Stream ciphers

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February 29, 2015

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- Consistency: $D(k, E(k,m)) = k \oplus (k \oplus m) = m$

Perfect secrecy

Definition

A cipher (E, D) over $(\mathcal{M}, \mathcal{C}, \mathcal{K})$ satisfies perfect secrecy if for all messages $m_1, m_2 \in \mathcal{M}$ of same length ($|m_1| = |m_2|$), and for all ciphertexts $c \in \mathcal{C}$

$$|Pr(E(k, m_1) = c) - Pr(E(k, m_2) = c)| \leq \epsilon$$

where $k \xleftarrow{r} \mathcal{K}$ and ϵ is some "negligible quantity".

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Thus, for all messages $m_1, m_2 \in \mathcal{M}$, and for all ciphertexts $c \in \mathcal{C}$

$$|Pr(E(k,m_1) = c) - Pr(E(k,m_2) = c)| \leq \left| \frac{1}{\#\mathcal{K}} - \frac{1}{\#\mathcal{K}} \right| = 0$$

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    given $m_1 \oplus k$ and $m_2 \oplus k$, we can compute
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  - OTP is malleable
    given the ciphertext $c = E(k, m)$ with $m = to\ bob : m_0$, it is possible to compute the ciphertext $c' = E(k, m')$ with
    $m' = to\ eve : m_0$
    $c' := c \oplus \text{"}to\ bob : 00 \ldots 00\text{"} \oplus \text{"}to\ eve : 00 \ldots 00\text{"}$

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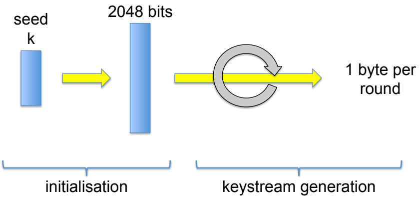

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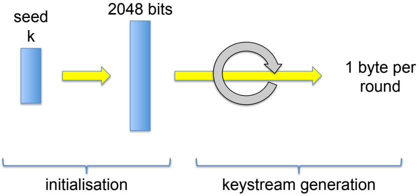

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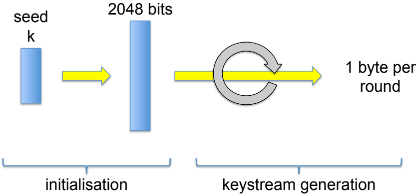

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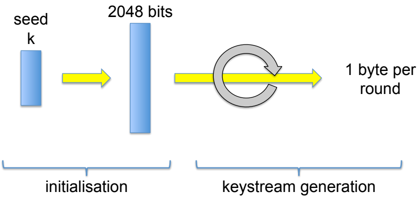

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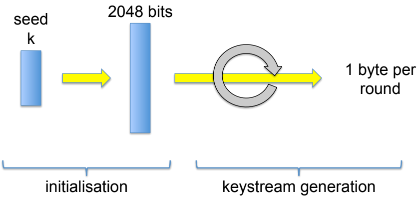

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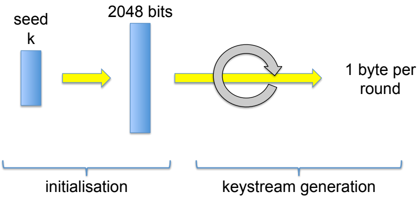

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  - subject to related keys attacks
    $\longrightarrow$ choose randomly generated keys as seeds

RC4: initialisation

```
for i := 0 to 255 do
    S[i] := i
end
j := 0
for i := 0 to 255 do
    j := (j + S[i] + K[i(mod |K|)])(mod 256)
    swap(S[i], S[j])
end
i := 0
j := 0
```

RC4: key stream generation

`while` generatingOutput

$i := i + 1 \pmod{256}$

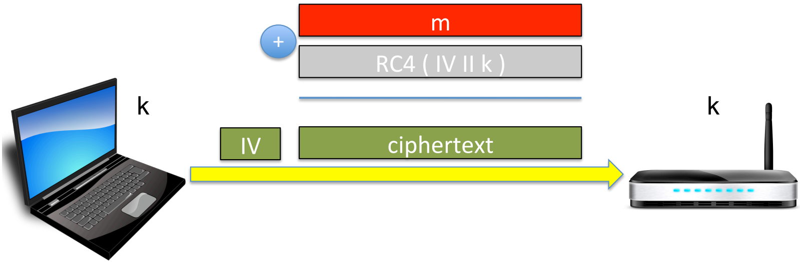
$j := j + S[i] \pmod{256}$

$swap(S[i], S[j])$

$output(S[S[i] + S[j] \pmod{256}])$

`end`

WEP uses RC4



Initialisation Vector (IV): 24-bits long string

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Remark

The FMS attack does not apply to RC4-based SSL (TLS), since SSL generates the encryption keys it uses for RC4 by hashing, meaning that different SSL sessions have unrelated keys

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  taps: $i_1, i_2, \ldots i_\ell$

  feedback bit: $R[i_1] \oplus R[i_2] \oplus \cdots \oplus R[i_\ell]$

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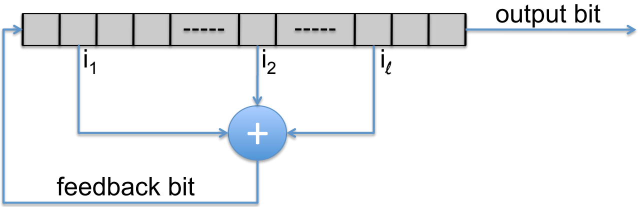

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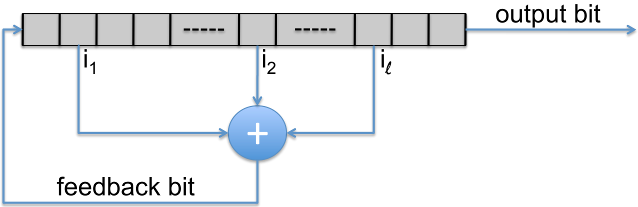

- Broken LFSR-based stream ciphers:
  - DVD encryption: CSS (2 LFSRs)
  - GSM encryption: A5 (3 LFSRs)
  - Bluetooth encryption: E0 (4 LFSRs)

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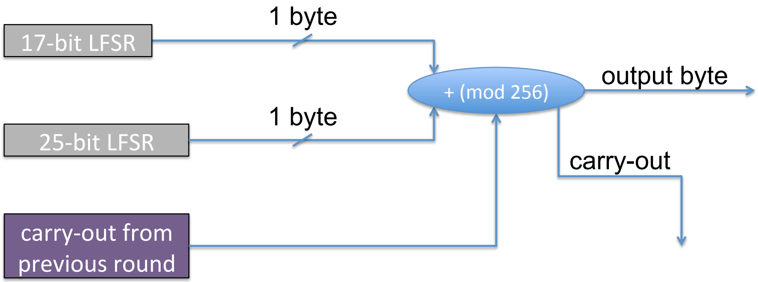

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- Given output of 17 bit LFSR, can deduce output of 25 bit LFSR by subtraction
- Hence try all 2^{17} possibilities for 17 bit LFSR and if generated 25 bit LFSR produces observed keystream, cipher is cracked

Modern stream ciphers

Project eStream: project to "identify new stream ciphers suitable for widespread adoption", organised by the EU ECRYPT network
$\longrightarrow$ HC-128, Rabbit, Salsa20/12, SOSEMANUK,
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Conjecture

These eStream stream ciphers are "secure"

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- Crypto primitives are secure under a precisely defined threat model.
  $\longrightarrow$ respect the security assumptions of the crypto primitives you use