Decision Making in Robots and Autonomous Agents: Homework Assignment 1 (Semester 2 - 2017/18)

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1 Instructions

- This homework assignment is to be done *individually*, without help from your classmates or others.
- Solve all problems and provide your **complete** solutions (with adequate reasoning behind each step, and citations where needed) in a computer-printed form.
- Submit electronically (including all electronic files associated with the assignment), **as well as** submitting a hard copy of your report to ITO.
- This assignment will count for 10% of your final course mark. It is due at 4 pm on 16 February 2018.

Good Scholarly Practice: Please remember the University requirement as regards all assessed work for credit. Details about this can be found at:

http://web.inf.ed.ac.uk/infweb/admin/policies/academic-misconduct

2 Questions

1. A government space project is conducting research on a certain engineering problem that must be solved before man can fly safely to Mars. Three research teams are currently trying three different approaches for solving this

Num. new scientists	Prob. Of failure of Team 1	Prob. Of failure of Team 2	Prob. Of failure of Team 3
0	0.4	0.6	0.8
1	0.2	0.4	0.5
2	0.15	0.2	0.3

Figure 1: Estimates of probability of failure with different number of new scientists.

problem. The estimate has been made that, under present circumstances, the probability that the respective teams - call them 1, 2 and 3 - will not succeed is 0.4, 0.6 and 0.8 respectively. Thus, the current probability that all three teams will fail is $0.4 \cdot 0.6 \cdot 0.8 = 0.192$. Since the objective is to minimise this probability, the decision has been made to assign twi more top scientists among the three teams to lower it as much as possible.
The table below gives the estimated probability that the respective teams will fail when 0,1 or 2 additional scientists are added to that team. Using a dynamic programming formulation, determine how to allocate the two additional scienists to minimise the probability that all three teams will fail.

2. Consider a decision problem involving five states, enumerated by $i = 1, ..., 5$, as illustrated in figure 2. Each edge is annotated by the cost associated with traversing it, in either direction. We wish to solve a shortest path problem on this graph (for all pairs, i.e., starting at each of states 1, 2, 3, 4 and going to 5), through the following steps:

   (a) Re-write this graph in an 'unrolled' form, as a deterministic finite-state system (similar to the diagram for the stagecoach problem in lectures).

   (b) Compute costs-to-go for each state, at each stage.

   (c) List the state sequence that is the shortest path through this system, for each case.

3. Consider the following model for how a rumour (or other piece of information) spreads through a closed population (of size $N + 1$) of agents. At time t, the total population can be classified into three categories:

   - x persons who are ignorant of the rumour;
   - y persons who are actively spreading the rumour;

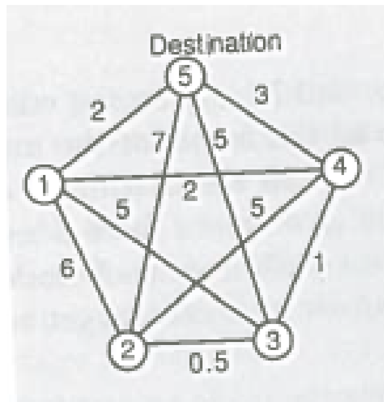

Figure 2: Shortest path problem data.

- z persons who have heard the rumour but have stopped spreading it: if two persons who are spreading the rumour meet then they stop spreading it.

The contact rate between any two categories is a constant μ. The following is the deterministic model of the problem:

$$\dot{x} = -\mu xy \tag{1}$$

$$\dot{y} = \mu[xy - y(y-1) - yz] \tag{2}$$

Note that, in a closed population, choice of x and y implicitly constrain z ($z = N + 1 - x - y$).

(a) Generate and sketch the phase plane diagram for this system (using Matlab or your favourite computational environment). Use an initial value of $\mu = 0.75$. [Note: In order to do that, select a grid of initial values in the $x - y$ phase space and iterate (i.e., solve initial value problems) over small time steps using the above differential equations to visualize the state evolution. You may use any standardly available ODE solver for this purpose, e.g., ode45 in Matlab (http://www.mathworks.com/access/helpdesk/help/techdoc/ref/ode113.html?BB=1)].

(b) Starting with the initial conditions $y = 1$ and $x = N = 75$, plot $x(t)$ and $y(t)$ over a time interval of 2.5 time units (seconds), assuming $z = 0$. Repeat for different initial conditions: $x = 71, y = 5, z = 0$; $x = 70, y = 1, z = 5$; $x = 66, y = 5, z = 5$.

(c) Change the free parameter μ and discuss the corresponding changes in observed behaviour.