Another proof method: Mathematical Induction

- Want to prove \( \forall n \in \mathbb{N} \ (P(n)) \)
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- Want to prove $\forall n \in \mathbb{N} \ (P(n))$

- BASIS STEP show $P(0)$
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- Want to prove $\forall n \in \mathbb{N} \ (P(n))$

- **BASIS STEP** show $P(0)$

- **INDUCTIVE STEP** show $P(k) \rightarrow P(k + 1)$ for all $k \in \mathbb{N}$
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- Want to prove $\forall n \in \mathbb{Z}^+ \ (P(n))$

- **BASIS STEP** show $P(1)$

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- Want to prove \( \forall n \geq m \in \mathbb{N} \ (P(n)) \)

- BASIS STEP show \( P(m) \)

- INDUCTIVE STEP show \( P(k) \rightarrow P(k + 1) \) for all \( k \geq m \in \mathbb{N} \)

- Assume \( k \geq m \) is arbitrary and \( P(k) \) is true. Show \( P(k + 1) \)
Another proof method: Mathematical Induction

- Want to prove $\forall n \in \mathbb{Q}^+ \ (P(n))$

Well ordering principle: every nonempty set $S \subseteq \mathbb{N}$ has a least element.
Another proof method: Mathematical Induction

- Want to prove $\forall n \in \mathbb{Q}^+ \ (P(n))$

- Can we use induction?
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- Want to prove $\forall x \in \mathbb{R}^+ \ (P(x))$
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- What justifies mathematical induction?
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Examples

\[ \sum_{j=1}^{n} j = \frac{n(n + 1)}{2} \]
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- for all \( n \in \mathbb{Z}^+ ((n^3 - n) \text{ is divisible by } 3) \)
- If \( S \) is a finite set with \( n \) elements then \( \mathcal{P}(S) \) contains \( 2^n \) elements
More examples

Odd Pie Fights An odd number of people stand in a room at mutually distinct distances. At the same time each person throws a pie at their nearest neighbour and hits them. Prove that at least one person is not hit by a pie
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- **All cats have the same colour**
Strong Induction

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- Prove that every amount of postage of 12p or more can be formed using just 4p and 5p stamps