# Block ciphers

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A block cipher with parameters $k$ and $\ell$ is a pair of deterministic algorithms $(E, D)$ such that

- Encryption $E : \{0,1\}^k \times \{0,1\}^\ell \rightarrow \{0,1\}^\ell$
- Decryption $D : \{0,1\}^k \times \{0,1\}^\ell \rightarrow \{0,1\}^\ell$

Examples:

$3DES: \ \ell = 64, \ k = 56$

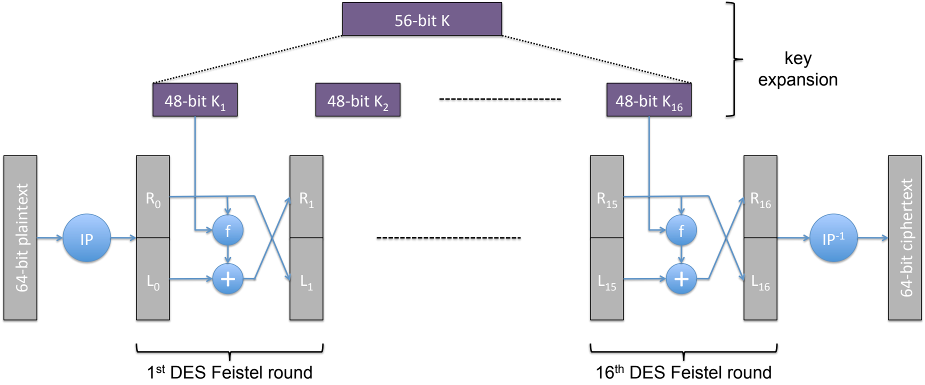
$AES: \ \ell = 64, \ k = 128, 192, 256$

# Data Encryption Standard (DES)

- Early 1970s: Horst Feistel designs Lucifer at IBM
  $k = 128$ bits, $\ell = 128$ bits
- 1973: NBS calls for block cipher proposals.
  $\longrightarrow$ IBM submits a variant of Lucifer.
- 1976: NBS adopts *DES* as a federal standard
  $k = 56$ bits, $\ell = 64$ bits
- 1997: *DES* broken by exhaustive search
- 2001: NIST adopts *AES* to replace *DES*
  $k = 128, 192, 256$ bits, $\ell = 128$ bits

Widely deployed in banking (ATM machines) and commerce

# *DES*: encryption circuit



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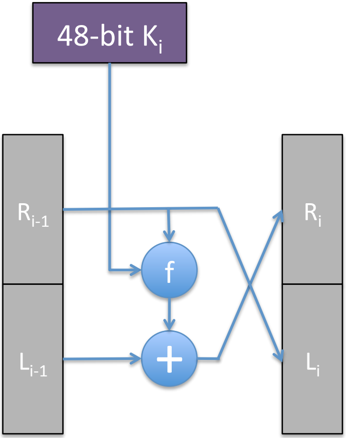

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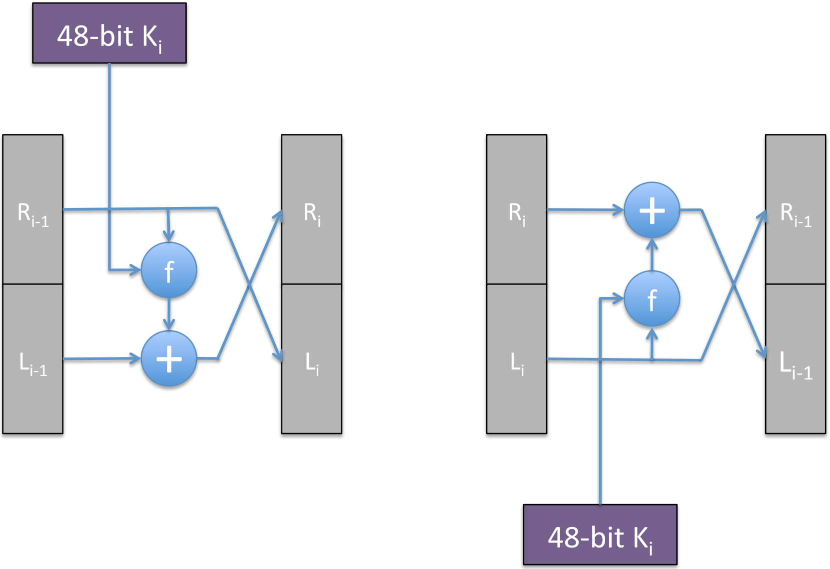
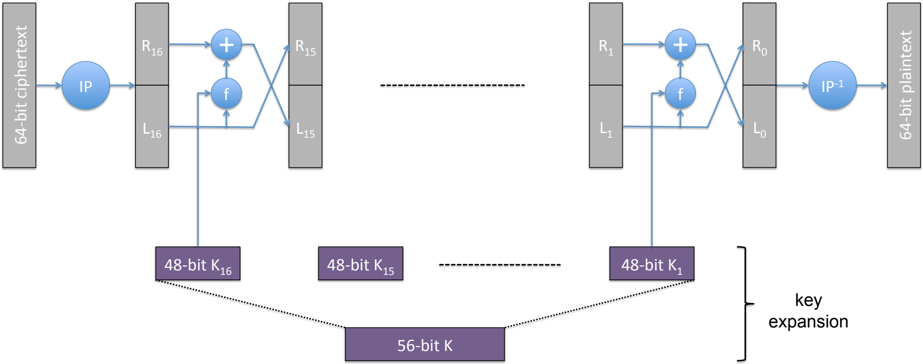
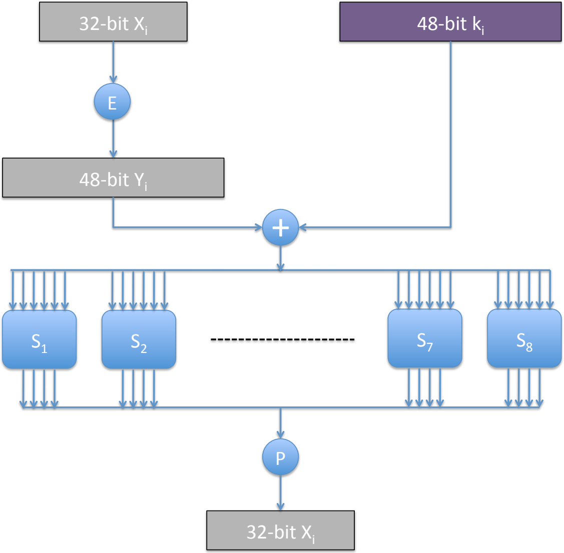


# *DES*: decryption circuit



# *DES*: the function *f*



# *DES*: $S_5$-box

| $S_5$ | | Middle 4 bits of input | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **0000** | **0001** | **0010** | **0011** | **0100** | **0101** | **0110** | **0111** | **1000** | **1001** | **1010** | **1011** | **1100** | **1101** | **1110** | **1111** |
| **Outer bits** | **00** | 0010 | 1100 | 0100 | 0001 | 0111 | 1010 | 1011 | 0110 | 1000 | 0101 | 0011 | 1111 | 1101 | 0000 | 1110 | 1001 |
| | **01** | 1110 | 1011 | 0010 | 1100 | 0100 | 0111 | 1101 | 0001 | 0101 | 0000 | 1111 | 1010 | 0011 | 1001 | 1000 | 0110 |
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$\longrightarrow$ Note that $S_5$ is not reversible as it maps 6 bits to 4 bits.

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$\Rightarrow$ *DES* is badly broken! Do not use it in new projects!!

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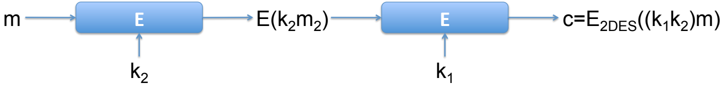

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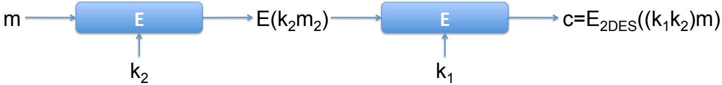


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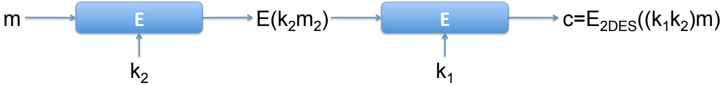


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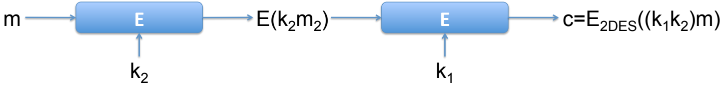


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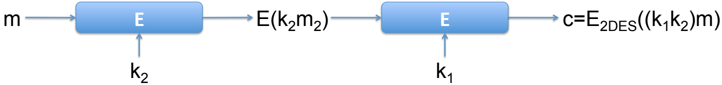


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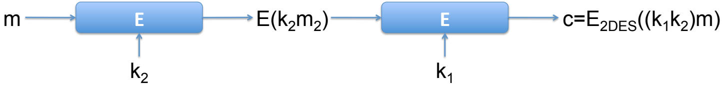


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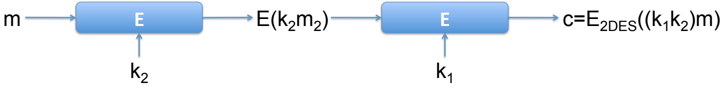


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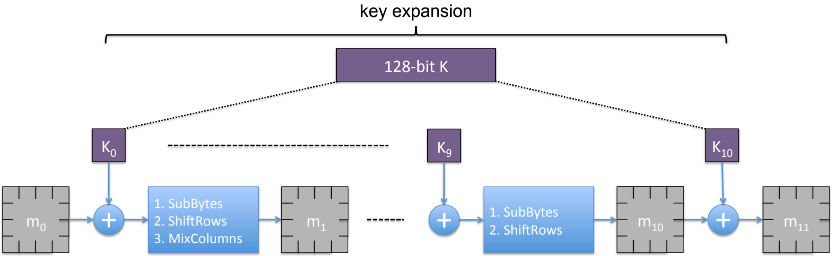
- Similar attack on $3DES$ in time $2^{118}$

# The Advenced Encryption Standard (AES)

- Goal: replace 3*DES* which is too slow (3*DES* is 3 times as slow as *DES*)
- 2001: NIST adopts Rijndael as AES
- Block size $\ell = 128$ bits, Key size $k = 128, 192, 256$ bits
- *AES* is Substitution-Permutation network (not a Feistel network)

# *AES*: encryption circuit




- $m_i$ : $4 \times 4$ byte matrix, $K_i$: 128-bit key
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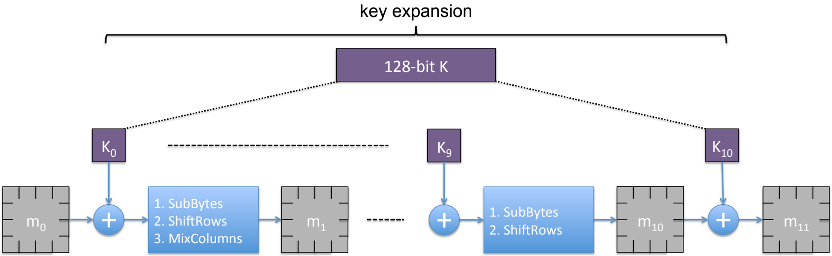


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$\longrightarrow$ As *AES* is not a Feistel network, each step needs to be reversible!

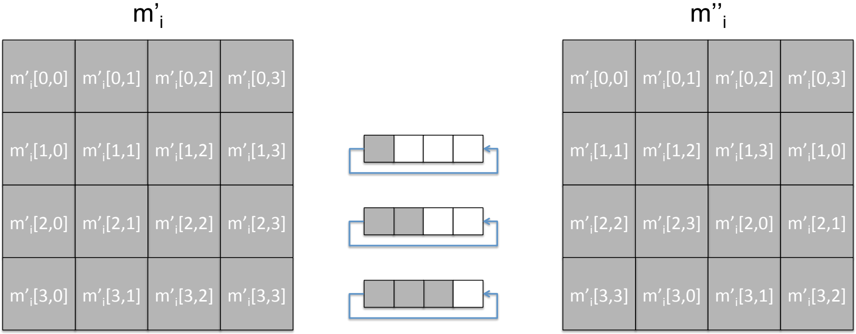
# *AES*: SubBytes

| | x0 | x1 | x2 | x3 | x4 | x5 | x6 | x7 | x8 | x9 | xa | xb | xc | xd | xe | xf |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0x | 63 | 7c | 77 | 7b | f2 | 6b | 6f | c5 | 30 | 01 | 67 | 2b | fe | d7 | ab | 76 |
| 1x | ca | 82 | c9 | 7d | fa | 59 | 47 | f0 | ad | d4 | a2 | af | 9c | a4 | 72 | c0 |
| 2x | b7 | fd | 93 | 26 | 36 | 3f | f7 | cc | 34 | a5 | e5 | f1 | 71 | d8 | 31 | 15 |
| 3x | 04 | c7 | 23 | c3 | 18 | 96 | 05 | 9a | 07 | 12 | 80 | e2 | eb | 27 | b2 | 75 |
| 4x | 09 | 83 | 2c | 1a | 1b | 6e | 5a | a0 | 52 | 3b | d6 | b3 | 29 | e3 | 2f | 84 |
| 5x | 53 | d1 | 00 | ed | 20 | fc | b1 | 5b | 6a | cb | be | 39 | 4a | 4c | 58 | cf |
| 6x | d0 | ef | aa | fb | 43 | 4d | 33 | 85 | 45 | f9 | 02 | 7f | 50 | 3c | 9f | a8 |
| 7x | 51 | a3 | 40 | 8f | 92 | 9d | 38 | f5 | bc | b6 | da | 21 | 10 | ff | f3 | d2 |
| 8x | cd | 0c | 13 | ec | 5f | 97 | 44 | 17 | c4 | a7 | 7e | 3d | 64 | 5d | 19 | 73 |
| 9x | 60 | 81 | 4f | dc | 22 | 2a | 90 | 88 | 46 | ee | b8 | 14 | de | 5e | 0b | db |
| ax | e0 | 32 | 3a | 0a | 49 | 06 | 24 | 5c | c2 | d3 | ac | 62 | 91 | 95 | e4 | 79 |
| bx | e7 | c8 | 37 | 6d | 8d | d5 | 4e | a9 | 6c | 56 | f4 | ea | 65 | 7a | ae | 08 |
| cx | ba | 78 | 25 | 2e | 1c | a6 | b4 | c6 | e8 | dd | 74 | 1f | 4b | bd | 8b | 8a |
| dx | 70 | 3e | b5 | 66 | 48 | 03 | f6 | 0e | 61 | 35 | 57 | b9 | 86 | c1 | 1d | 9e |
| ex | e1 | f8 | 98 | 11 | 69 | d9 | 8e | 94 | 9b | 1e | 87 | e9 | ce | 55 | 28 | df |
| fx | 8c | a1 | 89 | 0d | bf | e6 | 42 | 68 | 41 | 99 | 2d | 0f | b0 | 54 | bb | 16 |

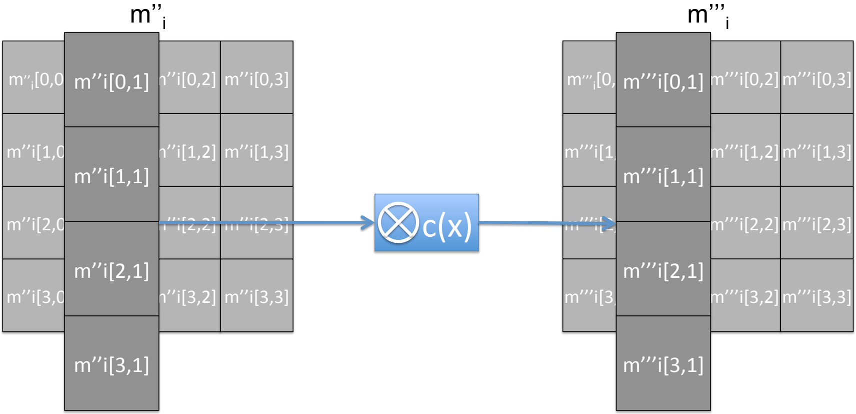
- $\forall j, k.\ m'_i[j,k] = S[m_i[j,k]]$
- rows: most significant 4 bits
- columns: least significant 4 bits

$\longrightarrow$ Note that SubBytes is reversible

# *AES*: ShiftRows



# *AES*: MixColumns



# Attacks on *AES*

- **Related-key attack** on the 192-bit and 256-bit versions of *AES*: exploits the *AES* key schedule [A. Biryukov, D. Khovratovich (2009)]
  $\longrightarrow$ key recovery in time $\sim 2^{99}$

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$\Rightarrow$ Existing attacks on $AES-128$ are still not practical, but use $AES-192$ and $AES-256$ in new projects!

# Using block ciphers

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University of Edinburgh

**Goal:** Encrypt $M$ using a block cipher operating on blocks of length $\ell$ when $|M| > \ell$

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- $M'$ is broken into $m$ blocks of length $\ell$
  $$\Rightarrow \; M' \; = M_1 \; || \; M_2 \; || \; \ldots \; || \; M_m$$

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To encrypt message $M$ under key $K$ using ECB mode:

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  $$\Rightarrow \ M' = M||P \text{ such that } |M'| = m \times \ell$$
- $M'$ is broken into $m$ blocks of length $\ell$
  $$\Rightarrow \ M' \ = M_1 \ || \ M_2 \ || \ \dots \ || \ M_m$$
- Each block $M_i$ is encrypted under the key $K$ using the block cipher
  $$\Rightarrow \ C_i = E(K, M_i) \text{ for all } i \in \{1, \dots, m\}$$

# Electronic Code Book (ECB) mode

$(E, D)$ a block cipher.

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- Each block $M_i$ is encrypted under the key $K$ using the block cipher
  $$\Rightarrow \; C_i = E(K, M_i) \text{ for all } i \in \{1, \ldots, m\}$$
- The ciphertext corresponding to $M$ is the concatenation of the $C_i$s
  $$\Rightarrow \; C \; = C_1 \; || \; C_2 \; || \; \ldots \; || \; C_m$$

# Weakness of ECB

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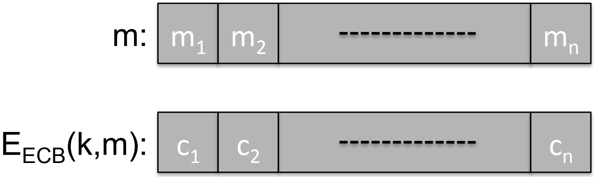

# Weakness of ECB


Problem: $\forall i, j.\ m_i = m_j \ \Rightarrow \ c_i = E(k, m_i) = E(k, m_j) = c_j$

$\Rightarrow$ Weak to frequency analysis!

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$\Rightarrow$ Weak to frequency analysis!

$\longrightarrow$ Playstation attack

# Weakness of ECB in pictures

Original image

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Original image

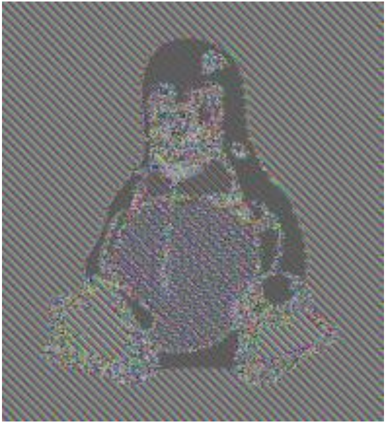
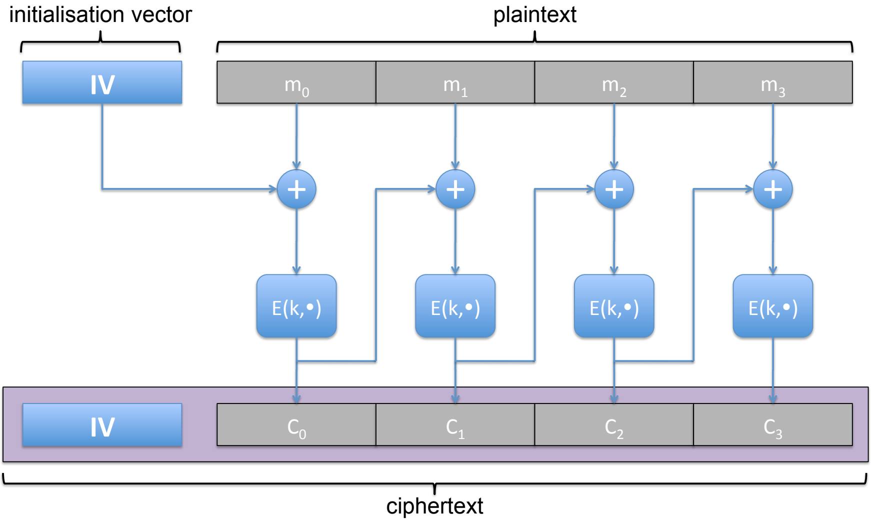


Image encrypted using ECB mode

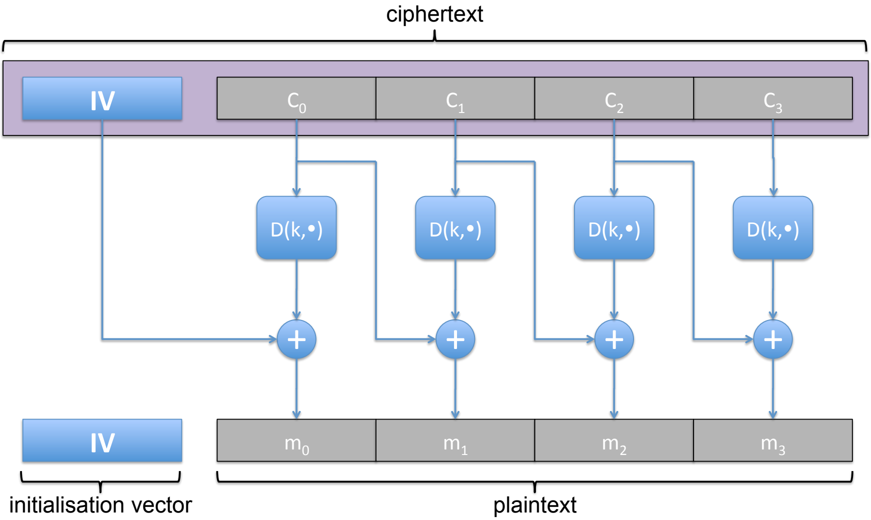
# Cipher-block chaining (CBC) mode: encryption

$(E, D)$ a block cipher that manipulates blocks of size $\ell$.



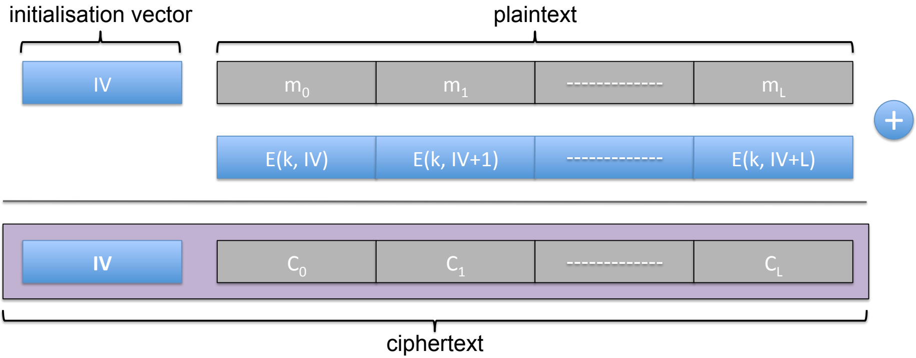

IV chosen at random in $\{0, 1\}^{\ell}$

# Cipher-block chaining (CBC) mode: decryption



# Counter (CTR) mode

$(E, D)$ a block cipher that manipulates blocks of size $\ell$.




IV chosen at random in $\{0, 1\}^\ell$