

# Discrete Mathematics & Mathematical Reasoning

## Course Overview

Colin Stirling

Informatics

# Teaching staff

## Lecturers:

- Colin Stirling, first half of course
- Kousha Etessami, second half of course

## Course Secretary (ITO):

- Kendall Reid ([kreid5@staffmail.ed.ac.uk](mailto:kreid5@staffmail.ed.ac.uk))

# Lectures

- Monday 16.10-17.00 Here
- Tuesday 10.00-10.50 Weeks 1 & 7 LT C DHT; other weeks LT 4 AT
- Thursday 16.10-17.00 Here

# Course web page (not on LEARN)

`http://www.inf.ed.ac.uk/teaching/courses/dmmr/`

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## Contains important information

- Lecture slides
- Tutorial sheet exercises
- Link to tutorial groups
- Course organization
- ...

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- If you change tutor groups for any reason, you must let Kendall and the ITO know (because your marked coursework is returned at the tutorial groups)
- Tutorial attendance is mandatory. If you miss two tutorials in a row, your PT will be notified

# Tutorials and (marked) exercises

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- **Exception: no tutorial in week 1**

# Textbook

- Kenneth Rosen, **Discrete Mathematics and its Applications**, 7th Edition, (Global Edition) McGraw-Hill, 2012
- Available at Blackwells
- For additional material see the course webpage



# Grading

- Written Examination: 85% **IMPORTANT CHANGE THIS YEAR: OPEN BOOK EXAM**
- Assessed Assignments: 15%. Each one of the 9 exercise sheets counts equally. (Actually, first 8 sheets are each out of 11, and the last is out of 12). **IMPORTANT CHANGE THIS YEAR: SUBMISSIONS ARE DONE ON DICE MACHINES**

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- Questions about course administration?

# Important themes

- mathematical reasoning
- combinatorial analysis
- discrete structures
- algorithmic thinking
- applications and modelling

# Foundations: proof

- Rudimentary predicate (first-order) logic: existential and universal quantification, basic algebraic laws of quantified logic (duality of existential and universal quantification)
- The structure of a well-reasoned mathematical proof; proof strategies: proofs by contradiction, proof by cases; examples of incorrect proofs (to build intuition about correct mathematical reasoning)

# Foundations: sets, functions and relations

- Sets (naive): operations on sets: union, intersection, set difference, the powerset operation, examples of finite and infinite sets (the natural numbers). Ordered pairs, n-tuples, and Cartesian products of sets
- Relations: (unary, binary, and n-ary) properties of binary relations (symmetry, reflexivity, transitivity).
- Functions: injective, surjective, and bijective functions, inverse functions, composition of functions
- Rudimentary counting: size of the Cartesian product of two finite sets, number of subsets of a finite set, (number of n-bit sequences), number of functions from one finite set to another

# Induction and recursion

- Principle of mathematical induction (for positive integers)
- Examples of proofs by (weak and strong) induction

# Basic number theory and some cryptography

- Integers and elementary number theory (divisibility, GCDs and the Euclidean algorithm, prime decomposition and the fundamental theorem of arithmetic)
- Modular arithmetic (congruences, Fermat's little theorem, the Chinese remainder theorem)
- Applications: public-key cryptography



# Basic algorithms

- Concept and basic properties of an algorithm
- Some examples of algorithms
- Basics of growth of function, and complexity of algorithms:  
comparison of growth rate of some common functions

# Counting

- Basics of counting
- Pigeon-hole principle
- Permutations and combinations
- Binomial coefficients, binomial theorem, and basic identities on binomial coefficients
- Generalizations of permutations and combinations (e.g., combinations with repetition/replacement)
- Stirling's approximation of the factorial function

# Graphs

- Directed and undirected graph: definitions and examples in Informatics
- Adjacency matrix representation
- Terminology: degree (indegree, outdegree), and special graphs: bipartite, complete, acyclic, ...
- Isomorphism of graphs; subgraphs
- Paths, cycles, and (strong) connectivity
- Euler paths/circuits, Hamiltonian paths (brief)
- Weighted graphs, and shortest paths (Dijkstra's algorithm)
- Bipartite matching: Hall's marriage theorem

# Trees

- Rooted and unrooted trees
- Ordered and unordered trees
- (Complete) binary (k-ary) tree
- Subtrees
- Examples in Informatics
- Spanning trees (Kruskal's algorithm, Prim's algorithm.)

# Discrete probability

- Discrete (finite or countable) probability spaces
- Events
- Basic axioms of discrete probability
- Independence and conditional probability
- Bayes' theorem
- Random variables
- Expectation; linearity of expectation
- Basic examples of discrete probability distributions, the birthday paradox and other subtle examples in probability
- The probabilistic method: a proof technique

# My proof

Colin's proof that  $1 = 2$

$$a = b \quad \text{Premise}$$

$$a^2 = ab \quad \text{Multiply both sides by } a$$

$$a^2 - b^2 = ab - b^2 \quad \text{Subtract } b^2 \text{ from both sides}$$

$$(a-b)(a+b) = b(a-b) \quad \text{Algebra}$$

$$a+b = b \quad \text{Divide both sides by } a-b$$

$$2b = b \quad \text{Replace } a \text{ by } b \text{ because } a=b$$

$$2 = 1 \quad \text{Divide both sides by } b$$

# My mark

$a = b$  Premise

$a^2 = ab$  Multiply both sides by  $a$


$a^2 - b^2 = ab - b^2$  Subtract  $b^2$  from both sides

$(a-b)(a+b) = b(a-b)$  Algebra

NO  $a+b = b$  Divide both sides by  $a-b$

$2b = b$  Replace  $a$  by  $b$  because  $a=b$

$2 = 1$  Divide both sides by  $b$

Rubbish  %11

# Reasoning 1

Given the following two premises

- All students in this class understand logic
- Colin is a student in this class



# Reasoning 1

Given the following two premises

- All students in this class understand logic
- Colin is a student in this class

Does it follow that

- Colin understands logic

# Reasoning 2

Given the following two premises

- Every computer science student takes discrete mathematics
- Helen is taking discrete mathematics

# Reasoning 2

Given the following two premises

- Every computer science student takes discrete mathematics
- Helen is taking discrete mathematics

Does it follow that

- Helen is a computer science student

# Reasoning 3

Given the following three premises

- All hummingbirds are richly coloured
- No large birds live on honey
- Birds that do not live on honey are dull in colour

# Reasoning 3

## Given the following three premises

- All hummingbirds are richly coloured
- No large birds live on honey
- Birds that do not live on honey are dull in colour

## Does it follow that

- Hummingbirds are small