DMMR Tutorial sheet 1

Propositional Logic, Predicate Logic, Proof techniques

September 19th, 2019

1. Construct the truth table for the formula $(A \to B) \to [((B \to C) \wedge \neg C) \to \neg A]$.

2. Let $P(m,n)$ be the statement "m divides n", where the domain for both variables is the positive integers (that is, integers $m, n > 0$). By "m divides n" we mean that $n = km$ for some integer k. Determine the truth values of each of these statements.

   (a) $P(4,5)$

   (b) $P(2,4)$

   (c) $\forall m \, \forall n \, P(m,n)$

   (d) $\exists n \, \forall m \, P(m,n)$

   (e) $\exists m \, \forall n \, P(m,n)$

   (f) $\forall n \, \exists m \, P(m,n)$

3. Assume the following predicates: $B(x)$ is "x is a baby", $C(x)$ is "x can manage crocodiles", $D(x)$ is "x is despised" and $L(x)$ is "x is logical".

   (a) Assume the domain consists of people. Express each of the following statements using quantifiers, logical connectives and the predicates $B(x)$, $C(x)$, $D(x)$ and $L(x)$.

      i. Babies are illogical

      ii. Nobody is despised who can manage crocodiles

      iii. Illogical people are despised

      iv. Babies cannot manage crocodiles

   (b) Prove that iv follows from i, ii and iii.

4. (a) Assume m and n are both integers. Prove by contraposition, if mn is even then m is even or n is even.

   (b) Prove by contradiction that the sum of an irrational number and a rational number is irrational.

   (c) Prove that there is not a rational number r such that $r^3 + r + 1 = 0$.

5. (a) Assume that the positive integers $1, 2, \ldots, 2n$ are written on a blackboard, where n is an odd integer. Choose any two of the integers j and k that are present on the blackboard and write $|j-k|$ on the board and erase j and k. Continue this process until only one integer is on the board. Prove that this integer must be odd.

   (b) Prove that if the first 10 positive integers are placed around a circle, in any order, there exists three integers in consecutive locations around the circle that have a sum greater than or equal to 17.