

Discrete Mathematics & Mathematical Reasoning

Course Overview

Colin Stirling

Informatics

Slides based on ones by Myrto Arapinis

Teaching staff

Lecturers:

- Colin Stirling, first half of course
- Kousha Etessami, second half of course

Course TA:

- Daniel Franzen & Weili Fu

Course Secretary (ITO):

- Kendall Reid (kr@inf.ed.ac.uk)

Course web page

`http://www.inf.ed.ac.uk/teaching/courses/dmmr/`

Contains important information

- Lecture slides
- Tutorial sheet exercises
- Course organization
- ...

Tutorials

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- Tutorial attendance is mandatory. If you miss two tutorials in a row, your PT will be notified

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- **Exception: no tutorial on week 1**

Textbook

- Kenneth Rosen, **Discrete Mathematics and its Applications**, 7th Edition, (Global Edition) McGraw-Hill, 2012
- Available at Blackwells
- For additional material see the course webpage

Grading

- Written Examination: 85%
- Assessed Assignments: 15%. Each one of the 9 exercise sheets counts equally, *i.e.* 1/9th

Important themes

- mathematical reasoning
- combinatorial analysis
- discrete structures
- algorithmic thinking
- applications and modelling

Foundations: proof

- Rudimentary predicate (first-order) logic: existential and universal quantification, basic algebraic laws of quantified logic (duality of existential and universal quantification)
- The structure of a well-reasoned mathematical proof; Proof strategies: proofs by contradiction, proof by cases; examples of incorrect proofs (to build intuition about correct mathematical reasoning)

Foundations: sets, functions and relations

- Sets (naive): operations on sets: union, intersection, set difference, the powerset operation, examples of finite and infinite sets (the natural numbers). Ordered pairs, n-tuples, and Cartesian products of sets
- Relations: (unary, binary, and n-ary) properties of binary relations (symmetry, reflexivity, transitivity).
- Functions: injective, surjective, and bijective functions, inverse functions, composition of functions
- Rudimentary counting: size of the Cartesian product of two finite sets, number of subsets of a finite set, (number of n-bit sequences), number of functions from one finite set to another

Basic number theory and cryptography

- Integers and elementary number theory (divisibility, GCDs and the Euclidean algorithm, prime decomposition and the fundamental theorem of arithmetic)
- Modular arithmetic (congruences, Fermat's little theorem, the Chinese remainder theorem)
- Applications: public-key cryptography

Basic algorithms

- Concept and basic properties of an algorithm
- Basics of growth of function, and complexity of algorithms: comparison of growth rate of some common functions

Induction and recursion

- Principle of mathematical induction (for positive integers)
- Examples of proofs by (weak and strong) induction
- Recursive definitions and Structural induction

Counting

- Basics of counting
- Pigeon-hole principle
- Permutations and combinations
- Binomial coefficients, binomial theorem, and basic identities on binomial coefficients
- Generalizations of permutations and combinations (e.g., combinations with repetition/replacement)
- Stirling's approximation of the factorial function

Graphs

- Directed and undirected graph: definitions and examples in Informatics
- Adjacency matrix representation
- Terminology: degree (indegree, outdegree), and special graphs: bipartite, complete, acyclic, ...
- Isomorphism of graphs; subgraphs
- Paths, cycles, and (strong) connectivity
- Euler paths/circuits, Hamiltonian paths (brief)
- Weighted graphs, and shortest paths (Dijkstra's algorithm)
- Bipartite matching: Hall's marriage theorem

Trees

- Rooted and unrooted trees
- Ordered and unordered trees
- (Complete) binary (k-ary) tree
- Subtrees
- Examples in Informatics
- Spanning trees (Kruskal's algorithm, Prim's algorithm.)

Discrete probability

- Discrete (finite or countable) probability spaces
- Events
- Basic axioms of discrete probability
- Independence and conditional probability
- Bayes' theorem
- Random variables
- Expectation; linearity of expectation
- Basic examples of discrete probability distributions birthday paradox, and other subtle examples in probability

“Proof” that $1 = 2$

Step

1. $a = b$

Reason

Premise

“Proof” that $1 = 2$

Step

1. $a = b$
2. $a^2 = ab$

Reason

Premise

Multiply both sides by a

“Proof” that $1 = 2$

Step

1. $a = b$

2. $a^2 = ab$

3. $a^2 - b^2 = ab - b^2$

Reason

Premise

Multiply both sides by a

Subtract b^2 from both sides

“Proof” that $1 = 2$

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1. $a = b$

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3. $a^2 - b^2 = ab - b^2$

4. $(a - b)(a + b) = b(a - b)$

Reason

Premise

Multiply both sides by a

Subtract b^2 from both sides

Algebra

“Proof” that $1 = 2$

Step

1. $a = b$
2. $a^2 = ab$
3. $a^2 - b^2 = ab - b^2$
4. $(a - b)(a + b) = b(a - b)$
5. $a + b = b$

Reason

- Premise
- Multiply both sides by a
- Subtract b^2 from both sides
- Algebra
- Divide both sides by $a - b$

“Proof” that $1 = 2$

Step

1. $a = b$
2. $a^2 = ab$
3. $a^2 - b^2 = ab - b^2$
4. $(a - b)(a + b) = b(a - b)$
5. $a + b = b$
6. $2b = b$

Reason

- Premise
- Multiply both sides by a
- Subtract b^2 from both sides
- Algebra
- Divide both sides by $a - b$
- Replace a by b because $a = b$

“Proof” that $1 = 2$

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1. $a = b$
2. $a^2 = ab$
3. $a^2 - b^2 = ab - b^2$
4. $(a - b)(a + b) = b(a - b)$
5. $a + b = b$
6. $2b = b$
7. $2 = 1$

Reason

- Premise
- Multiply both sides by a
- Subtract b^2 from both sides
- Algebra
- Divide both sides by $a - b$
- Replace a by b because $a = b$
- Divide both sides by b

“Proof” that $1 = 2$

Step

1. $a = b$

2. $a^2 = ab$

3. $a^2 - b^2 = ab - b^2$

4. $(a - b)(a + b) = b(a - b)$

5. $a + b = b$

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7. $2 = 1$

Reason

Premise

Multiply both sides by a

Subtract b^2 from both sides

Algebra

Divide both sides by $a - b$

Replace a by b because $a = b$

Divide both sides by b

Step 5. $a - b = 0$ by the premise and division by 0 is undefined!