GRAPH TRAVERSAL

PATH FINDING AND GRAPH TRAVERSAL

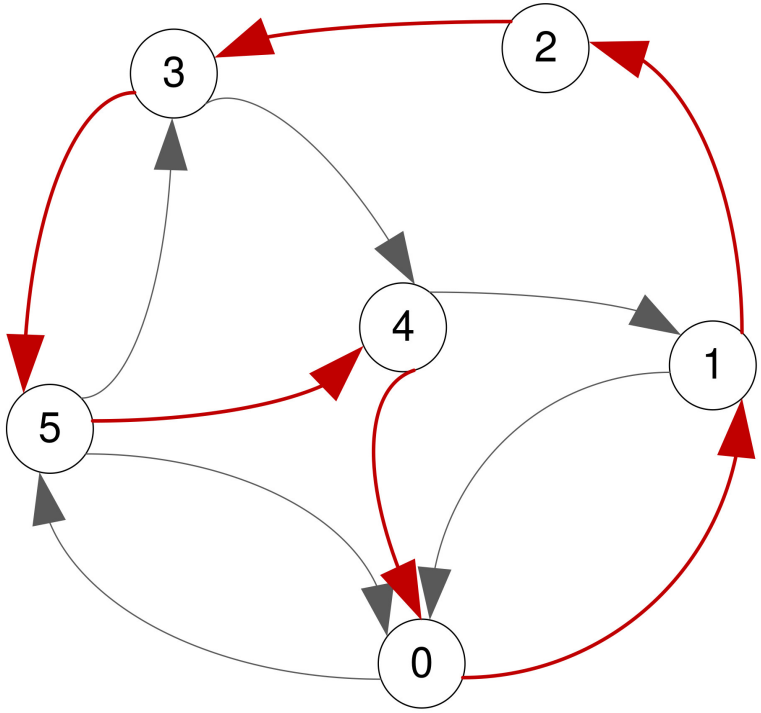
- **Path finding** refers to determining the shortest path between two vertices in a graph.
- We discussed the Floyd–Warshall algorithm previously, but you may achieve similar results with the Dijkstra or Bellman-Ford algorithms.
- **Graph traversal** refers to the problem of visiting a set of nodes in a graph in a certain way.
- You can use depth- or breadth-first search algorithms to traverse an entire graph starting from a given "root" (source vertex).

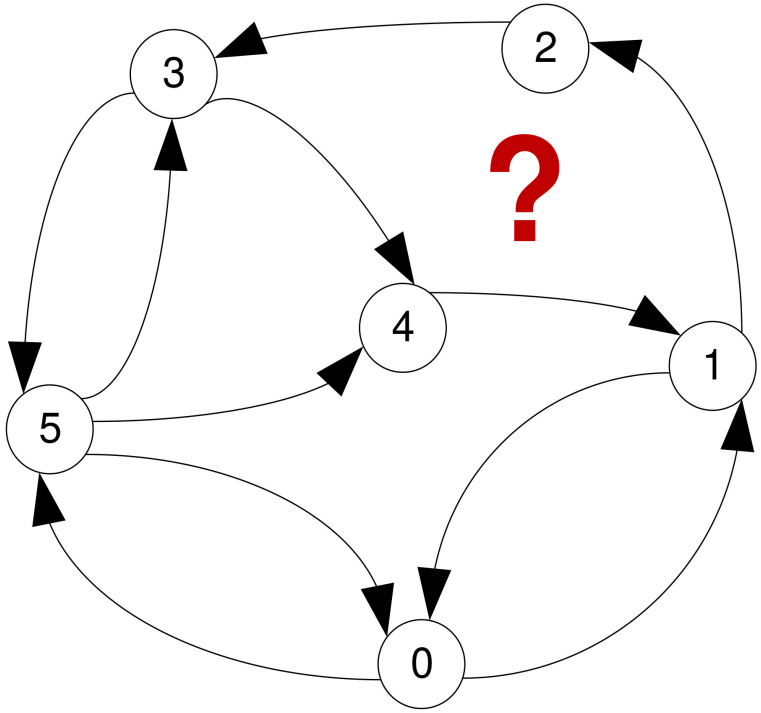
PATH FINDING AND GRAPH TRAVERSAL

- Today we will discuss some heuristics for finding circuits (tours) on graphs.
- We are particularly interested in finding circuits that also have minimum cost.
- Remember that circuits can include the same vertex twice, but arcs only once.
- You may want to perform some pre-processing on the graph to handle such restrictions.

TERMINOLOGY: HAMILTONIAN CIRCUIT

- A *Hamiltonian* circuit is a path that visits every vertex **exactly once** and starts at ends at the same vertex
- NB: Not all graphs may have a Hamiltonian circuit





MINIMUM COST HAMILTONIAN CIRCUIT

- In a weighted graph, the minimum cost Hamiltonian circuit is that where the sum of the arc weights is the smallest.
- Finding the minimum cost Hamiltonian circuit on your bin service graphs is one *option* for route planning.
- Finding a Hamiltonian circuit can be difficult. This is a known *NP-complete* problem. Simply put, the problem may not be solvable in polynomial time and the complexity increases with the number of vertices.

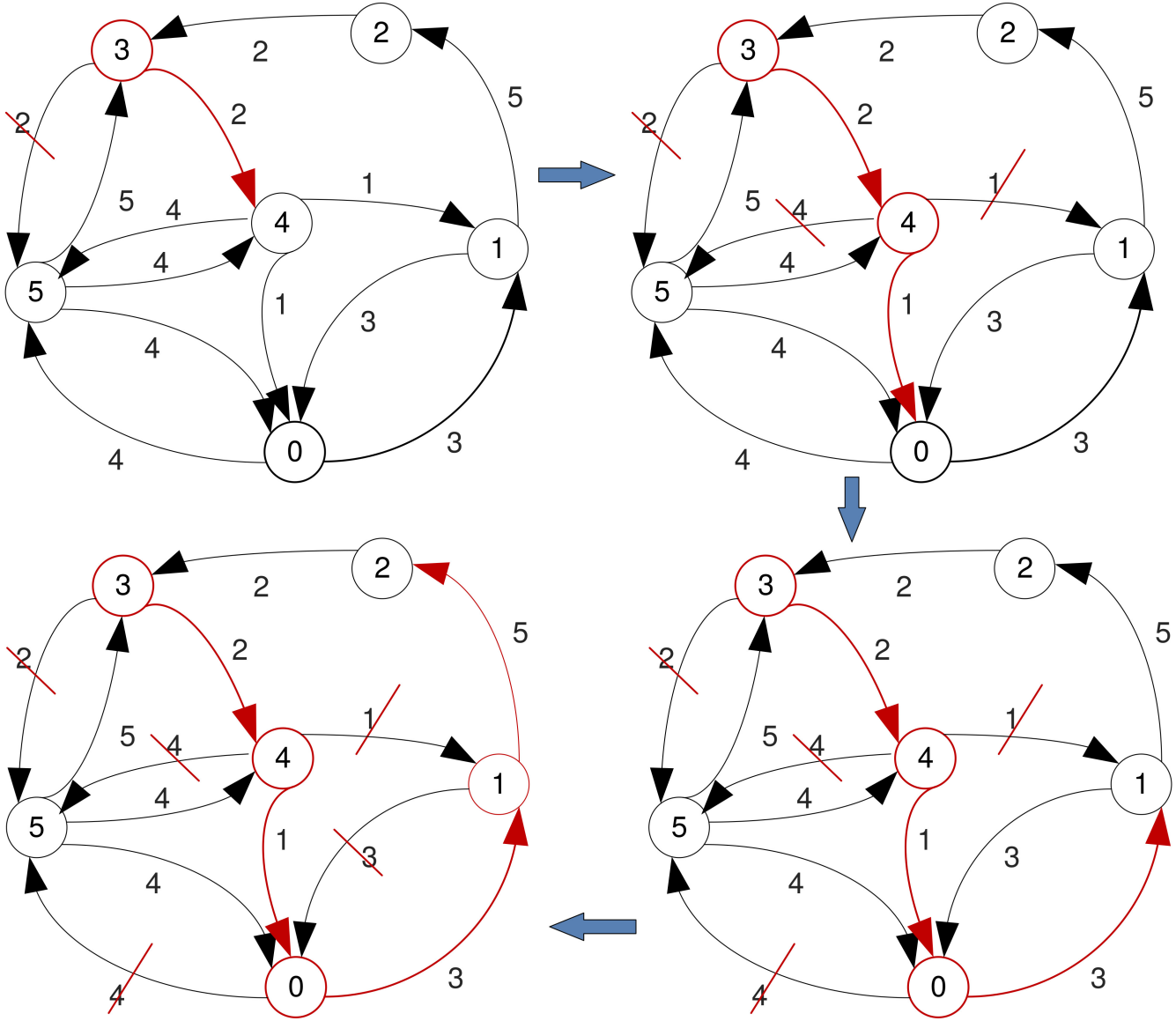
HEURISTIC ALGORITHMS

- Heuristics work quite well for finding (no always optimal, but good) solutions, most of the time.
- Work relatively fast.
- Popular heuristics for finding minimum cost Hamiltonian circuits:
  - Nearest Neighbour Algorithm
  - Sorted Edges Algorithm

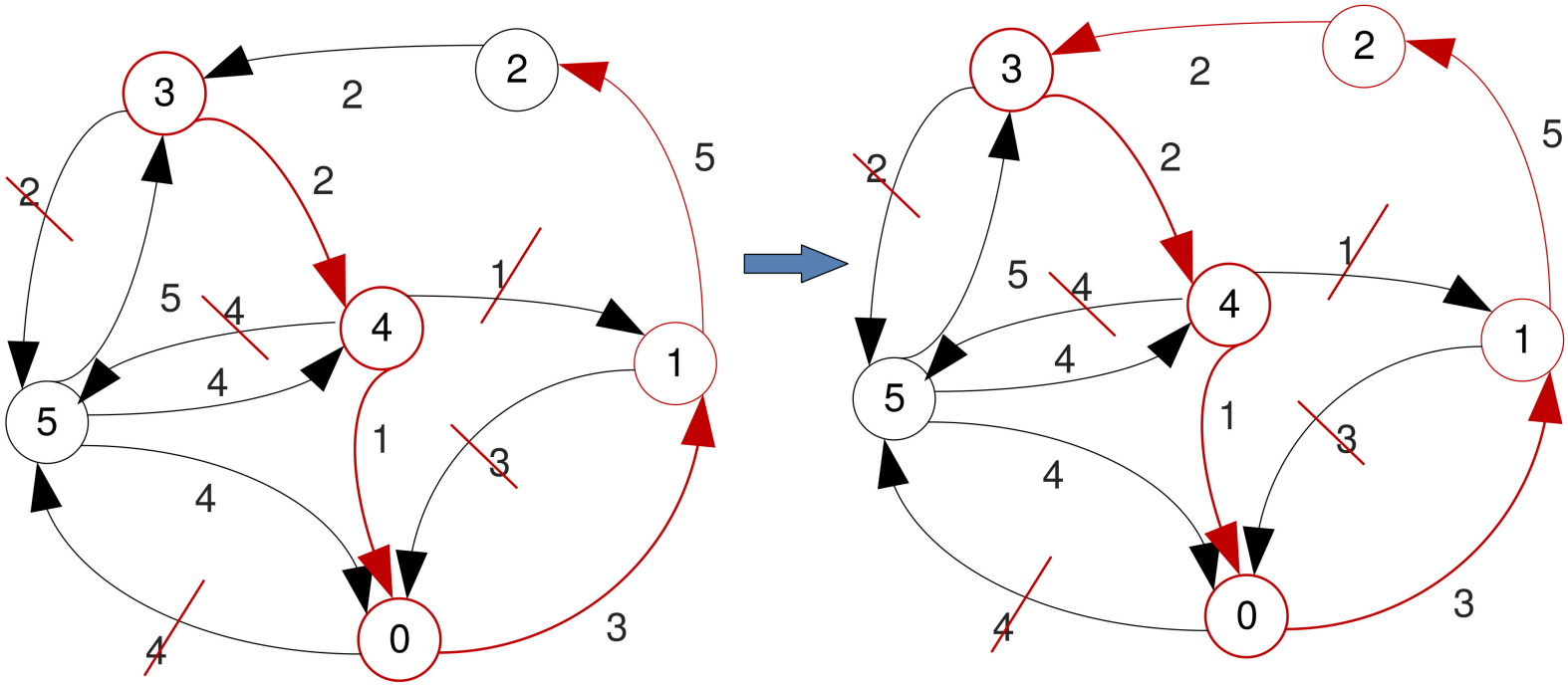
NEAREST NEIGHBOUR ALGORITHM

- Nearest Neighbour is a greedy algorithm – at every step it chooses as the next vertex the one connected to the current through the arc with the smallest weight.
- Only searches locally.
- Nodes already visited are ignored.
- Due to its greedy nature it may not find a solution.
- Finding a solution and its total cost depends on the start vertex chosen.

EXAMPLE, STARTING AT '3'

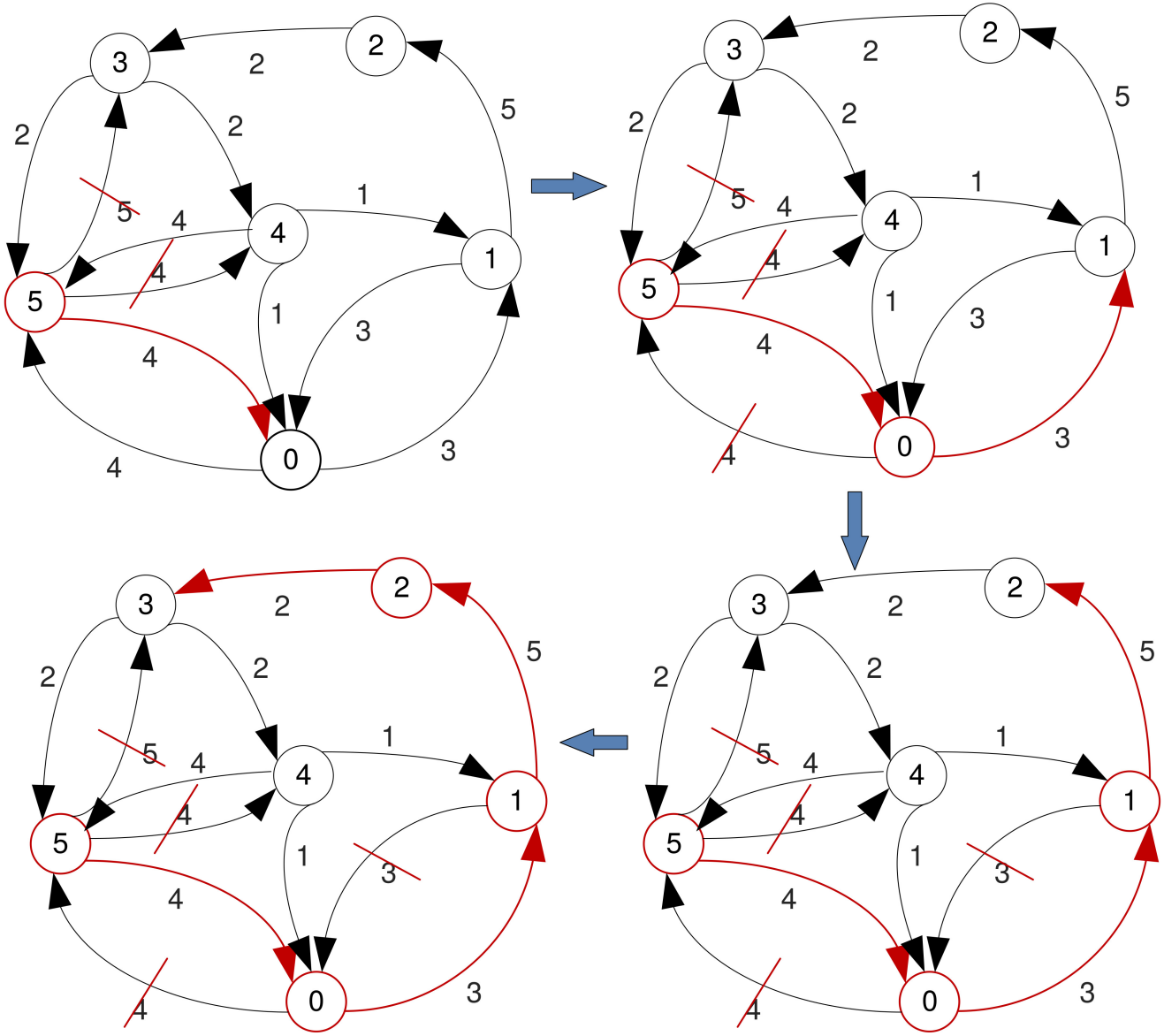


EXAMPLE, STARTING AT '3' (NO SOLUTION)

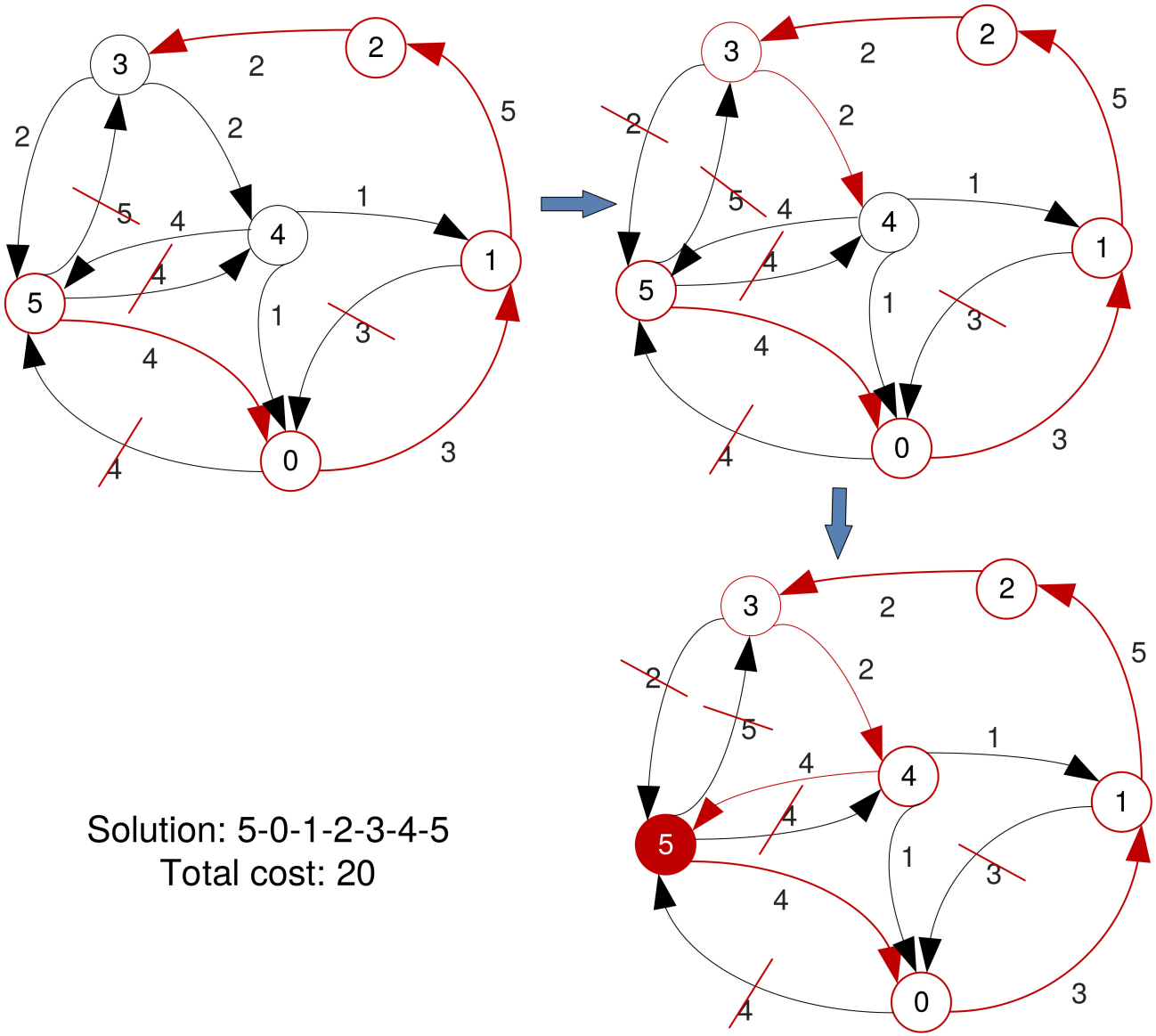


Arrived back at start point,
but vertex 5 not included

EXAMPLE, STARTING AT '5'



EXAMPLE, STARTING AT '5' (CONT'D)

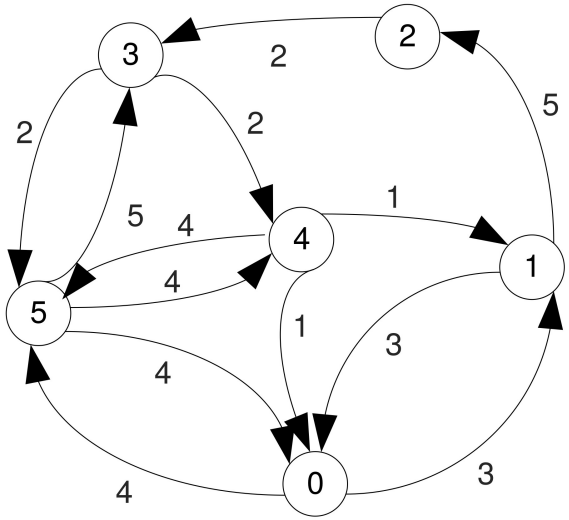


Solution: 5-0-1-2-3-4-5
Total cost: 20

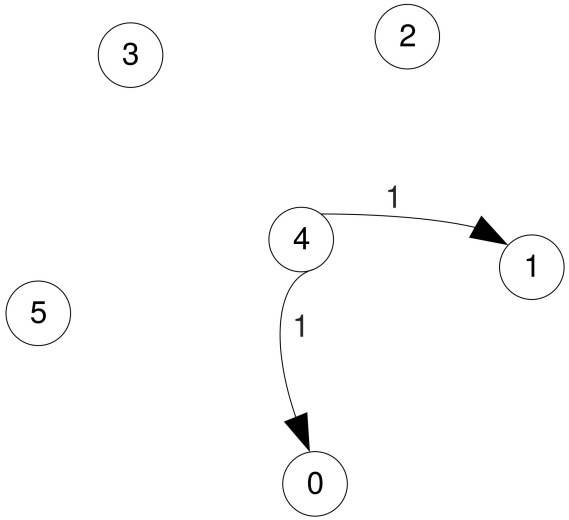
SORTED EDGES ALGORITHM

- Also greedy, but has a more global view → takes slightly more time to find a solution.
- First sorts all the (directed) edges in ascending order of their weights.
- Adds sorted edges one at the time, unless adding a new one leads to three arcs entering/leaving a node, or creates a circuit that does not include all vertices.
- Skips the arcs that violate these rules.
- Keeps adding arcs until finding a Hamiltonian circuit (or no more arcs left).
- Stops when a solution is found, even if there are arcs left.

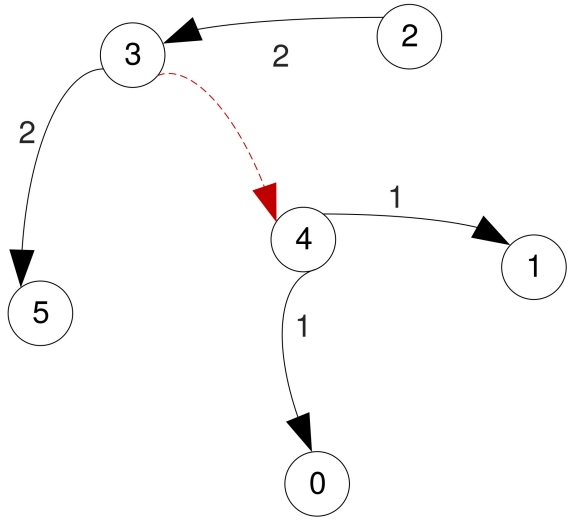
EXAMPLE



4-0	1	0-5	4
4-1	1	4-5	4
2-3	2	5-0	4
3-4	2	5-4	4
3-5	2	1-2	5
0-1	3	5-3	5
1-0	3		

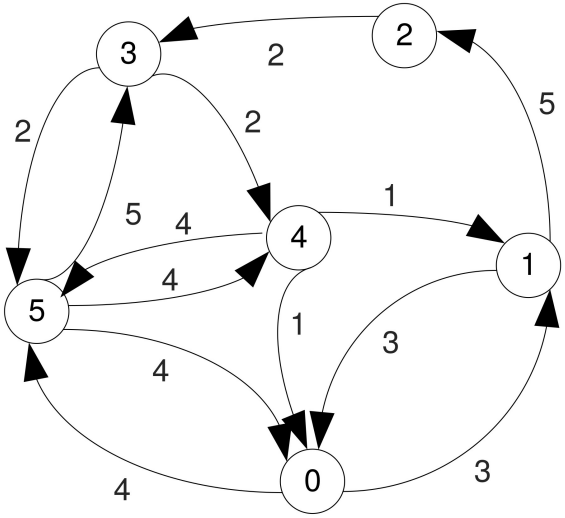


4-0	1
4-1	1

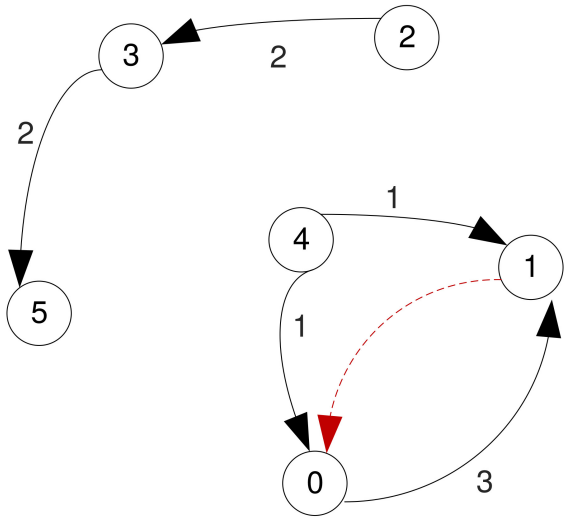


4-0	1
4-1	1
2-3	2
~~3-4~~	~~2~~ (“T” intersection)
3-5	2

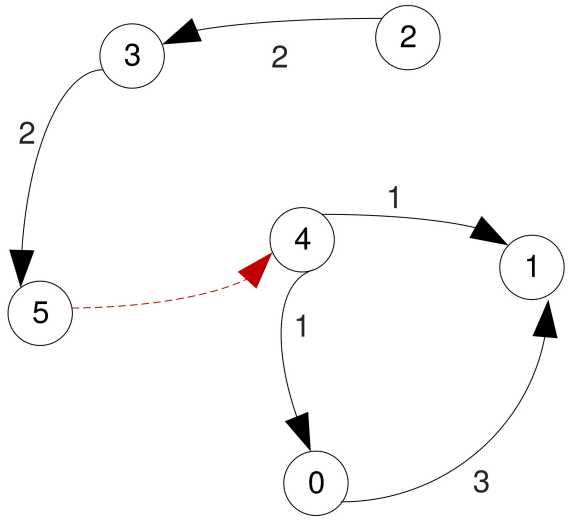
EXAMPLE (CONT'D)



4-0	1	0-5	4
4-1	1	4-5	4
2-3	2	5-0	4
3-4	2	5-4	4
3-5	2	1-2	5
0-1	3	5-3	5
1-0	3		



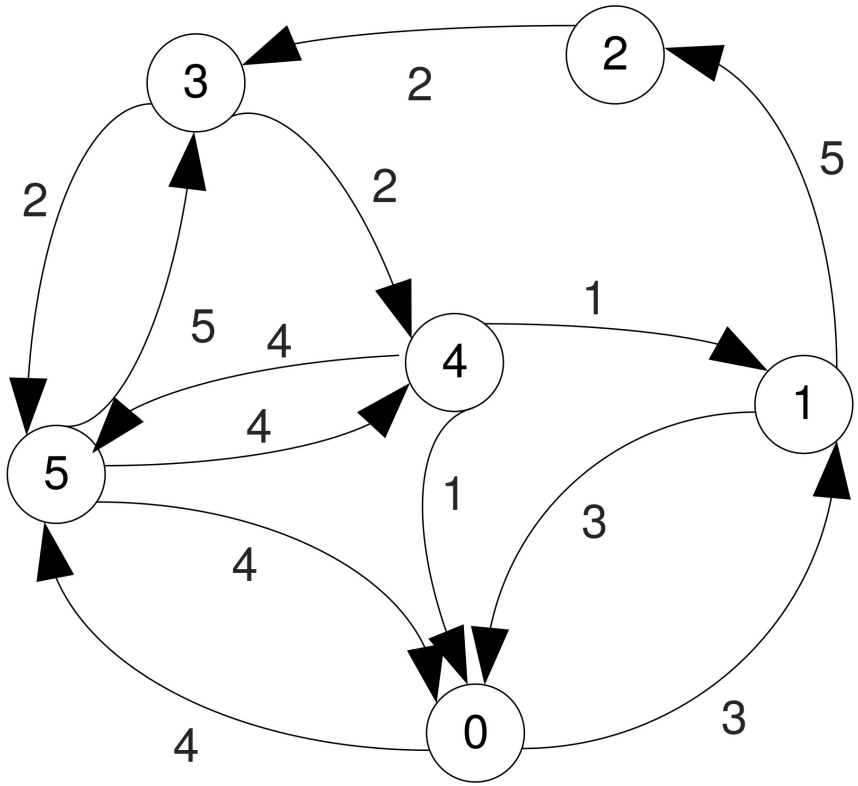
4-0	1
4-1	1
2-3	2
3-5	2
0-1	3
~~1-0~~	~~3~~ (0-1-0 cycle)



4-0	1
4-1	1
2-3	2
3-5	2
0-1	3

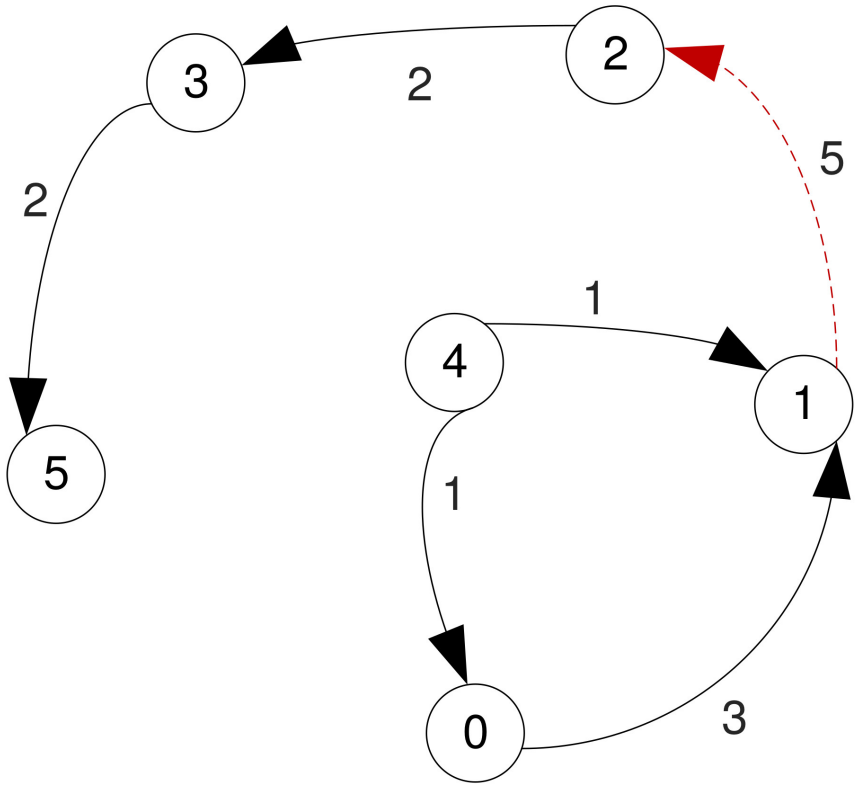
~~0-5~~	4 (3 arcs at ‘0’)
~~4-5~~	4 (3 arcs at ‘4’)
~~5-0~~	4 (3 arcs at ‘0’)
~~5-4~~	4 (3 arcs at ‘4’)

EXAMPLE (CONT'D)



4-0	1
4-1	1
2-3	2
3-4	2
3-5	2
0-1	3
1-0	3

0-5	4
4-5	4
5-0	4
5-4	4
1-2	5
5-3	5



4-0	1
4-1	1
2-3	2
3-5	2
0-1	3
~~1-2~~	~~5~~ (3 arcs at '1')
~~5-3~~	~~5~~ (5-3-3 cycle)

No more arcs left

No solution found.

BRUTE FORCE ALGORITHM

- When the number of vertices is small, a 'brute force' approach could be feasible.
- Find all paths that visit all vertices once and pick the one with the lowest cost.
- Guaranteed to find a solution (if there exists one), and this will be optimal.

OTHER APPROACHES

- A Hamiltonian circuit may **not always** be the path that visits all the vertices **and has the lowest cost**.
- Sometimes visiting a node more than once can give circuits that are more cost effective.
- The problem could also be seen as an instance of finding an Eulerian circuit (cycle).

EULERIAN CIRCUITS

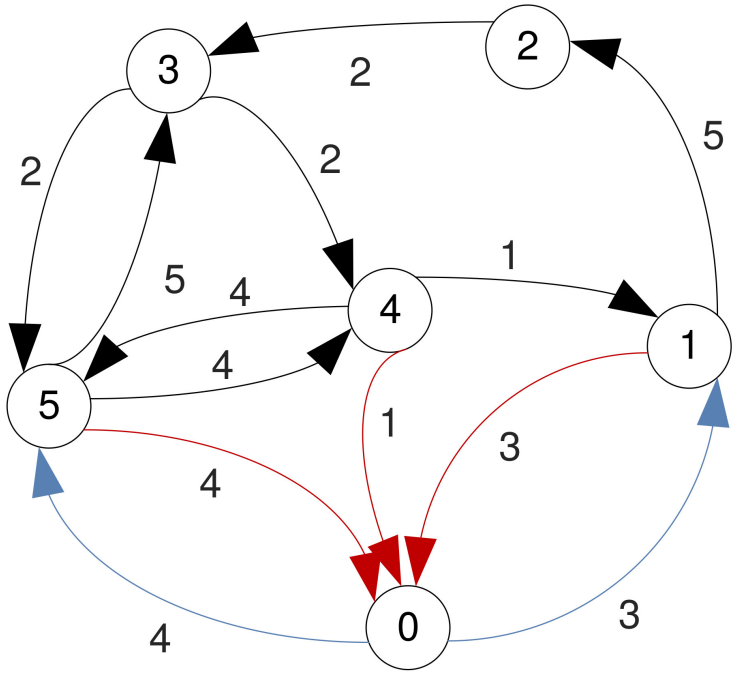
- An Eulerian path visits every edge (arc) exactly once.
- The idea dates back to birth of Graph Theory, when Leonhard Euler was asked to determine a path that traverses all of the 7 bridges in Koenigsberg exactly once.

Source: Nature Biotechnology

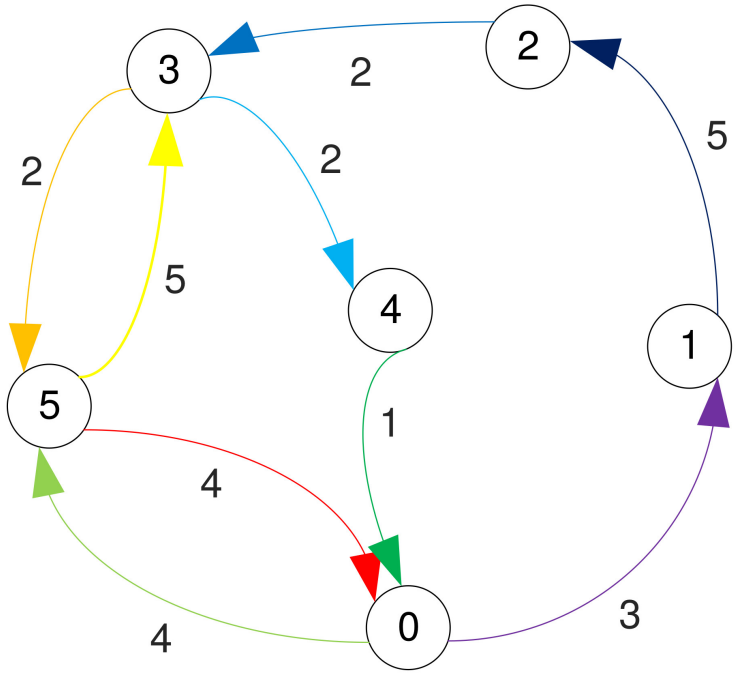
EULERIAN CIRCUITS

- An Eulerian circuit begins and ends at the same vertex.
- Euler realised a solution to the original problem did not exist, but found conditions that a graph must fulfil to have Eulerian paths.
- For your problem (directed graphs), it is important to know Eulerian circuits exist if and only if every vertex has equal in degree and out degree.

EXAMPLES



No solution



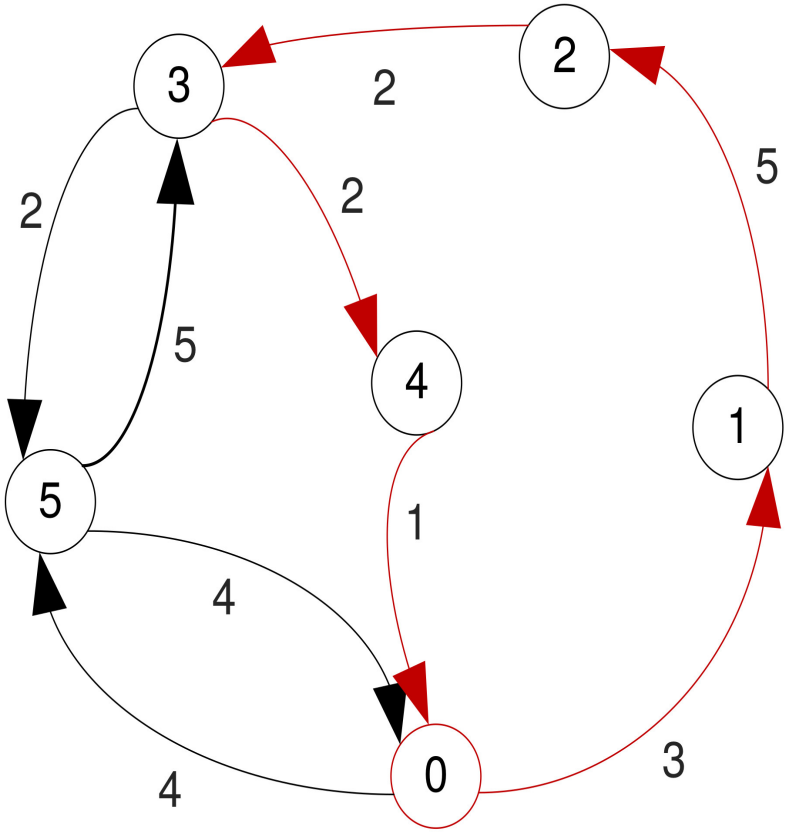
0-1-2-3-4-0-5-3-5-0

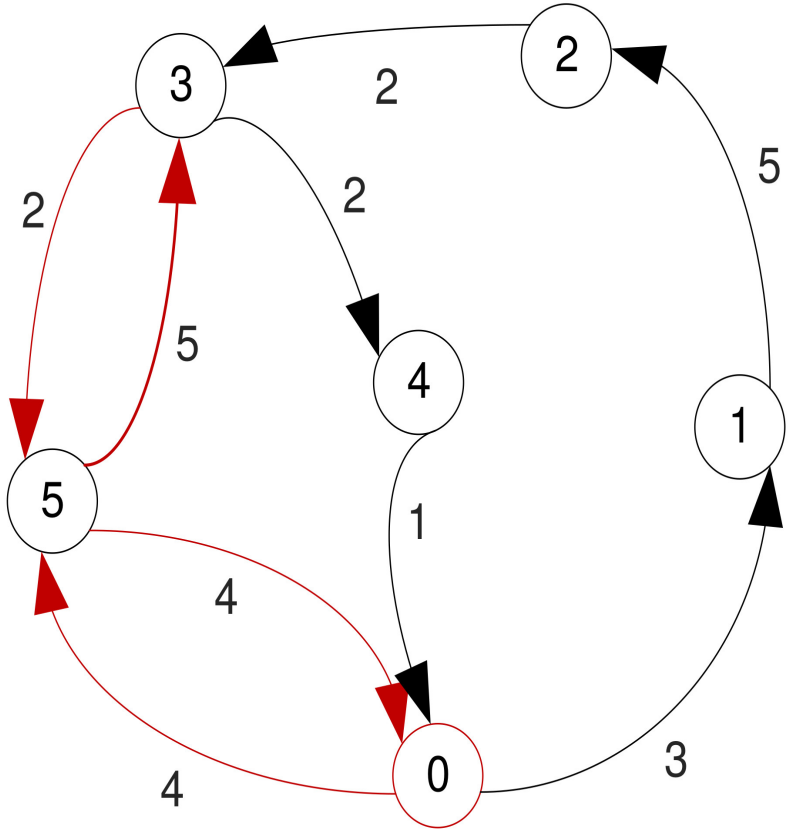
Total cost: 28

HIERHOLZER'S ALGORITHM

- Assuming you have verified the necessary condition the graph must meet to have an Eulerian circuit.
- Pick any vertex as a starting point.
- Add arcs to build trail, until returning to the start. You cannot get stuck at other vertices.
- If there are vertices on this tour but have adjacent arcs not included, start new trail from these.
- Join the tours formed this way.

EXAMPLE





THE PROBLEM YOU ARE TRYING TO SOLVE

- The graphs you will be given may not have Hamiltonian or Eulerian circuits.
- Even if they do, at start of service in an area you may not need to visit ALL the bins (vertices).
- It is likely you will need to do some pre-processing, e.g.
  - Build complete graphs starting from the original ones.
  - Remove some arcs.
- And work only with a subset of vertices (the bins that require service).

THE PROBLEM YOU ARE TRYING TO SOLVE

- There is no BEST solution.
- Depending on the graph structure/size, some may return more cost effective circuits than others.
- But may also take more time.
- Try to experiment with several, but aim to have at least ONE decent implementation.