Discrete Mathematics & Mathematical Reasoning
Algorithms

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Informatics

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$765 = 3 \cdot 3 \cdot 5 \cdot 17 = 3^2 \cdot 5 \cdot 17$

Use the sieve of Eratosthenes

Find all primes between 2 and n

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2. Remove all strict multiples of i from the list
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4. If $i > \sqrt{n}$ then stop else go to step 2

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Is there a quicker algorithm? WHAT DOES THIS MEAN?

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- Effective: it must be possible to perform each step correctly and in a finite amount of time
- Generality: it should work for all problems of the desired form

Recursive algorithm

Euclidian algorithm

```
algorithm gcd(x,y)
    if y = 0
    then return(x)
    else return(gcd(y,x mod y))
```

Euclidian algorithm (proof of correctness)

Lemma

If $a = bq + r$, where a, b, q, and r are positive integers, then $\gcd(a, b) = \gcd(b, r)$

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Proof.

($\Rightarrow$) Suppose that d divides both a and b. Then d also divides $a - bq = r$. Hence, any common divisor of a and b must also be a common divisor of b and r
($\Leftarrow$) Suppose that d divides both b and r. Then d also divides $bq + r = a$. Hence, any common divisor of b and r must also be a common divisor of a and b.
Therefore, $\gcd(a, b) = \gcd(b, r)$ $\square$

Description of algorithms in pseudocode

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- Intermediate step between English prose and formal coding in a programming language
- Focus on the fundamental operation of the program, instead of peculiarities of a given programming language
- Analyze the time required to solve a problem using an algorithm, independent of the actual programming language

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procedure maximum(a_1,...,a_n)
max:=a_1
for i:=2 to n
    if max < a_i
    then max:=a_i
return max
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How to prove correctness?

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Describe an algorithm for locating an item in a sequence of integers

Input integer x and finite sequence of distinct integers $a_1, \ldots, a_n$

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Make use of property that sequence is of increasing integers

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procedure binary_search(x,a_1,...,a_n)
i:=1
j:=n
while i<j
    m:=⌊(i + j)/2⌋
    if x > a_m
    then i:=m+1
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Big-O notation for function growth

Definition

Let $f, g : \mathbb{N} \to \mathbb{R}$ or $f, g : \mathbb{R} \to \mathbb{R}$. Then f is $O(g)$ if there is a constant k and a positive constant c such that

$$\forall x > k \; (|f(x)| \leq c|g(x)|)$$

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- Show $f(x)$ is also $O(g)$ where $g(x) = x^3$
- Show $f(x)$ is not $O(h)$ where $h(x) = x$

Examples

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- f is $\Omega(g)$ if and only if g is $O(f)$

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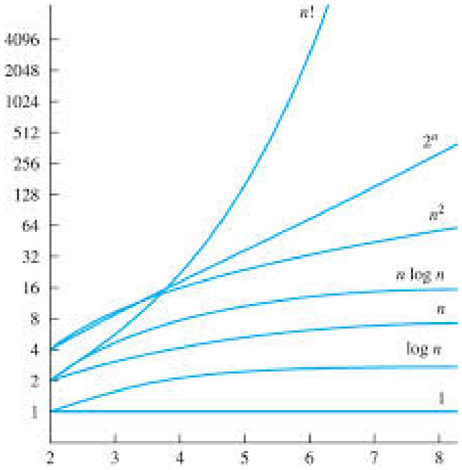
- For factorisation input size of integer n is its binary representation $log\ n$

Growth



n!
2^n
n^2
n log n
n
log n
1
4096
2048
1024
512
256
128
64
32
16
8
4
2
1
2
3
4
5
6
7
8

Linear search

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procedure linear_search(x,a_1,...,a_n)
i:=1
while i≤n and x≠a_i
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- If x is not in the list, $2n + 2$ comparisons are made which is the worst case
- This means that the complexity is $\Theta(n)$

Binary search

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i:=1
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while i<j
    m:=⌊(i + j)/2⌋
    if x > a_m
    then i:=m+1
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- Hence, at most $2k + 2 = 2log_2 n + 2$ comparisons are made
- This means that complexity is $\Theta(log\, n)$

Computer time

TABLE 2 The Computer Time Used by Algorithms.

Problem Size	*Bit Operations Used*					
n	$\log n$	n	$n \log n$	n^2	2^n	$n!$
10	3×10^{-9} sec	10^{-8} sec	3×10^{-8} sec	10^{-7} sec	10^{-6} sec	3×10^{-3} sec
10^2	7×10^{-9} sec	10^{-7} sec	7×10^{-7} sec	10^{-5} sec	4×10^{13} yr	*
10^3	1.0×10^{-8} sec	10^{-6} sec	1×10^{-5} sec	10^{-3} sec	*	*
10^4	1.3×10^{-8} sec	10^{-5} sec	1×10^{-4} sec	10^{-1} sec	*	*
10^5	1.7×10^{-8} sec	10^{-4} sec	2×10^{-3} sec	10 sec	*	*
10^6	2×10^{-8} sec	10^{-3} sec	2×10^{-2} sec	17 min	*	*

However, the time required for an algorithm to solve a problem of a specified size can be determined if all operations can be reduced to the bit operations used by the computer. Table 2 displays the time needed to solve problems of various sizes with an algorithm using the indicated number of bit operations. Times of more than 10^{100} years are indicated with an asterisk. (In Section 2.4 the number of bit operations

Further topics

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- Open problem: NP $\subseteq$ P ?

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- It is very unlikely to be NP complete