ▶ strategy  $\pi_i : \mathbb{H} \times A_i \times \Theta_i \rightarrow [0, 1]$   
 $\mathbb{H}$  is set of histories  $H^t = \langle s^0, a^0, \dots, s^t \rangle$  s.t.  $s^t \in S, a^t \in A$
- ▶ state transition function  $T : S \times A \times S \rightarrow [0, 1]$
- ▶ **type distribution**  $\Delta : \Theta \rightarrow [0, 1]$

(Albrecht & Ramamoorthy, 2014)

# Harsanyi-Bellman Ad Hoc Coordination (HBA)

Canonical formulation **HBA**:

$$a_i^t \sim \arg \max_{a_i \in A_i} E_{s^t}^{a_i}(H^t)$$

where

$$E_s^{a_i}(\hat{H}) = \sum_{\theta_{-i}^* \in \Theta_{-i}^*} \Pr(\theta_{-i}^* | H^t) \sum_{a_{-i} \in A_{-i}} Q_s^{a_i, -i}(\hat{H}) \prod_{j \neq i} \pi_j(\hat{H}, a_j, \theta_j^*)$$

$$Q_s^a(\hat{H}) = \sum_{s' \in S} T(s, a, s') \left[ u_i(s, a, \alpha) + \gamma \max_{a_i} E_{s'}^{a_i}(\langle \hat{H}, a, s' \rangle) \right]$$

(Albrecht & Ramamoorthy, 2014)

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