Digital signatures

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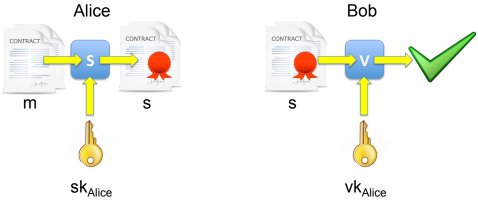
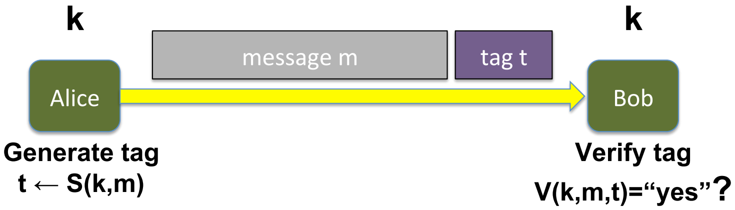
Goal

Data integrity and origin authenticity in the public-key setting



- key generation algorithm: $G : \to \mathcal{K} \times \mathcal{K}$
- signing algorithm $S : \mathcal{K} \times \mathcal{M} \to \mathcal{S}$
- verification algorithm $V : \mathcal{K} \times \mathcal{M} \times \mathcal{S} \to \{\top, \bot\}$
- s.t. $\forall (sk, vk) \in G$, and $\forall m \in \mathcal{M}$, $V(vk, m, S(sk, m)) = \top$

Advantages of digital signatures over MACs



k

k

message m

tag t

Alice

Bob

Generate tag
t ← S(k,m)

Verify tag
V(k,m,t)="yes"?

MACs

- are not publicly verifiable (and so not transferable)
  No one else, except Bob, can verify t.
- do not provide non-repudiation
  t is not bound to Alice's identity only. Alice could later claim she didn't compute t herself. It could very well have been Bob since he also knows the key k.

Advantages of digital signatures over MACs

Digital signatures

- are publicly verifiable - anyone can verify a signature
- are tansferable - due to public verifiability
- provide non-repudiation - if Alice signs a document with her secret key, she cannot deny it later

Security

A good digital signature schemes should satisfy existential unforgeabitliy.

Existential unforgeability

- Given $(m_1, S(sk, m_1))$, …, $(m_n, S(sk, m_n))$ (where m_1, …, m_n chosen by the adversary)
- It should be hard to computer a valid pair $(m, S(sk, m))$ without knowing sk for any $m \notin \{m_1, \ldots, m_n\}$

Textbook RSA signatues

- $G_{RSA}() = (pk, sk)$ where $pk = (N, e)$ and $sk = (N, d)$ and $N = p \cdot q$ with p, q random primes and $e, d \in \mathbb{Z}$ st. $e \cdot d \equiv 1 \pmod{\phi(N)}$

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  Proof: exactly as proof of consistency of RSA encryption/decryption

Problems with “textbook RSA sinatures”

Textbook RSA sinatures are not secure

The “textbook RSA sinature” scheme does not provide existential unforgeabitlity

- Suppose Eve has two valid signatures $\sigma_1 = M_1^d \mod n$ and $\sigma_2 = M_2^d \mod n$ from Bob, on messages M_1 and M_2.
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$$\sigma = \sigma_1 \cdot \sigma_2 \mod n = M_1^d \cdot M_2^d \mod n = (M_1 \cdot M_2)^d \mod n$$

which is a valid signature from Bob on message $M_1 \cdot M_2$.

How to use RSA for signatures

Solution

Before computing the RSA function, apply a hash function H.

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