Discrete Mathematics & Mathematical Reasoning
Course Overview

Colin Stirling
Informatics
Teaching staff

Lecturers:
- Colin Stirling, first half of course
- Kousha Etessami, second half of course

Course Secretary (ITO):
- Kendall Reid (kreid5@staffmail.ed.ac.uk)
Course web page

http://www.inf.ed.ac.uk/teaching/courses/dmmr/

Contains important information

- Lecture slides
- Tutorial sheet exercises
- Link to tutorial groups
- Course organization
- …
Tutorials

- You should receive email from the ITO informing you of preliminary allocation of tutorial groups

See link on course web page for current assignment of tutorial groups

If you can't make the time of your allocated group, please email Kendall suggesting some groups you can manage

If you change tutor groups for any reason, you must let Kendall and the ITO know (because your marked coursework is returned at the tutorial groups)

Tutorial attendance is mandatory. If you miss two tutorials in a row, your PT will be notified
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Tutorials and (marked) exercises

- Weekly exercise sheets, available previous Wednesday (except for the first) on the course web page

The last question on every sheet will be graded. The coursework grade contributes 15% to the total course grade, and every one of the 9 exercise sheets counts 1/9th of the coursework grade.

Starting in week 2, deadline for submission of each tutorial sheet is Wednesday at 4:00pm at the ITO (they also have a collection box).

Solutions will be discussed in tutorials the following week. Graded sheets are returned in tutorials (or collected later from the ITO).

Exception: no tutorial in week 1.
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Textbook


- Available at Blackwells

- For additional material see the course webpage
Grading

- **Written Examination**: 85%

- **Assessed Assignments**: 15%. Each one of the 9 exercise sheets counts equally. *(Actually, first 8 sheets are each out of 11, and the last is out of 12)*

To pass course need 40% or more overall *(No separate exam/coursework hurdle)*
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Important themes

- mathematical reasoning
- combinatorial analysis
- discrete structures
- algorithmic thinking
- applications and modelling
Foundations: proof

- Rudimentary predicate (first-order) logic: existential and universal quantification, basic algebraic laws of quantified logic (duality of existential and universal quantification)

- The structure of a well-reasoned mathematical proof; proof strategies: proofs by contradiction, proof by cases; examples of incorrect proofs (to build intuition about correct mathematical reasoning)
Foundations: sets, functions and relations

- Sets (naive): operations on sets: union, intersection, set difference, the powerset operation, examples of finite and infinite sets (the natural numbers). Ordered pairs, n-tuples, and Cartesian products of sets.

- Relations: (unary, binary, and n-ary) properties of binary relations (symmetry, reflexivity, transitivity).

- Functions: injective, surjective, and bijective functions, inverse functions, composition of functions.

- Rudimentary counting: size of the Cartesian product of two finite sets, number of subsets of a finite set, (number of n-bit sequences), number of functions from one finite set to another.
Induction and recursion

- Principle of mathematical induction (for positive integers)
- Examples of proofs by (weak and strong) induction
- Recursive definitions and structural induction
Basic number theory and some cryptography

- Integers and elementary number theory (divisibility, GCDs and the Euclidean algorithm, prime decomposition and the fundamental theorem of arithmetic)

- Modular arithmetic (congruences, Fermat’s little theorem, the Chinese remainder theorem)

- Applications: public-key cryptography
Basic algorithms

- Concept and basic properties of an algorithm
- Some examples of algorithms
- Basics of growth of function, and complexity of algorithms: comparison of growth rate of some common functions
Counting

- Basics of counting
- Pigeon-hole principle
- Permutations and combinations
- Binomial coefficients, binomial theorem, and basic identities on binomial coefficients
- Generalizations of permutations and combinations (e.g., combinations with repetition/replacement)
- Stirling’s approximation of the factorial function
Graphs

- Directed and undirected graph: definitions and examples in Informatics
- Adjacency matrix representation
- Terminology: degree (indegree, outdegree), and special graphs: bipartite, complete, acyclic, ...
- Isomorphism of graphs; subgraphs
- Paths, cycles, and (strong) connectivity
- Euler paths/circuits, Hamiltonian paths (brief)
- Weighted graphs, and shortest paths (Dijkstra’s algorithm)
- Bipartite matching: Hall’s marriage theorem
Trees

- Rooted and unrooted trees
- Ordered and unordered trees
- (Complete) binary (k-ary) tree
- Subtrees
- Examples in Informatics
- Spanning trees (Kruskal’s algorithm, Prim’s algorithm.)
Discrete probability

- Discrete (finite or countable) probability spaces
- Events
- Basic axioms of discrete probability
- Independence and conditional probability
- Bayes’ theorem
- Random variables
- Expectation; linearity of expectation
- Basic examples of discrete probability distributions, the birthday paradox and other subtle examples in probability
- The probabilistic method: a proof technique
**“Proof” that 1 = 2**

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**Colin Stirling (Informatics)**

**Discrete Mathematics**

Today 18 / 19
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2. $x + 3 = x^2 - 6x + 9$  
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5. $x = 1$ or $x = 6$  
   **Reason** If $ab = 0$ then $a = 0$ or $b = 0$
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