Cryptographic hash functions

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Introduction

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What about authenticity and integrity against an active attacker?
$\longrightarrow$ cryptographic hash functions and Message authentication codes
$\longrightarrow$ this lecture

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Multiplication of large primes IS a OWF:
integer factorization is a hard problem - given $p \times q$ (where p and q are primes) it is hard to retrieve p and q

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Multiplication of large primes IS a CRF:
  every positive integer has a unique prime factorization

Cryptographic hash functions

A cryptographic hash function takes messages of arbitrary length end returns a fixed-size bit string such that any change to the data will (with very high probability) change the corresponding hash value.

Definition (Cryptographic hash function)

A cryptographic hash function $H : \mathcal{M} \to \mathcal{T}$ is a function that satisfies the following 4 properties:

- $|\mathcal{M}| >> |\mathcal{T}|$
- it is easy to compute the hash value for any given message
- it is hard to retrieve a message from it hashed value (OWF)
- it is hard to find two different messages with the same hash value (CRF)

Examples: MD4, MD5, SHA-1, SHA-256, Whirlpool, . . .

Cryptographic hash functions: applications

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- **Building block of other crypto primitives** - Used to build MACs, block ciphers, PRG, ...

Collision resistance and the birthday attack

Theorem

Let $H: \mathcal{M} \rightarrow \{0,1\}^n$ be a cryptographic hash function ($|\mathcal{M}| >> 2^n$)
Generic algorithm to find a collision in time $O(2^{n/2})$ hashes:

1. *Choose $2^{n/2}$ random messages in $\mathcal{M}$: $m_1, \ldots, m_{2^{n/2}}$*
2. *For $i = 1, \ldots, 2^{n/2}$ compute $t_i = H(m_i)$*
3. *If there exists a collision ($\exists i, j.\ t_i \neq t_j$)*
   then return (t_i, t_j)
   else go back to 1

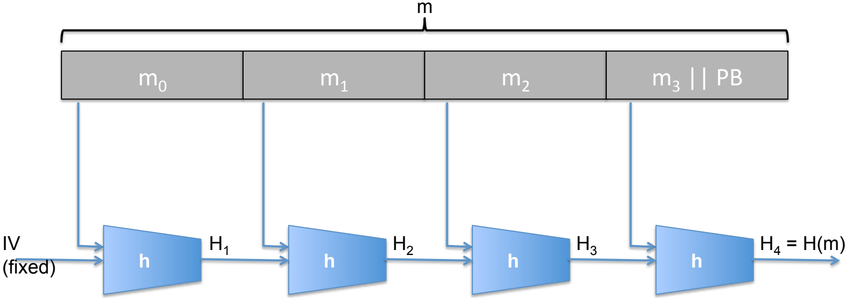
Birthday paradox Let $r_1, \ldots, r_n \in \{1, \ldots, N\}$ be independent variables. For $n = 1.2 \times \sqrt{N}$, $Pr(\exists i \neq j.\ r_i = r_j) \geq \frac{1}{2}$

$\Rightarrow$ the expected number of iteration is 2

$\Rightarrow$ running time $O(2^{n/2})$

$\Rightarrow$ Cryptographic function used in new projects should have an output size $n \geq 256$!

The Merkle-Damgard construction



- Compression function: $h: \mathcal{T} \times \mathcal{X} \rightarrow \mathcal{T}$
- PB: $1000 \ldots 0 \| \text{mes-len}$ (add extra block if needed)

Theorem

Let H be built using the MD construction to the compression function h. If H admits a collision, so does h.

Example of MD constructions: MD5, SHA-1, SHA-2, ...

Compression functions from block ciphers

Let $E: \mathcal{K} \times \{0,1\}^n \rightarrow \{0,1\}^n$ be a block cipher

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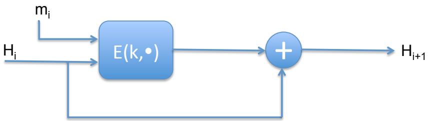

Davies-Meyer

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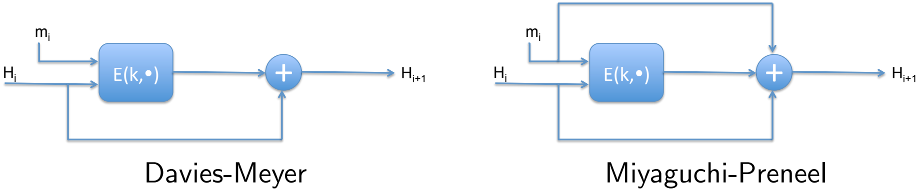
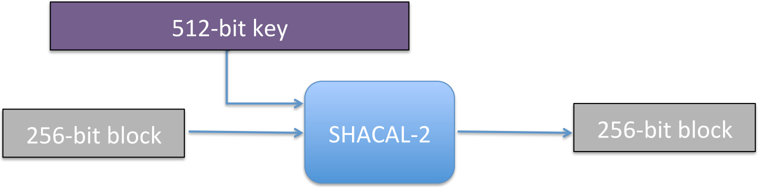


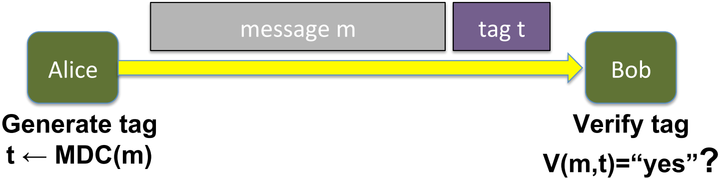
Example of cryptographic hash function: SHA-256

- Structure: Merkle-Damgard
- Compression function: Davies-Meyer
- Bloc cipher: SHACAL-2



Message Authentication Codes (MACs)

Goal: message integrity



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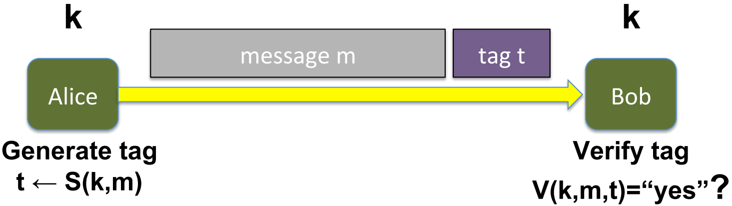


A MAC is a pair of algorithms (S, V) defined over $(\mathcal{K}, \mathcal{M}, \mathcal{T})$:

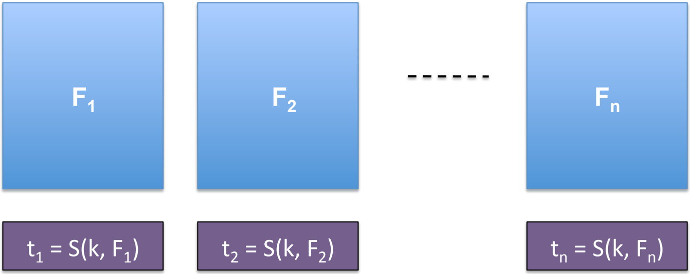
- $S : \mathcal{K} \times \mathcal{M} \to \mathcal{T}$
- $V : \mathcal{K} \times \mathcal{M} \times \mathcal{T} \to \{\top, \bot\}$
- Consistency: $V(k, m, S(k, m)) = T$

and such that

- It is hard to computer a valid pair $(m, S(k, m))$ without knowing k

File system protection

- At installation time



k derived from user password

- To check for virus file tampering/alteration:
  - reboot to clean OS
  - supply password
  - any file modification will be detected

Block ciphers and message integrity

Block ciphers and message integrity

Let (E, D) be a block cipher. We build a MAC (S, V) using (E, D) as follows:

- $S(k, m) = E(k, m)$
- $V(k, m, t) =$ if $m = D(k, t)$
  then return $\top$
  else return $\perp$

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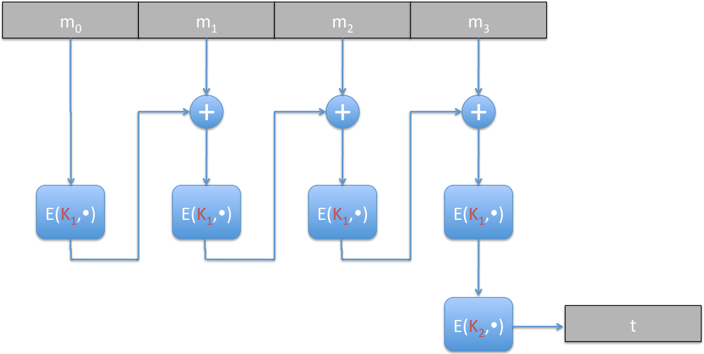
Our goal now: construct MACs for long messages

ECBC-MAC



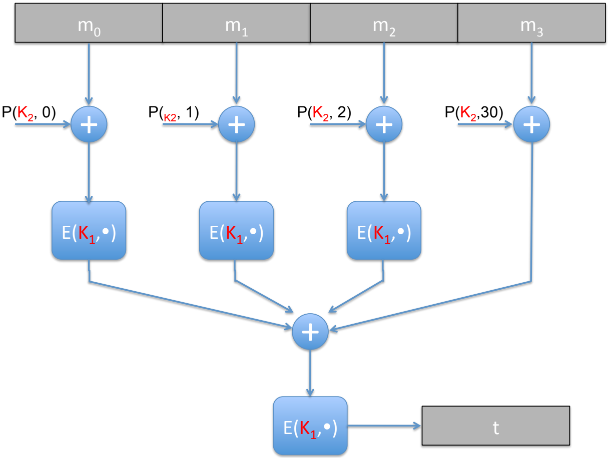
- $E : \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ a block cipher
- $ECBC\text{-}MAC : \mathcal{K}^2 \times \{0,1\}^* \to \{0,1\}^n$

$\to$ the last encryption is crucial to avoid forgeries!!

(details on the board)

Ex: 802.11i uses AES based ECBC-MAC

PMAC



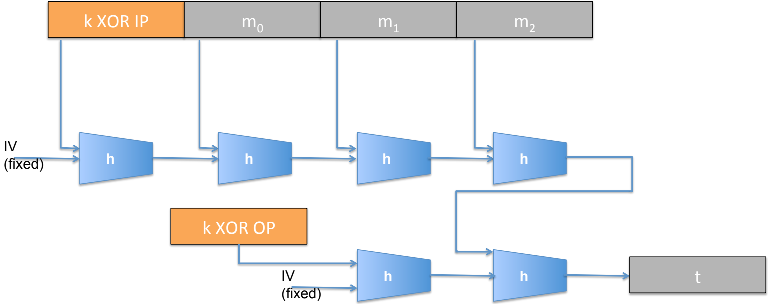
- $E: \mathcal{K} \times \{0,1\}^n \to \{0,1\}^n$ a block cipher
- $P: \mathcal{K} \times \mathbb{N} \to \{0,1\}^n$ any easy to compute function
- $PMAC : \mathcal{K}^2 \times \{0,1\}^* \to \{0,1\}^n$

HMAC

MAC built from cryptographic hash functions

$$HMAC(k, m) = H(k \oplus OP || H(k \oplus IP || m))$$

IP, OP: publicly known padding constants



Ex: SSL, IPsec, SSH, . . .