# Discrete Mathematics & Mathematical Reasoning
# Course Overview


Colin Stirling

Informatics

# Teaching staff

**Lecturers:**

- Colin Stirling, first half of course
- Kousha Etessami, second half of course

**Course Secretary (ITO):**

- Kendall Reid (kreid5@staffmail.ed.ac.uk)

# Course web page

http://www.inf.ed.ac.uk/teaching/courses/dmmr/

Contains important information

- Lecture slides
- Tutorial sheet exercises
- Link to tutorial groups
- Course organization
- ...

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- Tutorial attendance is mandatory. If you miss two tutorials in a row, your PT will be notified

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- Exception: no tutorial in week 1

# Textbook

- Kenneth Rosen, **Discrete Mathematics and its Applications**, 7th Edition, (Global Edition) McGraw-Hill, 2012
- Available at Blackwells
- For additional material see the course webpage

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- To pass course need 40% or more overall (No separate exam/coursework hurdle)

# Important themes

- mathematical reasoning
- combinatorial analysis
- discrete structures
- algorithmic thinking
- applications and modelling

# Foundations: proof

- Rudimentary predicate (first-order) logic: existential and universal quantification, basic algebraic laws of quantified logic (duality of existential and universal quantification)
- The structure of a well-reasoned mathematical proof; proof strategies: proofs by contradiction, proof by cases; examples of incorrect proofs (to build intuition about correct mathematical reasoning)

# Foundations: sets, functions and relations

- Sets (naive): operations on sets: union, intersection, set difference, the powerset operation, examples of finite and infinite sets (the natural numbers). Ordered pairs, n-tuples, and Cartesian products of sets
- Relations: (unary, binary, and n-ary) properties of binary relations (symmetry, reflexivity, transitivity).
- Functions: injective, surjective, and bijective functions, inverse functions, composition of functions
- Rudimentary counting: size of the Cartesian product of two finite sets, number of subsets of a finite set, (number of n-bit sequences), number of functions from one finite set to another

# Induction and recursion

- Principle of mathematical induction (for positive integers)
- Examples of proofs by (weak and strong) induction
- Recursive definitions and structural induction

# Basic number theory and some cryptography

- Integers and elementary number theory (divisibility, GCDs and the Euclidean algorithm, prime decomposition and the fundamental theorem of arithmetic)
- Modular arithmetic (congruences, Fermat's little theorem, the Chinese remainder theorem)
- Applications: public-key cryptography

# Basic algorithms

- Concept and basic properties of an algorithm
- Some examples of algorithms
- Basics of growth of function, and complexity of algorithms: comparison of growth rate of some common functions

# Counting

- Basics of counting
- Pigeon-hole principle
- Permutations and combinations
- Binomial coefficients, binomial theorem, and basic identities on binomial coefficients
- Generalizations of permutations and combinations (e.g., combinations with repetition/replacement)
- Stirling's approximation of the factorial function

# Graphs

- Directed and undirected graph: definitions and examples in Informatics
- Adjacency matrix representation
- Terminology: degree (indegree, outdegree), and special graphs: bipartite, complete, acyclic, ...
- Isomorphism of graphs; subgraphs
- Paths, cycles, and (strong) connectivity
- Euler paths/circuits, Hamiltonian paths (brief)
- Weighted graphs, and shortest paths (Dijkstra's algorithm)
- Bipartite matching: Hall's marriage theorem

# Trees

- Rooted and unrooted trees
- Ordered and unordered trees
- (Complete) binary (k-ary) tree
- Subtrees
- Examples in Informatics
- Spanning trees (Kruskal's algorithm, Prim's algorithm.)

# Discrete probability

- Discrete (finite or countable) probability spaces
- Events
- Basic axioms of discrete probability
- Independence and conditional probability
- Bayes' theorem
- Random variables
- Expectation; linearity of expectation
- Basic examples of discrete probability distributions, the birthday paradox and other subtle examples in probability
- The probabilistic method: a proof technique

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Step 5. $a - b = 0$ by the premise and division by 0 is undefined!

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Is this reasoning correct?