Decision Making in Robots and Autonomous Agents: Homework Assignment 1 (Semester 2 - 2018/19)

Subramanian Ramamoorthy

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1 Instructions

- This homework assignment is to be done *individually*, without help from your classmates or others.
- Solve all problems and provide your **complete** solutions (with adequate reasoning behind each step, and citations where needed) in a computer-printed form.
- Submit electronically (including all electronic files associated with the assignment), **as well as** submitting a hard copy of your report to ITO.
- This assignment will count for 10% of your final course mark. It is due at 4 pm on 15 February 2019.

Good Scholarly Practice: Please remember the University requirement as regards all assessed work for credit. Details about this can be found at:

http://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct

2 Questions

1. Consider the third-order dynamical system described by the equations:

$$\dot{x_1} = x_1^2 - x_2 + x_3 \tag{1}$$

$$\dot{x_2} = x_1 - x_2 \tag{2}$$

$$\dot{x_3} = 2x_2^2 + x_3 - 2 \tag{3}$$

Find all the equilibrium points of the system. At each equilibrium point, write down the linear approximation of the dynamical system equations.
4 marks

2. Consider the following model for how a rumour (or other piece of information) spreads through a closed population (of size $N + 1$) of agents. At time t, the total population can be classified into three categories:

   - x persons who are ignorant of the rumour;
   - y persons who are actively spreading the rumour;
   - z persons who have heard the rumour but have stopped spreading it: if two persons who are spreading the rumour meet then they stop spreading it.

   The contact rate between any two categories is a constant μ. The following is the deterministic model of the problem:

$$\dot{x} = -\mu xy \tag{4}$$

$$\dot{y} = \mu[xy - y(y - 1) - yz] \tag{5}$$

   Note that, in a closed population, choice of x and y implicitly constrain z ($z = N + 1 - x - y$).
   6 marks

   (a) Generate and sketch the phase plane diagram for this system (using Matlab or your favourite computational environment). Use an initial value of $\mu = 0.75$. [Note: In order to do that, select a grid of initial values in the $x - y$ phase space and iterate (i.e., solve initial value problems) over small time steps using the above differential equations to visualize the state evolution. You may use any standardly available ODE solver for this purpose, e.g., ode45 in Matlab (http://www.mathworks.com/access/helpdesk/help/techdoc/ref/ode113.html?BB=1)].

   (b) Starting with the initial conditions $y = 1$ and $x = N = 75$, plot $x(t)$ and $y(t)$ over a time interval of 2.5 time units (seconds), assuming $z = 0$. Repeat for different initial conditions: $x = 71, y = 5, z = 0$; $x = 70, y = 1, z = 5$; $x = 66, y = 5, z = 5$.

   (c) Change the free parameter μ and discuss the corresponding changes in observed behaviour.